

Two Stage Fuzzy Transportation Problem Using Symmetric Trapezoidal Fuzzy Number

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Abstract: -The transportation problem is one of the earliest applications of linear programming problems. In the literature, several methods are proposed for solving transportation problems in fuzzy environment but in all the proposed methods, the parameters are normal fuzzy numbers. In this paper, a general fuzzy transportation problem is discussed. In the proposed method, transportation cost, availability and demand of the product are represented by symmetric trapezoidal fuzzy numbers. Robust ranking technique is used for finding fuzzy optimal solution of fuzzy transportation problem. A numerical example is given to show the efficiency of the method

Keywords: *Fuzzy set; Symmetric Trapezoidal fuzzy number; Robust ranking method; zero suffix method*

I. INTRODUCTION

The Transportation problem is a special type of linear programming problem which deals with the distribution of single product (raw or finished) from various sources of supply to various destination of demand in such a way that the total transportation cost is minimized. There are effective algorithms for solving the transportation problems when all the decision parameters, i.e. the supply available at each source, the demand required at each destination as well as the unit transportation costs are given in a precise way

In some circumstances due to storage constraints, designations are unable to receive the quantity in excess of their minimum demand. After consuming part of whole of this initial shipment, they are prepared to receive the excess quantity in the second stage. According to Sonia and Rita Malhotra [1], in such situations the product transported to the destination has two stages. Just enough of the product is shipped in stage-I so that the minimum requirements of the destinations are satisfied and having done this the surplus quantities (if any) at the sources are shipped to the destinations according to cost consideration. In both the stages the transportation of the product from sources to the destinations is done in parallel. The aim is to minimize the sum of the transportation costs in two stages. Here, a ranking technique is used for solving two stage fuzzy transportation problem, where transportation cost, demand and supply are in the form of symmetric trapezoidal fuzzy numbers.

II. PRELIMINARIES

Zadeh[2] in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

2.1 Definition:

A *fuzzy set* is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0, 1].

(i.e) $A = \{(x, \mu_A(x)) ; x \in X\}$, Here $\mu_A : X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0,1]

2.2 Definition:

A fuzzy set A of the universe of discourse X is called a *normal* fuzzy set implying that there exist at least one $x \in X$ such that $\mu_A(x) = 1$.

2.3 Definition (Symmetric trapezoidal fuzzy number):

A fuzzy number $A = (a, b, c, c)$ is said to be symmetric Trapezoidal fuzzy number if its membership function $\mu_A(x)$ is given by

$$\mu_A(x) = \begin{cases} \frac{x + (c - a)}{c}, & (c - a) \leq x \leq c \\ 1, & a < x < b \\ \frac{-x + (b + c)}{c}, & b \leq x \leq (b + c) \\ 0, & \text{otherwise} \end{cases}$$

III. ROBUST RANKING TECHNIQUES:

Robust ranking technique[6] which satisfies compensation, linearity, and additivity properties and provides results which are consistent with human intuition. Give a convex fuzzy number \tilde{a} , the Robust Ranking Index is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U) d\alpha, \text{ where } (a_\alpha^L, a_\alpha^U) \text{ is a } \alpha\text{-level cut of a fuzzy number } \tilde{a}.$$

In this paper we use this method for ranking the objective values. The Robust ranking index $R(\tilde{a})$ gives the representative value of the fuzzy number \tilde{a} .

IV. ZERO SUFFIX METHOD

The zero suffix method proceeds as follows.

Step 1: Construct the transportation table for the given TP and check the balanced condition. If not, convert it into balanced one.

Step 2: Subtract each row entries of the transportation table from the row minimum.

Step 3: subtract each column entries of the resulting transportation table after using the Step 2 from the column minimum.

Step 4: In the reduced cost matrix there will be atleast one zero in each row and column, then find the suffix value of all the zeros in the reduced cost matrix by following simplification, the suffix value is denoted by S,

$$S = \text{Add the costs of nearest adjacent sides of zero / No. of costs added}$$

Step 5: Choose the maximum of S, if it has one maximum value then supply to that demand corresponding to the cell. If it has more equal values then select $\{a_i, b_j\}$ and supply to that demand maximum possible.

Step 6: After the above step, the exhausted demands (column) or supplies (row) are to be trimmed. If $a_i = b_j$, cross out either i th row or j th column but not both. The resultant matrix must possess atleast one zero in each row and each column, else repeat step2 and step3.

Step 7: Repeat Step 4 to Step 6 until the optimal cost is obtained.

V. NUMERICAL EXAMPLE

Consider the Fuzzy transportation problem. Here cost value, supplies and demands are trapezoidal fuzzy number. Here a_i and b_j are Fuzzy Supply and Fuzzy Demand. Robust Ranking technique is used to finding the initial basic feasible solution.

