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Single Phase Inverter with Selective Harmonics Elimination PWM Based on Secant Method

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Abstract: *Selective harmonics can be eliminated by controlling the inverter switches at optimized switching angles in single phase dc-ac inverters. These switching angles are estimated by solving a set of nonlinear equations. This paper presents a novel technique for solving these equations based on secant method. The proposed algorithm simplifies the numerical solution of the nonlinear equations, solves them fast and achieves convergence of solution at different starting vector of angles. The proposed method is applied with different modulation index and different number of equations, i.e. different number of switching angles to achieve the elimination of certain harmonics. High performance of the inverter is validated through simulation results.*

Keywords:Selective harmonics elimination, Secant method, Switching angles.

I. INTRODUCTION

Single phase dc-ac inverters are widely used in many applications such as AC motor drives, power supplies and uninterruptible power supplies [1]. Half bridge and full bridge topologies can be found but full bridge topology are more common. The output voltage of the inverter is required to be as close as possible to sinusoidal shape. Conventional sinusoidal pulse width modulation SPWM can be applied but selective harmonics elimination pulse width modulation (SHEPWM) technique is more advantageous than SPWM as listed in the literature [1-5].In SHEPWM, number of low order harmonics is selected to be eliminated based on the requirements of application. Then a set of nonlinear equations called “transcendental equations” are solved numerically to get the switching angles. These switching angles are applied to generate the control signals of the inverter switches. Equal area algorithm, Newton-Raphson algorithm, pattern search method and genetic algorithm can be applied to solve these equations [2, 3, 6]. The main challenges are the starting vector of solution and the convergence of the solution. This paper proposes a novel technique for solving the transcendental equations based on secant method. The proposed algorithm simplifies the numerical solution of the nonlinear equations, solves them fast and achieves convergence of solution at different starting vector of angles. The switching angles are estimated using the proposed method. The switching angles are thenutilized to generate the control signals which are applied to unipolar single phase full bridge inverter. High performance of the inverter is validated through simulation results.

II. PRINCIPLE OF SHEPWM INVERTERS

Schematic of full bridge single phase inverter is shown in Fig. 1.The states of the four switches and the corresponding load voltage are summarizedin Table 1.Two techniques for selective harmonics elimination

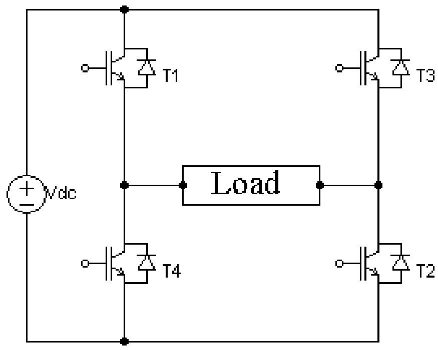

Fig. 1 Single phase inverter

Table 1 States of the inverter switches

Switches state	V_o
T_1 and T_2 are "on" - T_3 and T_4 are "off"	V_{dc}
T_3 and T_4 are "on" - T_1 and T_2 are "off"	$-V_{dc}$
T_1 and T_3 are "on" - T_2 and T_4 are "off"	0
T_2 and T_4 are "on" - T_1 and T_3 are "off"	0

PWM can be applied; bipolar PWM technique and unipolar PWM technique [1]. In the bipolar PWM technique, the first two states in Table 1 are only utilized. The four states are applied in unipolar PWM technique which is more effective than bipolar PWM technique when harmonics elimination is required. Therefore unipolar PWM technique is applied in this paper. The output voltage of the unipolar PWM inverter is shown in Fig. 2 with N switching angles (θ_1 , θ_1 , θ_1 … θ_N) , (N-1) harmonics can be eliminated. The output voltage can be represented in Fourier series as a function of the switching angles as

$$v_0(\omega t) = \frac{4V_s}{\pi}\left(\sum_{k=1,3,5}^{\infty} \frac{\sin(k\omega t)}{k}[\cos k\theta_1 - \cos k\theta_2 + \cos k\theta_3 - \cdots..]\right) \qquad (1)$$

