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MHD Stagnation Point Flow of A MICROPOLAR Fluid over a Stretching Surface with Heat Source/Sink, Chemical Reaction and Viscous Dissipation

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Abstract: *The steady laminar two dimensional stagnation point flow of an incompressible electrically conducting magneto-micropolar fluid over a permeable stretching surface with heat source/sink and viscous dissipation in the presence of mass transfer and chemical reaction has been studied. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, forth order Runge-Kutta method along with shooting technique has been used for solving it. Numerical results are obtained for the skin-friction coefficient, the local Nusselt number and Sherwood number as well as the velocity, temperature and concentration profiles for different values of the governing parameters, namely, velocity ratio parameter, boundary parameter, material parameter, magnetic parameter, Prandtl number, Eckert number, heat source/sink parameter, Schmidt number and chemical reaction parameter.*
Keywords: *micropolar fluid, MHD, Stretching surface, viscous dissipation, heat source or sink and chemical reaction.*

I. Introduction

Micropolar fluids are a subset of the micromorphic fluid theory introduced in a pioneering paper by Eringen[1]. Micropolar fluids are those consisting of randomly oriented particles suspended in a viscous medium, which can undergo a rotation that can affect the hydrodynamics of the flow, making it a distinctly non-Newtonian fluid. They constitute an important branch of non-Newtonian fluid dynamics where microrotation effects as well as microinertia are exhibited. Eringen's theory has provided a good model for studying a number of complicated fluids, such as colloidal fluids, polymeric fluids and blood.

The study of flow and heat transfer past stretching sheet problems has gained considerable interest because of its extensive engineering applications, such as in the extrusion of plastic sheets, paper production, crystal growing and glass blowing. Crane [2] presented an exact similarity solution in closed analytical form for the laminar boundary flow of an incompressible, steady viscous fluid over a stretching surface with a velocity varying linearly with the distance from a fixed point. Gupta and Gupta [3] extended the Crane's problem to include suction or blowing and studied its influence on the heat and mass transfer in the boundary layer over a stretching surface. Chakrabarti and Gupta [4] extended Pavlov's work to study the heat transfer when a uniform suction is applied at the stretching surface. Hady [5] presented analytical solutions for the problem of heat transfer to micropolar fluid from a non-isothermal permeable stretching sheet. Hassanien and Gorla [6] numerically studied the effects of suction and blowing on the flow and heat transfer of a micropolar fluid over a non-isothermal stretching surface. Hayat et al. [7] analytically studied the problem of a steady two-dimensional mixed convection flow of a micropolar fluid over a non-linear stretching surface.

The problem of MHD flow and heat transfer over a stretching surface has gained considerable interest because of its applications in industry. For example in the extrusion of a polymer sheet from a die, the sheet is sometimes stretched. During this process, the properties of the final products depend considerably on the rate of cooling. By drawing such sheet in an electrically conducting fluid subjected to a magnetic field, the rate of cooling can be controlled and the final product can be obtained with desired characteristics. Pavlov [8] studied the boundary layer flow of an electrically conducting fluid due to a stretching of a plane elastic surface in the presence of a uniform transverse magnetic field. Muhammad and Kiran [9] studied the MHD flow and heat transfer of a micropolar fluid over a stretchable disk. Jat and Chaudhary [10] studied the MHD flow and heat transfer over a stretching sheet.

Hiemenz [11] first reported the stagnation point flow towards a flat plate. It is worthwhile to note that the stagnation flow appears whenever the flow impinges to any solid object and the local fluid velocity at a point (called the stagnation-point) is zero. Chiam [12] extended the works of Hiemenz [11] replaced the solid body a stretching sheet with equal stretching and straining velocities and he was unable to obtain any boundary layer near the sheet. Whereas, Mahapatra and Gupta [13] reinvestigated the stagnation-point flow towards a stretching sheet considering different stretching and straining velocities and they found two different kinds of boundary layers near the sheet depending on the ratio of the stretching and straining constants. The study of a steady two dimensional stagnation point flow of a micropolar fluid over a stretching sheet when the sheet was stretched in its own plane and the stretching velocity was proportional to the distance from the stagnation point was examined by Nazar *et al*. [14]. The resulting coupled equations of nonlinear ordinary differential equations were solved numerically. Hayat et al. [15] investigated the two-dimensional magnetohydrodynamic (MHD) stagnation-point flow of an incompressible micropolar fluid over a nonlinear stretching surface. Hayat *et al*. [16] analyzed the steady two dimensional MHD stagnation point flow of an upper convected Maxwell fluid over the stretching surface. The governing nonlinear partial differential equations were reduced to ordinary ones using the similarity transformation. The homotopy analysis method (HAM) was used to solve these equations. Bhattacharyya [17] investigated the boundary layer stagnation-point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet.

Vajravelu and Rollins [18] studied the heat transfer characteristics in an electrically conducting fluid over a stretching sheet with variable wall temperature and internal heat generation or absorption. Mahmoud and Waheed [19] investigated the effect of slip velocity on mixed convection flow of a micropolar fluid over a heated stretching surface in the presence of a uniform magnetic field and heat generation or absorption. Mostafa [20] studied the MHD flow and heat transfer of a micropolar fluid over a stretching surface with heat generation (absorption) and slip velocity. Recently, Jat et al. [21] studied the MHD flow and heat transfer near the stagnation point of a micropolar fluid over a stretching surface with heat generation/absorption.

In all the above papers viscous dissipation is neglected. But when the motion is under strong gravitational field, or flow field is of extreme size, the viscous dissipative heat cannot be neglected. Rahman [22] analyzed the steady laminar free-forced convective flow and heat transfer of micropolar fluids past a vertical radiate isothermal permeable surface in the presence of viscous dissipation and Ohmic heating. Abel *et al.*[23] studied the MHD flow, and heat transfer with effects of buoyancy, viscous and Joules dissipation over a nonlinear vertical stretching porous sheet with partial slip. Hsiao and Lee [24] analyzed the conjugate heat and mass transfer for MHD mixed convection with viscous dissipation and radiation effect for viscoelastic fluid past a stretching sheet. Gebhart [25] has shown that the viscous dissipation effect plays an important role in natural convection in various devices processes on large scales (or large planets). Also, he pointed out that when the temperature is small, or when the gravitational field is of high intensity, viscous dissipations is more predominant in vigorous natural convection processes. The radiation effect on steady free convection flow near isothermal stretching sheet in the presence of a magnetic field is studied by Ahmed [26]. Govardhan et al. [27] studied the radiation effect on MHD steady free convection flow of a gas at a stretching surface with a uniform free stream with viscous dissipation.