	D₁	D₂	D₃	D₄	SUPPLY
S₁	(1,2,3,3)	(1,3,4,4)	(9,11,12,12)	(5,7,8,8)	(1,6,7,7)
S₂	(0,1,2,2)	(-1,0,1,1)	(5,6,7,7)	(0,1,2,2)	(2,1,3,3)
S₃	(3,5,6,6)	(5,8,9,9)	(12,15,16,16)	(7,9,10,10)	(5,10,12,12)
DEMAND	(5,7,8,8)	(1,5,6,6)	(1,3,4,4)	(1,2,3,3)	

Here, $\sum_{i=1}^n a_i = \sum_{j=1}^m b_j$ the problem is balanced fuzzy transportation problem. There exists a fuzzy initial basic feasible solution. Now apply Robust Ranking Technique for the above fuzzy transportation problem.

$$R(\tilde{a}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U) d\alpha$$

Where $(a_\alpha^L, a_\alpha^U) = [c\alpha - (c - a), (b + c) - c\alpha]$

$$\begin{aligned} R(1,2,3,3) &= \int_0^1 (0.5)[3\alpha - (3 - 1), (2 + 3) - 3\alpha] d\alpha \\ &= \int_0^1 (0.5)(-2 + 5) d\alpha \\ &= \int_0^1 (0.5)(3) d\alpha = 1.5 \end{aligned}$$

Similarly

$$\begin{aligned} R(1,3,4,4) &= 2, \quad R(9,11,12,12) = 10, \quad R(5,7,8,8) = 6, \quad R(0,1,2,2) = 0.5, \\ R(-1,0,1,1) &= -0.5, \quad R(5,6,7,7) = 5.5, \quad R(0,1,2,2) = 0.5, \quad R(3,5,6,6) = 4, \\ R(5,8,9,9) &= 6.5, \quad R(12,15,16,16) = 13.5, \quad R(7,9,10,10) = 8 \end{aligned}$$

Rank of all supply

$$R(1,6,7,7) = 3.5, \quad R(2,1,3,3) = 1.5, \quad R(5,10,12,12) = 7.5$$

Rank of all demand

$$R(5,7,8,8) = 6, \quad R(1,5,6,6) = 3, \quad R(1,3,4,4) = 2, \quad R(1,2,3,3) = 1.5$$

Table after ranking

	D ₁	D ₂	D ₃	D ₄	SUPPLY
S ₁	1.5	2	10	6	3.5
S ₂	0.5	-0.5	5.5	0.5	1.5
S ₃	4	6.5	13.5	8	7.5
DEMAND	6	3	2	1.5	

Stage I:

Take $a_1 = 2, a_2 = 1, a_3 = 4$

$b_1 = 4, b_2 = 1, b_3 = 1, b_4 = 1$

	D ₁	D ₂	D ₃	D ₄	SUPPLY
S ₁	1.5	2	10	6	2
S ₂	0.5	-0.5	5.5	0.5	1
S ₃	4	6.5	13.5	8	4
DEMAND	4	1	1	1	

After applying Zero suffix method, we get

	D ₁	D ₂	D ₃	D ₄	SUPPLY
S ₁	1.5	2 1	10 1	6	2
S ₂	0.5	-0.5	5.5	0.5 1	1
S ₃	4 4	6.5	13.5	8 0	4
DEMAND	4	1	1	1	

$$\begin{aligned} \text{Min } Z &= 2 \times 1 + 10 \times 1 + 0.5 \times 1 + 4 \times 4 + 8 \times 0 \\ &= 28.5 \end{aligned}$$

Stage II:

Take $a_1 = 1.5, a_2 = 0.5, a_3 = 3.5$

$b_1 = 2, b_2 = 2, b_3 = 1, b_4 = 0.5$

	D ₁	D ₂	D ₃	D ₄	SUPPLY
S ₁	1.5	2	10	6	1.5
S ₂	0.5	-0.5	5.5	0.5	0.5
S ₃	4	6.5	13.5	8	3.5
DEMAND	2	2	1	0.5	

After applying Zero suffix method, we get

	D ₁	D ₂	D ₃	D ₄	SUPPLY
S ₁	1.5	2 1.5	10	6	1.5
S ₂	0.5	-0.5	5.5	0.5 0.5	0.5
S ₃	4 2	6.5 0.5	13.5 1	8	3.5
DEMAND	2	2	1	0.5	

$$\begin{aligned} \text{Min } Z &= 2 \times 1.5 + 0.5 \times 0.5 + 4 \times 2 + 6.5 \times 0.5 + 13.5 \times 1 \\ &= 28 \end{aligned}$$

Therefore the optimal value for the given problem is
Minimum $(28.5+28) = 56.5$

VI. CONCLUSION

The transportation cost is considered as imprecise fuzzy numbers in this paper. Here, the fuzzy transportation has been changed into crisp transportation using Robust ranking technique[6]. Later, an optimal solution has been originated from crisp and fuzzy optimal total cost of given example. Moreover, using Robust ranking Method and Zero suffix method can conclude that the solution of fuzzy problem obtained more accurately and effectively.

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