Only odd harmonics exist while even harmonics do not exist since the output voltage is even function in its nature. The conditions for fully elimination of certain number (N-1) of odd harmonics are:

$$\sum_{i=1}^{N}(-1)^{i+1}\cos(\theta_i) - \frac{\pi}{4}M_a = 0 \qquad (2)$$

$$\sum_{k=1}^{N}(-1)^{k+1}\cos(n\theta_i) = 0 \qquad (3)$$

Where M_a is the modulation index which is defined as

$M_a = V_1 / V_{dc}$, n = 3, 5, 7, ….., 2N – 1 and $\theta_1 < \theta_2 < \theta_3$ …. $< \theta_N < 90^o$

For instance if it is desired the 3rd,5th,7th,9th, 11th and 13th harmonics, equations (2) and (3) can be written as:

$$\sum_{i=1}^{7}(-1)^{i+1}\cos(\theta_i) - \frac{\pi}{4}M_a = 0 \qquad (4)$$

$$\sum_{k=1}^{7}(-1)^{k+1}\cos(n\theta_i) = 0 \qquad (5)$$

Where 'n' takes the values 3, 5, 7, 9, 11, 13. These will lead to the following seven equations which should be solved simultaneously:

$\cos(\theta_1) - \cos(\theta_2) + \cos(\theta_3) - \cos(\theta_4) + \cos(\theta_5) - \cos(\theta_6) + \cos(\theta_7) - \frac{\pi}{4}M_a = 0$ (6-a)

$\cos(3\theta_1) - \cos(3\theta_2) + \cos(3\theta_3) - \cos(3\theta_4) + \cos(3\theta_5) - \cos(3\theta_6) + \cos(3\theta_7) = 0$ (6-b)

$\cos(5\theta_1) - \cos(5\theta_2) + \cos(5\theta_3) - \cos(5\theta_4) + \cos(5\theta_5) - \cos(5\theta_6) + \cos(5\theta_7) = 0$ (6-c)

$\cos(7\theta_1) - \cos(7\theta_2) + \cos(7\theta_3) - \cos(7\theta_4) + \cos(7\theta_5) - \cos(7\theta_6) + \cos(7\theta_7) = 0$ (6-d)

$\cos(9\theta_1) - \cos(9\theta_2) + \cos(9\theta_3) - \cos(9\theta_4) + \cos(9\theta_5) - \cos(9\theta_6) + \cos(9\theta_7) = 0$ (6-e)

$\cos(11\theta_1) - \cos(11\theta_2) + \cos(11\theta_3) - \cos(11\theta_4) + \cos(11\theta_5) - \cos(11\theta_6) + \cos(11\theta_7) = 0$ (6-e)

$\cos(13\theta_1) - \cos(13\theta_2) + \cos(13\theta_3) - \cos(13\theta_4) + \cos(13\theta_5) - \cos(13\theta_6) + \cos(13\theta_7) = 0$ (6-f)

These nonlinear transcendental equations are solved numerically using the proposed algorithm by applying secant method.

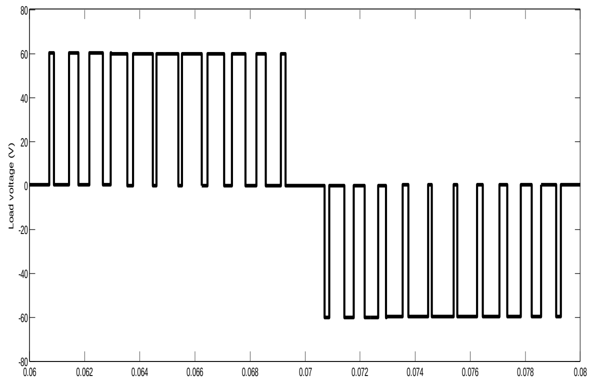

Fig. 2 Output voltage of single phase full bridge inverter with unipolar PWM

III. PROPOSED ALGORITHM USING SECANT METHOD

As a brief description of Newton-Raphson method to solve systems of non-linear equations, it is required to determine a Jacobian matrix "J" which is defined for N equations as follows:

$$\mathbf{J} = \begin{bmatrix} \frac{\partial f_1}{\partial \theta_1} & \frac{\partial f_1}{\partial \theta_2} & \frac{\partial f_1}{\partial \theta_3} & \vdots & \frac{\partial f_1}{\partial \theta_N} \\ \frac{\partial f_2}{\partial \theta_1} & \frac{\partial f_2}{\partial \theta_2} & \frac{\partial f_2}{\partial \theta_3} & \vdots & \frac{\partial f_2}{\partial \theta_N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_N}{\partial \theta_1} & \frac{\partial f_N}{\partial \theta_2} & \frac{\partial f_N}{\partial \theta_3} & \vdots & \frac{\partial f_N}{\partial \theta_N} \end{bmatrix} \qquad (7)$$

This Jacobian matrix is very complex since it needs a large number of partial differentiations especially when several number of harmonics are desired to be eliminated i.e. high value of "N". Therefore very complex calculations will be needed and divergence of solution may occur. With the proposed method, each partial differentiation is replaced by the following equation:

$$\frac{\partial f_i}{\partial \theta_k} \approx \frac{f_i(\theta_1,\theta_2,\ldots.\theta_k+\Delta\theta,\ldots..,\theta_N)}{\Delta\theta}\frac{-f_i(\theta_1,\theta_2,\ldots.,\theta_k,\ldots..,\theta_N)}{\Delta\theta} \qquad (8)$$

i and k take the values 1, 2, …., N $\partial f_2/\partial\theta_3$represents element J_{23}in the Jacobean matrix

$$\frac{\partial f_2}{\partial \theta_3} \approx \frac{f_2(\theta_1,\theta_2,\ldots.\theta_3+\Delta\theta,\ldots..,\theta_N)}{\Delta\theta}\frac{-f_i(\theta_1,\theta_2,\ldots.,\ \theta_3,\ldots..,\theta_N)}{\Delta\theta}$$

Where "$\Delta\theta$" is a small increment. The unknowns θ_1, θ_2 , … θ_N are assigned initial values as:

$\boldsymbol{\theta_0} = [\theta_1\theta_2\ \theta_3\ \ldots.\ \theta_N]^T$

Flow chart that summarizes the proposed algorithm is given in Fig. 3. Equation (8) is applied to determine each element of the Jacobian matrix, then the elements are combined together to estimate the Jacobian matrix "J" as mentioned in (7). An update of the vector θ is carried out using the relation:

$$\boldsymbol{\theta_{n+1}} = \boldsymbol{\theta_0} + \boldsymbol{J}^{-1}.\boldsymbol{f}(\theta_n)$$

The algorithm utilizes $\boldsymbol{\theta_1}$as an initial vector for the next cycle which will repeat until the stopping criteria is achieved. this criteria is

$$\max|f_k(\boldsymbol{\theta})| \le \varepsilon_1 \ , \qquad k = 1, 2, \ldots, N,$$
$$\max|(\theta_k)_{n+1} - (\theta_k)_n| \le \varepsilon_2 \ , \qquad k = 1, 2, \ldots, N\ ,$$

Or the maximum number of iterations is exceeded which means divergence of solution and another initial solution vector $\boldsymbol{\theta_0}$ should be selected.