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries, For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Chambre and Young [28] have presented a first order chemical reaction in the neighborhood of a horizontal plate. Dekha *et al.*[29] investigated the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past a vertical plate with a constant heat and mass transfer. Muthcumaraswamy and Ganesan [30] studied effect of the chemical reaction and injection on the flow characteristics in an unsteady upward motion of an isothermal plate. Chamkha [31] studied the MHD flow of a numerical of uniform stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Seddeek *et al.*[32] analyzed the effects of chemical reaction, radiation and variable viscosity on hydromagnetic mixed convection heat and mass transfer Hiemenz flow through porous media.

The present study investigates the steady magnetohydrodynamic (MHD) laminar flow of an incompressible electrically conducting micropolar fluid impinging on a permeable stretching surface by taking heat generation or absorption, chemical reaction and viscous dissipation into account. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, fourth order Runge-Kutta method along with shooting technique has been used for solving it. The results for velocity, microrotation, temperature and concentration functions are carried out for the wide range of important parameters namely, material parameter, magnetic parameter, boundary parameter, velocity ratio parameter, Prandtl number, Eckert number, heat source/sink parameter, Schmidt number and chemical reaction parameter. The skin friction, the rate of heat transfer and the rate of mass transfer have also been computed.

II. Mathematical Formulation

Consider a steady, two-dimensional flow of an incompressible electrically conducting micropolar fluid (in the region $y>0$) near a stagnation point at the origin over a flat sheet coinciding with the x-axis such that the sheet is stretched in its own plane with uniform velocity proportional to the distance from the stagnation point in the presence of an externally applied normal magnetic field of constant strength B_0. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible. The stretching sheet has uniform temperature T_w and concentration C_w and a linear velocity U_w whereas the temperature and concentration of the micropolar fluid for away from the sheet is T_∞ and C_∞, the velocity of the flow external to the boundary layer is $U(x)$ respectively. The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Under these assumptions along with the Boussinesq and boundary layer approximations, the system of equations, which models the flow is given by

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

Linear momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\left(\frac{\mu+h}{\rho}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u+\frac{h}{\rho}\frac{\partial N}{\partial y} \tag{2.2}$$

Angular momentum equation

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\gamma\frac{\partial^2 N}{\partial y^2}-h\left(2N+\frac{\partial u}{\partial y}\right) \tag{2.3}$$

Energy equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}+q(T-T_\infty)+\left(\mu+h\right)\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2u^2 \tag{2.4}$$

Species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-k_0(C-C_\infty) \tag{2.5}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u=U_w=ax, v=0, N=-m\frac{\partial u}{\partial y}, T=T_w, C=C_w \quad \text{at} \quad y=0$$

$$u=U(x)=bx, N=0, T=T_\infty, C=C_\infty \quad \text{as} \quad y\to\infty \tag{2.6}$$

where m is the boundary parameter with $0 \le m \le 1$, u and v are the velocity components in the x - and y - directions, respectively, T is the fluid temperature, C is the fluid concentration, N is the microrotation or angular velocity whose direction of rotation is in the xy-plane, q is the volumetric rate of heat source/sink, κ is the thermal conductivity, c_p is the specific heat at the constant pressure, a(>0) and $b \ge 0$ are constants, and j, γ, μ, h, ρ, and α are the microinertia per unit mass, spin gradient viscosity, dynamic viscosity, vortex viscosity, fluid density and thermal diffusivity, respectively.

Here, γ, j and h are assumed to be constants and γ is assumed to be given by Nazar et al [14]

$$\gamma=\left(\mu+\frac{h}{2}\right)j$$

We take $j = \frac{\upsilon}{a}$ as a reference length, where υ is the kinematic viscosity.
The continuity equation (2.1) is satisfied by the Cauchy Riemann equations

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{2.7}$$

where $\psi(x, y)$ is the stream function.
In order to transform equations (2.2) to (2.6) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced.

$$\psi(x,y) = x(a\upsilon)^{1/2} f(\eta), N(x,y) = ax\left(\frac{a}{\upsilon}\right)^{1/2} g(\eta), \eta = y\left(\frac{a}{\upsilon}\right)^{1/2}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, K = \frac{h}{\mu}(\geq 0), c = \frac{b}{a}, M = \frac{\sigma B_0^2}{\rho a} \tag{2.8}$$

$$\Pr = \frac{\mu c_p}{\kappa}, Q = \frac{q}{a\rho c_p}, Ec = \frac{U_w^2}{c_p\left(T_w - T_\infty\right)}, Sc = \frac{\upsilon}{D}, Kr = \frac{k_0}{a}$$

where $f(\eta)$ is the dimensionless stream function, θ - dimensionless temperature, ϕ - dimensionless concentration, η - similarity variable, M - the Magnetic parameter, *K*- the material parameter, *c* - the velocity ratio parameter (Stretching parameter), *Sc*-Schmidt number, *Ec*-Eckert number, *Q*- the heat source/sink parameter, *Pr*- the Prandtl number and *Kr*-the chemical reaction parameter.

In view of Equations (2.7) and (2.8), the Equations (2.2) to (2.6) transform into

$$(1+K)f''' + ff'' - f'^2 + Kg' - Mf' + c^2 = 0 \tag{2.9}$$

$$\left(1+\frac{K}{2}\right)g'' + fg' - f'g - K(2g + f'') = 0 \tag{2.10}$$

$$\frac{1}{\Pr}\theta'' + f\theta' + Q\theta + (1+K)Ecf''^2 + MEcf'^2 = 0 \tag{2.11}$$

$$\frac{1}{Sc}\phi'' + f\phi' - Kr\phi = 0 \tag{2.12}$$

The corresponding boundary conditions are:

$$f(0) = 0,\ f'(0) = 1, g = -mf''(0), \theta(0) = 1, \phi(0) = 1$$
$$f' \to c, g \to 0, \theta \to 0, \phi \to 0 \quad \text{as} \quad \eta \to \infty \tag{2.13}$$

where the primes denote differentiation with respect to η

The physical quantities of interest are the local skin friction coefficient C_f, the local Nusselt number Nu and local Sherwood number Sh which are defined as

$$C_f = \frac{\tau_w}{\rho U_w^{\ 2}}, \quad Nu = \frac{xq_w}{\kappa(T_w - T_\infty)}, \ Sh = \frac{xq_m}{\kappa(C_w - C_\infty)} \tag{2.14}$$

where the wall shear stress τ_w, the heat flux q_w and mass flux q_m are given by

$$\tau_w = \left[(\mu + h)\frac{\partial u}{\partial y} + hN\right]_{y=0}, \quad q_w = -\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} \quad q_m = -\kappa\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

Thus, we get

$$C_f = \frac{\left(1+\frac{K}{2}\right)f''(0)}{\mathrm{Re}_x^{1/2}}, \quad Nu = -\mathrm{Re}_x^{1/2}\,\theta'(0) \text{ and } Sh = -\mathrm{Re}_x^{1/2}\,\phi'(0) \tag{2.15}$$

where $\mathrm{Re}_x = \frac{U_w x}{\upsilon}$ is the local Reynolds number

Our main aim is to investigate how the values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ vary in terms of the governing parameters.

III. Solution Of The Problem

The set of coupled non-linear governing boundary layer equations (2.9) - (2.12) together with the boundary conditions (2.13) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (2.9) - (2.12) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain *et al.*[33]). The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta$ =0.05 is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

IV. Results And Discussion

The governing equations (2.9) - (2.12) subject to the boundary conditions (2.13) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the parameters encountered in the problem. The effects of various parameters on velocity profiles in the boundary layer are depicted in Figs. 1-3. The effects of various parameters on angular velocity profiles in the boundary layer are depicted in Figs.4-6. The effects of various parameters on temperature profiles in the boundary layer are depicted in Figs. 7-11. The effects of various parameters on concentration profiles in the boundary layer are depicted in Figs. 12-14. Fig. 1 shows the dimensionless velocity for different values of magnetic parameter (*M*). It is seen that, as expected, the velocity increases with an increase of magnetic parameter. The magnetic parameter is found to retard the velocity at all points of the flow field. It is because that the application of transverse magnetic field will result in a resistive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness increases with an increase in the magnetic parameter. The effect of material parameter (*K*) on the velocity is illustrated in Fig.2. It is noticed that the velocity increases with increasing values of the material parameter. . Fig. 3 shows the variation of the velocity with the velocity ratio parameter (*c*) and material parameter (*K*). It is noticed that the velocity thickness increases with an increase in the parameters *c* and *K*.

Fig.4 illustrates the effect of magnetic parameter on the angular velocity. It is noticed that as the magnetic parameter increases, the angular velocity increases. Fig. 5 shows the variation of the angular velocity with the parameters *c* and *K*. It is noticed that the angular velocity thickness decreases with an increase in the parameters *c* and *K*. Fig. 6 depicts the angular velocity with the parameters *c* and *m*. It is noticed that the angular velocity thickness increases with an increase in the boundary parameter (*m*). The effect of the magnetic parameter on the temperature is illustrated in Fig.7. It is observed that as the magnetic parameter increases, the temperature increases. Fig. 8 depicts the thermal boundary-layer with the parameters *c* and *K*. It is noticed that the thermal boundary layer thickness decreases with an increase in the parameters *c* and *K*. Fig. 9 depicts the thermal boundary-layer with the parameters *c* and *Pr*. It is noticed that the thermal boundary layer thickness decreases with an increase in the parameters *c* and *K*. Fig.10 illustrates the effect of the velocity ratio parameter and heat source/sink parameter (*Q*) on the temperature. It is noticed that as the heat source/sink parameter increases, the temperature increases. Fig. 11 shows the variation of the thermal boundary-layer with the parameters *c* and *Ec*. It is noticed that the thermal boundary layer thickness increases with an increase in the Eckert number. The effect of magnetic parameter on the concentration field is illustrated Fig.12. As the magnetic parameter increases, the concentration is found to be increasing. The effect of parameters *c* and *Sc* on the concentration field is illustrated Fig. 13. It is noticed that the concentration boundary layer thickness decreases with an increase in the parameters c and *Ec*. The effect of parameters *c* and *Kr* on the concentration field is illustrated Fig. 14. It is noticed that the concentration boundary layer thickness decreases with an increase in the parameters *c* and *Kr*.

To assess the present method, comparisons with the previous reported data available in the open literature are made for several values of *M* and *c* when *K=Q=Ec=Sc=Kr=0*, as given in table 1. Table 2 shows the variation of the skin friction with for different values of *M, K* or *c*. It is observed that the skin friction decreases with an increase in the *M* and increases with increasing the *K* or *c*. Table.3 depicts the variation of the heat transfer rate with for different values of *M, Q, Pr, Ec* and *c*. It is noticed that the heat transfer rate decreases with an increase in the *M* or *Q* or *Ec* and increases with increasing the *Pr* or *c*. Table 4 shows the variation of the mass transfer rate with for different values of *M, Sc, Kr* and *c*. It is observed that the mass transfer rate decreases with an increase in the *M*, where as increases with an increase in the *Sc* or *Kr* or *c*.

V. Conclusions

In the present chapter, a steady magnetohydrodynamic (MHD) laminar flow of an incompressible electrically conducting micropolar fluid impinging on a permeable stretching surface with heat generation or absorption by taking viscous dissipation and chemical reaction into account, are analyzed. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

1. The velocity decreases as well as the angular velocity, temperature and concentration increases with an increase in the magnetic parameter.
2. The velocity increases as well as the angular velocity, temperature and concentration decreases with an increase in the material parameter.
3. The heat source/sink and viscous dissipation enhances the temperature.
4. The chemical reaction reduces the concentration.
5. The skin friction reduces the magnetic parameter and increases the material parameter or velocity ratio parameter.
6. The heat source/sink or magnetic field or Eckert number reduces the heat transfer rate.
7. The velocity ratio parameter or Schmidt number or chemical reaction enhances the mass transfer rate.