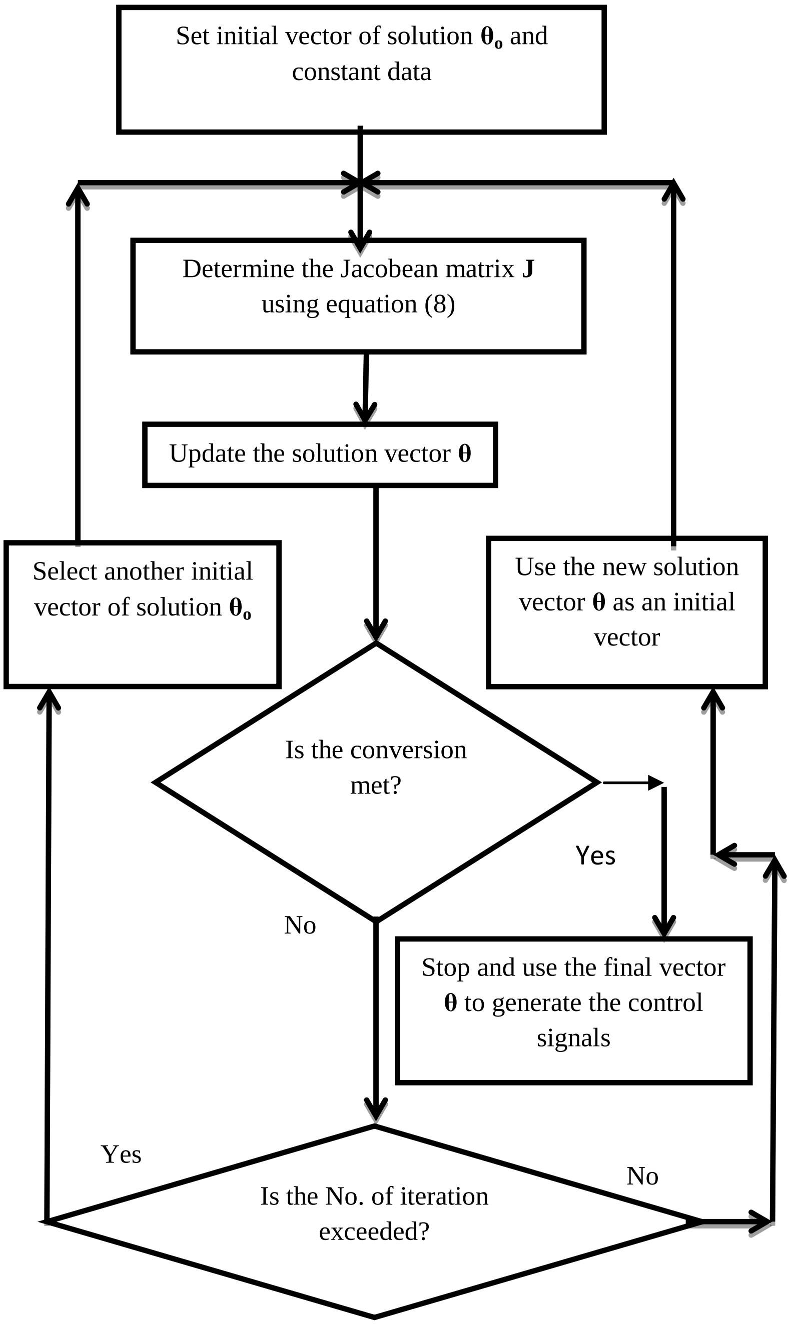

Fig. 3 Flow chart of the proposed algorithm

IV. SIMULATION RESULTS

The proposed method estimates the switching angles $\theta_1\ \theta_2\ \theta_3\ \ldots\ \theta_N$. These switching angles are then utilized in an m-file in Matlab software program to generate the control signals for the inverter switches. Two Matlab m-files are built to carry out the algorithm at different number of harmonics that are desired to be eliminated. The first file is for constant data including dc voltage, desired output frequency and initial vectors of solution. The second file is for the proposed described algorithm. The dc source is 60 V and the load is R-L where R = 10 Ω and L = 40 mH. The increment "Δθ" is chosen as 0.05 rad The control signals are adjusted so that the load frequency is 50 Hz. Different cases are included from N = 3 to N = 11. The switching angles are given in Table 2. Selected cases are studied; N=7, N=10 and N=11. Load voltage and current for N=7, N=10 and N=11 are shown in Figures 4-6 respectively. The modulation index "M_a" is adjusted at 0.85 The associated spectrum analysis of load voltage is presented in Figures 7-9 respectively.It is obvious the harmonics of order 6, 9 and 10 are eliminated for N=7, N=10 and N=11 respectively. The load current is very close to sinusoidal shape as "N" increases. To expect the switching angles using the proposed method and reduce the iteration cycles especially with large values of "N", variation of switching angles with "N" is given in Fig. 10. Therefore the initial values of the switching angles can be selected properly