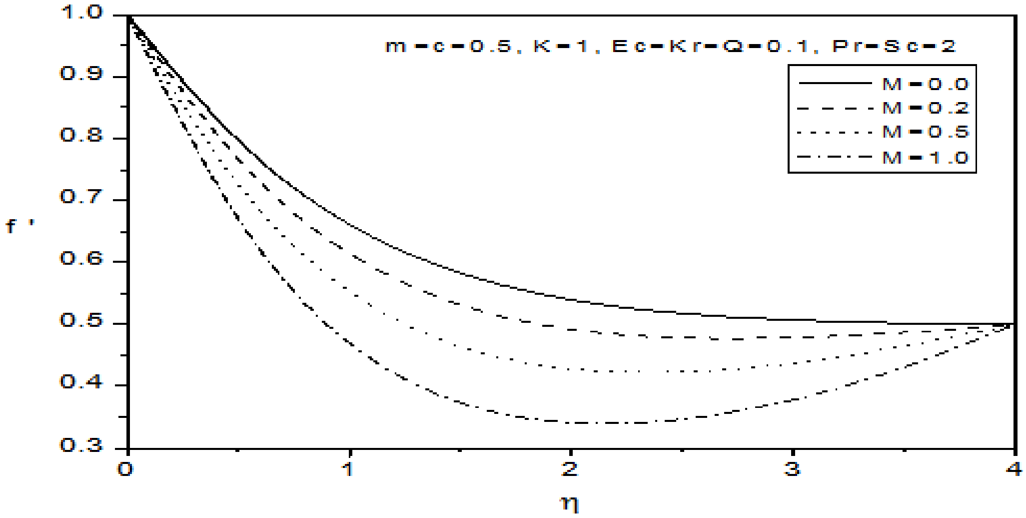

Fig.1 Velocity for different values of *M*

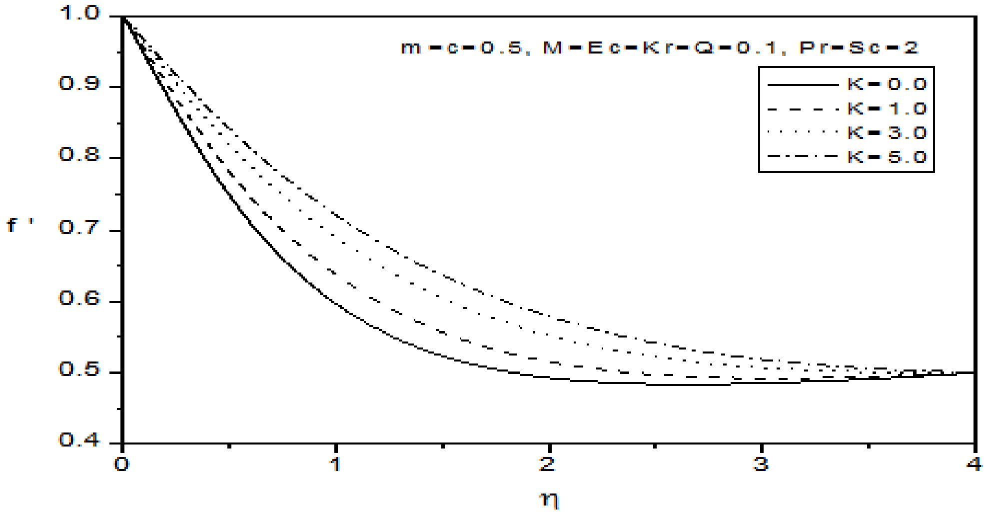

Fig.2 Velocity for different values of *K*

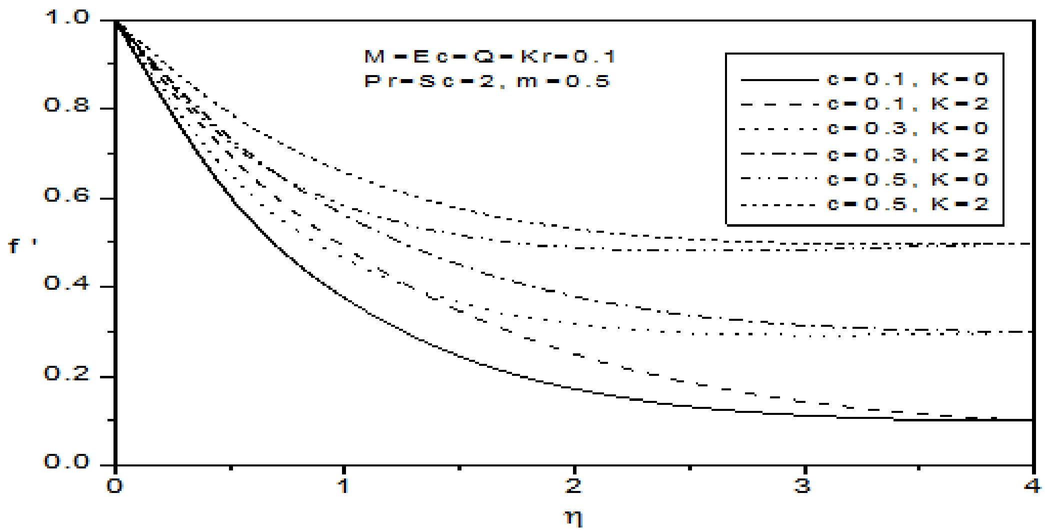

Fig.3 Velocity for different values of *c* and *K*

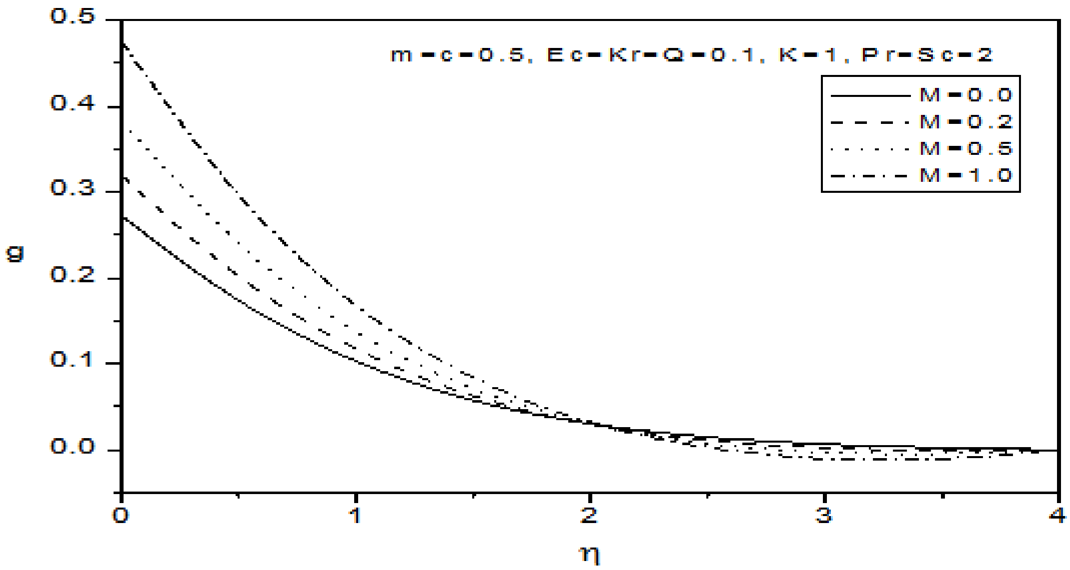

Fig.4 Angular velocity for different values of *M*

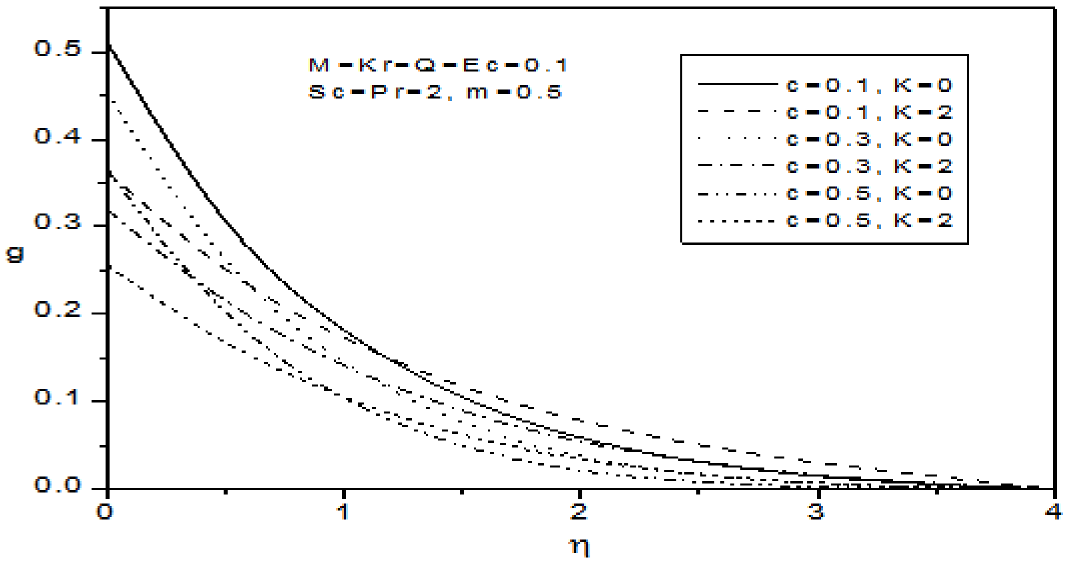

Fig.5 Angular velocity for different values of *c* and *K*

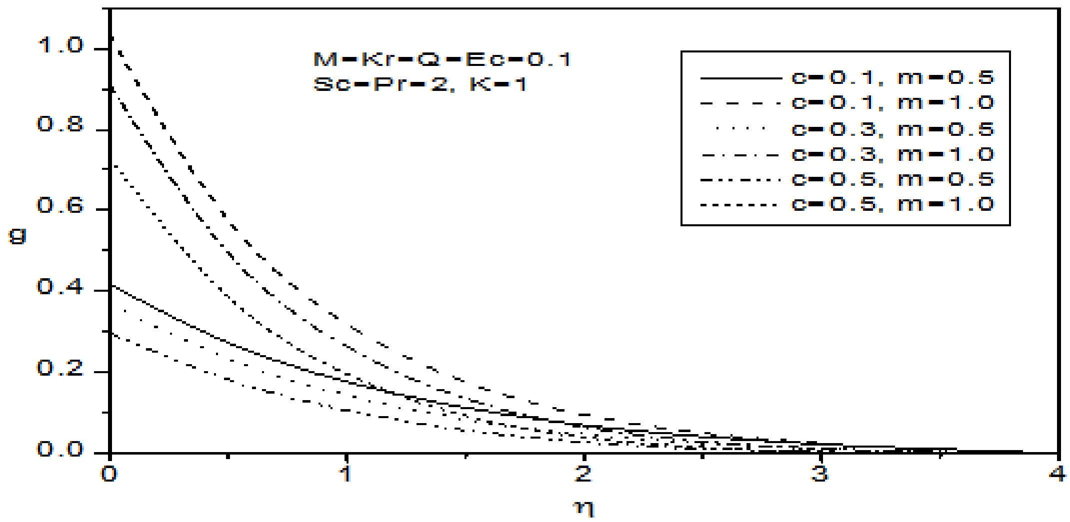

Fig.6 Angular velocity for different values of *c* and *m*

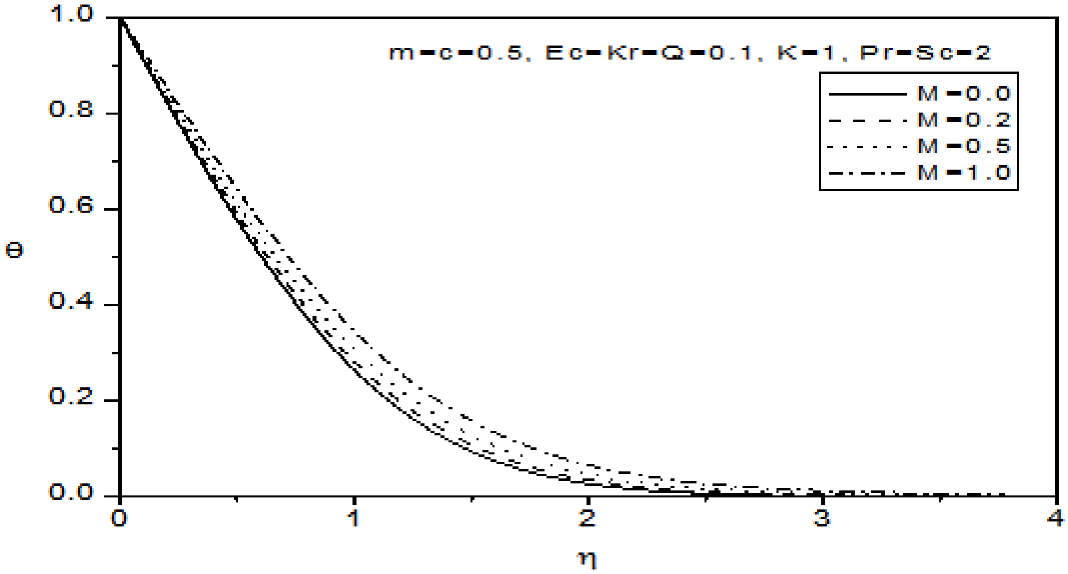

Fig.7 Temperature for different values of *M*

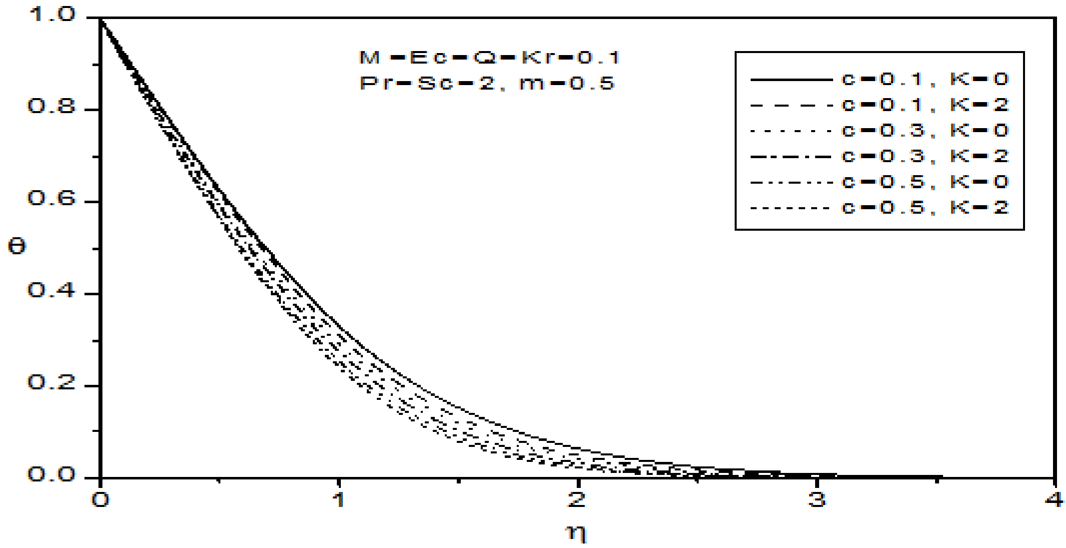

Fig.8 Temperature for different values of *c* and *K*

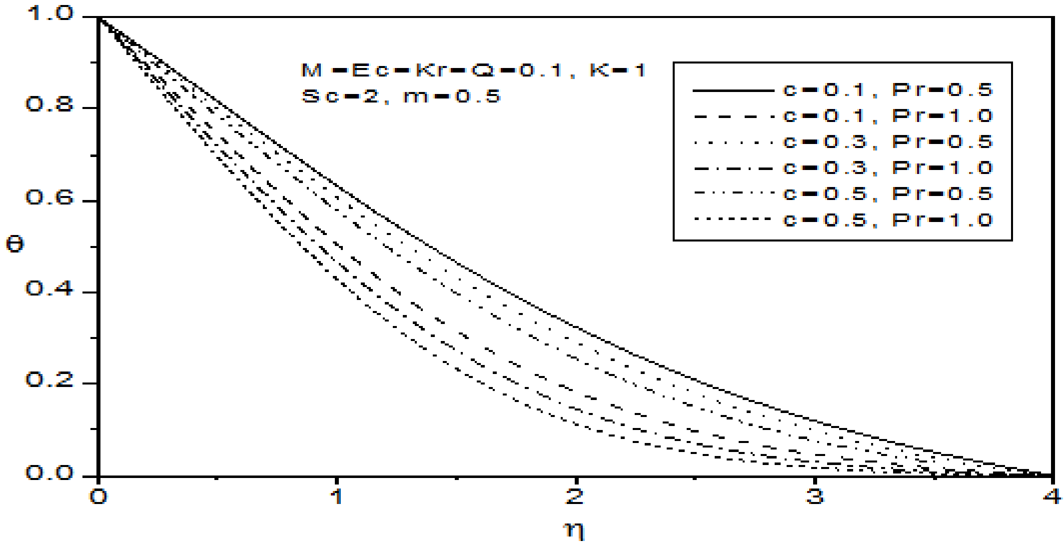

Fig.9 Temperature for different values of *c* and *Pr*

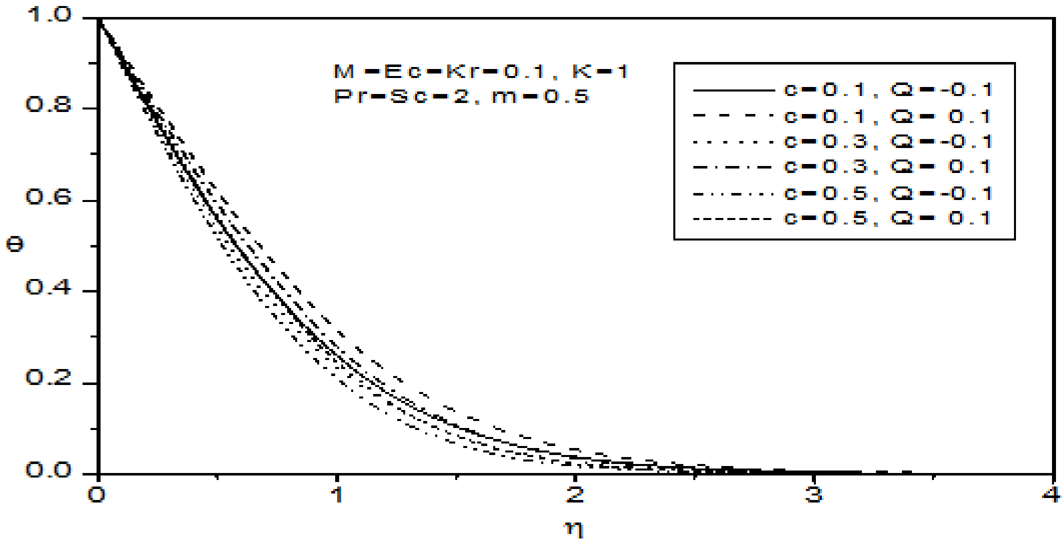

Fig.10 Temperature for different values of *c* and *Q*

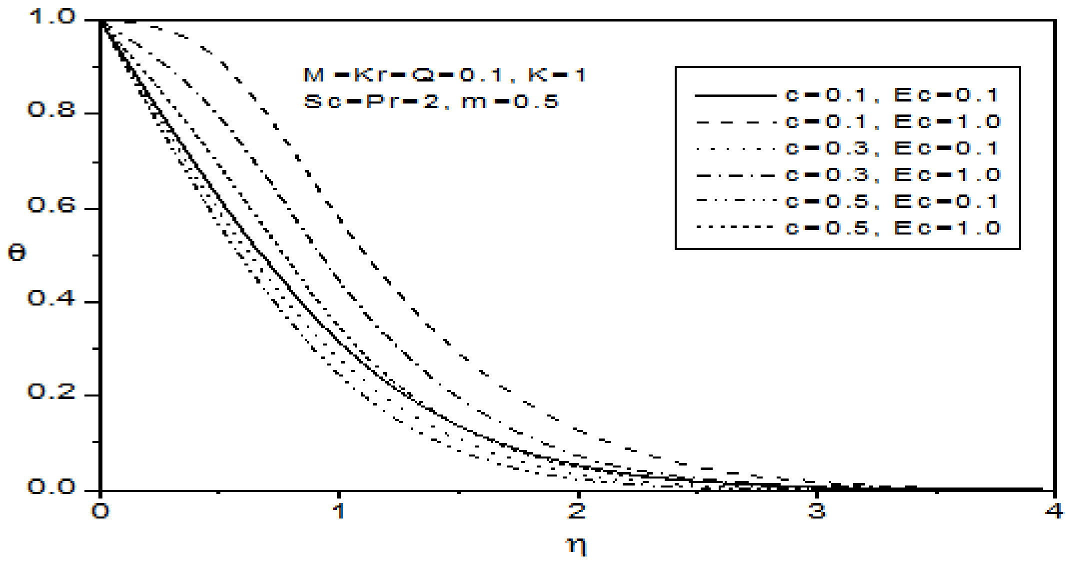

Fig.11 Temperature for different values of *c* and *Ec*

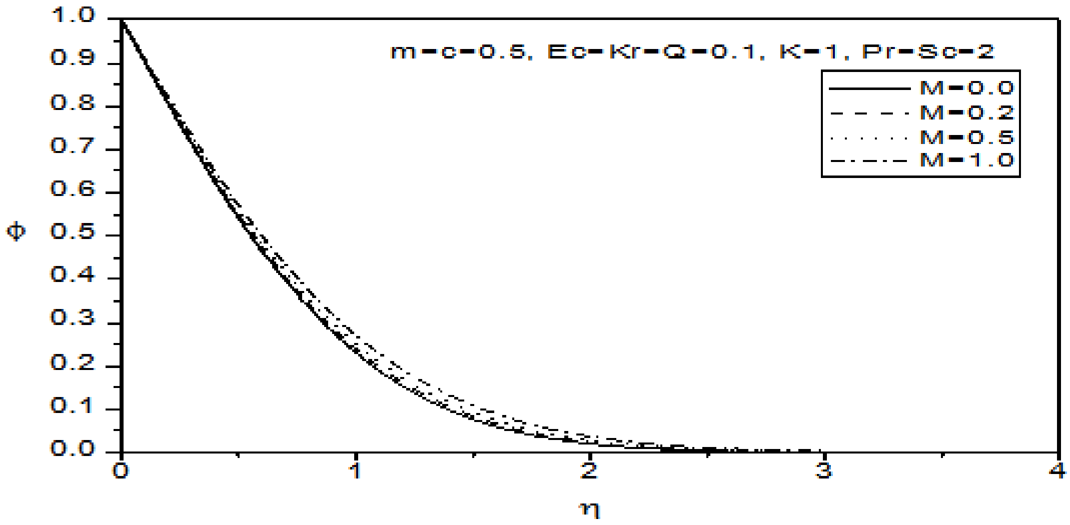

Fig.12 Concentration for different values of *M*

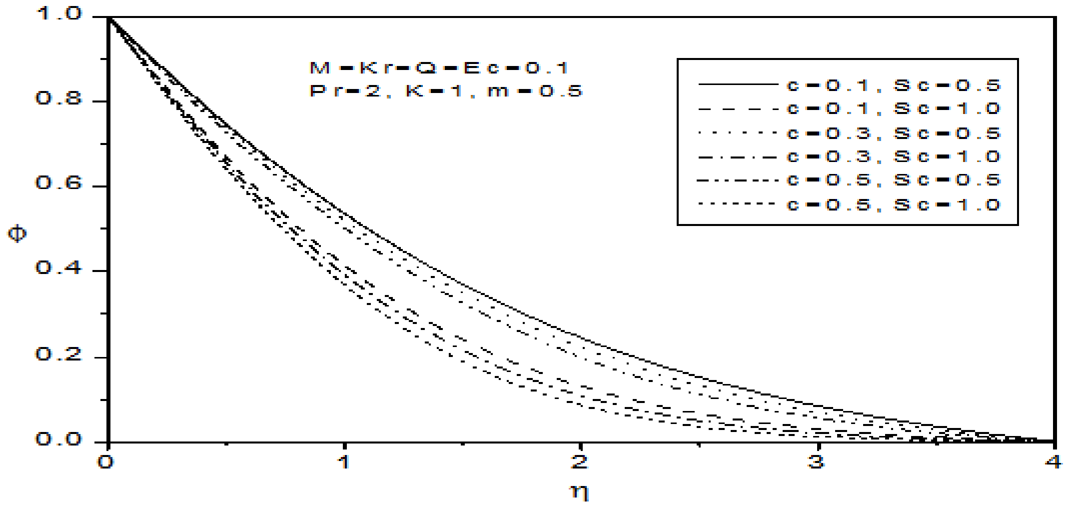

Fig.13 Concentration for different values of *c* and *Sc*

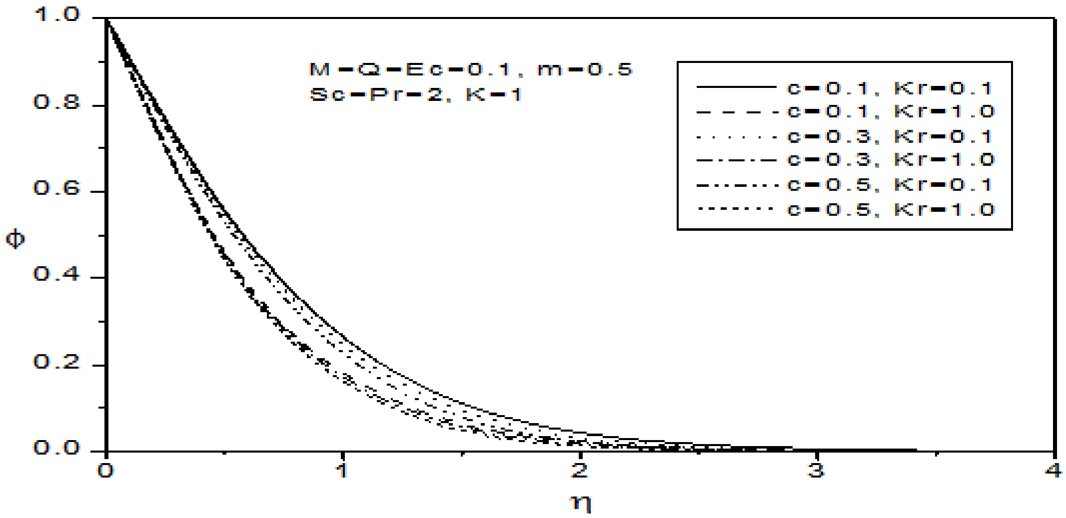

Fig.14 Concentration for different values of *c* and *Kr*

Table 1 Numerical values of $f''(0)$ at the sheet for different values of *M* and *C* when *K=1*and *Q=0*, omparison of the present results with that of Jat et al.[21].

M	Jat et al.[21]				Present results			
	$f''(0)$				$f''(0)$			
	c=0.1	*c=0.2*	*c=0.3*	*c=0.5*	*c=0.1*	*c=0.2*	*c=0.3*	*c=0.5*
0.0	-0.7917	-0.7496	-0.6935	-0.5448	-0.792787	-0.750087	-0.693713	-0.544840