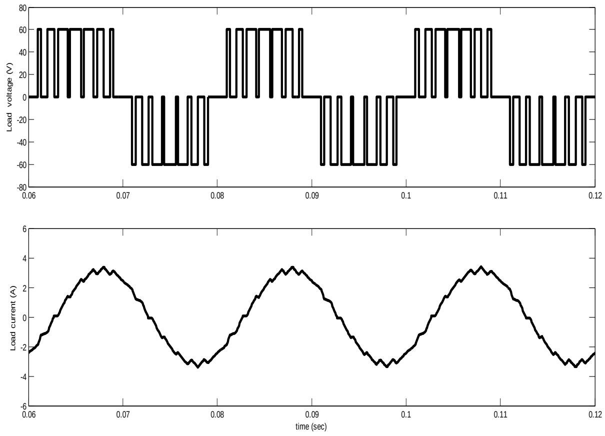

Fig. 4 Load voltage and current, N = 7, M_a = 0.85

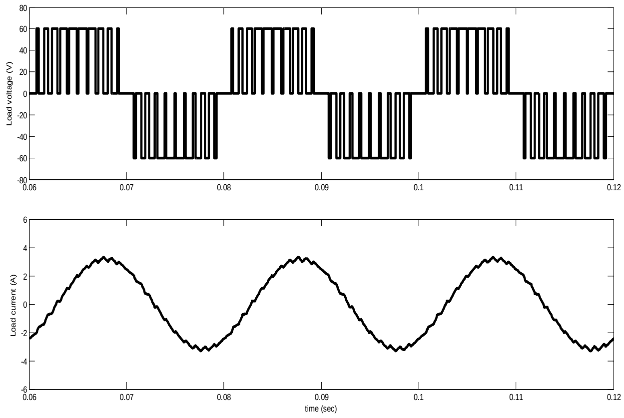

Fig. 5 Load voltage and current, N = 10, Ma = 0.85

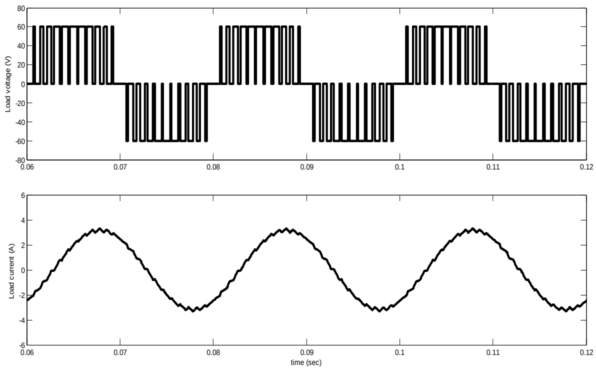

Fig. 6 Load voltage and current, N = 11, Ma = 0.85

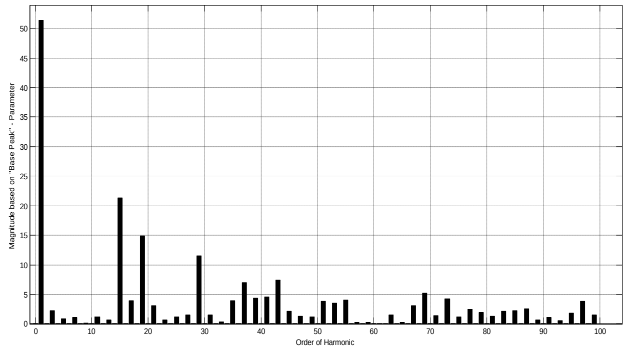

Fig. 7 spectrum analysis of load voltage, N = 7, Ma = 0.85

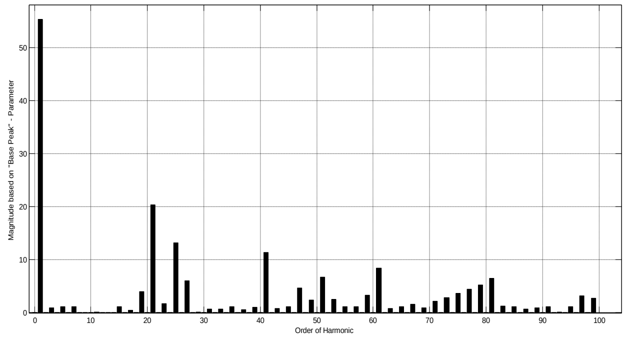

Fig. 8 spectrum analysis of load voltage, N = 10, Ma = 0.85

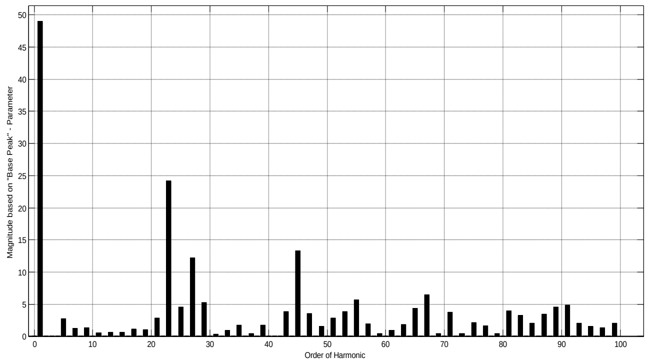

Fig. 9 spectrum analysis of load voltage, N = 11, Ma = 0.85

Table 2 Switching angles with different number of harmonics: Ma = 0.85

N	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9	θ_{10}	θ_{11}
3	30.45	54.28	67.1								
4	25.9	41.5	54.6	86.5							
5	22.6	33.6	46.6	68.5	75.0						
6	20	28.3	40.9	57.12	64.1	87.7					
7	18	24.4	36.6	49.1	56.5	74.8	79.1				
8	16.4	21.5	33.1	43.2	50.7	65.3	69.9	88.3			