0.1	-0.8342	-0.7951	-0.7415	-0.5973	-0.832740	-0.792727	-0.738942	-0.594844
0.2	-0.8747	-0.8382	-0.7872	-0.6475	-0.870889	-0.833385	-0.782093	-0.642712
0.5	-0.9858	-0.956	-0.9118	-0.7853	-0.975945	-0.944882	-0.900349	-0.774479
1.0	-1.145	-1.123	-1.0871	-0.9795	-1.130210	-1.109270	-1.075300	-0.970779

Table 2 Numerical values of $f''(0)$ at the sheet for different values of *M, K* and *c* when *Pr=Sc=2, Kr=Q=0.1.*

M	*K*	$f''(0)$			
		c=0.1	*c=0.2*	*c=0.3*	*c=0.5*
0.0	1	-0.796378	-0.751991	-0.694694	-0.545080
0.2	1	-0.871563	-0.831749	-0.779099	-0.638599
0.5	1	-0.974222	-0.940277	-0.893842	-0.766110
1.0	1	-1.124100	-1.097550	-1.059420	-0.949956
0.1	0	-1.021580	-0.972588	-0.906636	-0.729858
0.1	2	-0.726201	-0.687905	-0.639078	-0.512192
0.1	3	-0.653681	-0.617696	-0.572764	-0.457765
0.1	5	-0.561344	-0.528033	-0.487797	-0.387570

Table 3 Numerical values of $-\theta'(0)$ at the sheet for different values of *M, Q, Pr, Ec* and *c* when *Sc=2,Kr=0.1.*

M	*Q*	*Pr*	*Ec*	$-\theta'(0)$			
				c=0.1	*c=0.2*	*c=0.3*	*c=0.5*
0.0	0.1	2.0	0.1	0.739473	0.775684	0.814067	0.891728
0.2	0.1	2.0	0.1	0.695748	0.729918	0.766825	0.842985
0.5	0.1	2.0	0.1	0.634545	0.665703	0.700260	0.773587
1.0	0.1	2.0	0.1	0.542850	0.569460	0.600131	0.667910
0.1	-0.1	2.0	0.1	0.933973	0.960816	0.990455	1.053020
0.1	-0.05	2.0	0.1	0.883698	0.912174	0.943421	1.008960
0.1	0.0	2.0	0.1	0.831056	0.861422	0.894506	0.963383
0.1	0.05	2.0	0.1	0.775724	0.808301	0.843502	0.916156
0.1	0.1	2.0	0.1	0.717306	0.752498	0.790158	0.867115
0.1	0.1	0.5	0.1	0.348134	0.364023	0.381504	0.418642
0.1	0.1	1.0	0.1	0.479355	0.506381	0.535274	0.594127
0.1	0.1	1.5	0.1	0.605622	0.637858	0.672110	0.741492