9	15	19.2	30.2	38.5	46.1	58.1	62.9	78.2	81.5		
10	13.8	17.3	27.8	34.8	42.3	52.3	57.4	70.3	73.6	88.6	
11	12.8	15.8	25.8	31.6	39.1	47.7	52.8	63.8	67.4	80.4	83

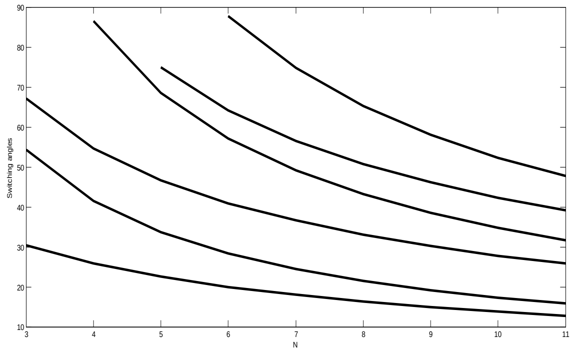

Fig. 10 variation of switching angles versus "N"

V. CONCLUSION

Selective harmonics elimination is achieved by adjusting the switching angles of the inverter. The switching angles have been estimated using the proposed algorithm which is based on secant method. The switching angles are utilized to generate the control signals of the inverter switches. Simulation results verify the effectiveness of the proposed algorithm. Selective harmonics in the inverter output are eliminated and the load is very close to sinusoidal shape. The proposed algorithm can be applied for any number of harmonics. A guide of the initial vector of switching angles is proposed to minimize the iterations especially with large number of harmonics.

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