0.1	0.1	2.0	0.2	0.591239	0.543274	0.698301	0.808489
0.1	0.1	2.0	0.5	0.213035	0.315602	0.422730	0.632610
0.1	0.1	2.0	1.0	-0.417304	-0.230518	-0.0365556	0.339478

Table 4 Numerical values of $-\phi'(0)$ at the sheet for different values of *M, Sc, Kr* and *c* when *Pr=2, Ec=0.1.*

M	*Sc*	*Kr*	$-\phi'(0)$			
			c=0.1	*c=0.2*	*c=0.3*	*c=0.5*
0.0	2.0	0.1	1.009060	1.022780	1.038320	1.072890
0.2	2.0	0.1	0.994129	1.007170	1.022240	1.056350
0.5	2.0	0.1	0.973514	0.985538	0.999835	1.033060
1.0	2.0	0.1	0.943309	0.953776	0.966762	0.998193
0.1	0.5	0.1	0.537214	0.545246	0.554506	0.575682
0.1	1.0	0.1	0.706500	0.718811	0.732719	0.763458
0.1	1.5	0.1	0.861954	0.875387	0.890566	0.924114
0.1	2.0	0.2	1.049420	1.061630	1.075720	1.107680
0.1	2.0	0.5	1.182450	1.192020	1.203310	1.229580
0.1	2.0	1.0	1.377160	1.384070	1.392410	1.412390

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