

## **Heat Transfer on Steady Laminar Mixed Convective MHD Flow through an Inclined Channel**

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**Abstract:** We have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time. The expressions for the velocity field, the temperature field, the pressure drop, the friction factors and the Nusselt number at any plane parallel to the walls are obtained analytically. The effects of various emerging parameters on the velocity field, the temperature field, the pressure drop, the friction factors and the Nusselt number are discussed through graphically.

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### **I. INTRODUCTION**

The laminar mixed convection flow in an inclined channel has been the subject of much investigation due its possible application in many industrial and engineering processes. These include cooling of electronic equipment, heating of the Trombe wall system, gas cooling nuclear reactors geological system, agricultural engineering metallurgy, oil technology and others. The theory developed by viscous flow through porous media is useful in analyzing the influence of temperature and pressure on the flow of soil water. It is made use of in the energy extraction from geothermal regions. The flow of viscous fluid in an inclined channel with free surface has applications in coating to paper rolls, hydraulic engineering, and in the designs of drainage and irrigation canals. Until the eighteenth century the flow rate in a channel, the channels were related quantitatively on sound principles. A comparatively rapid development in the understanding of open channel phenomena has yielded a body of knowledge that is very successfully applied to the design of irrigation, flood control and sewerage systems as well as to the design of aqueducts, dam spillways and other structures in which water flows with a surface in contact with the atmosphere. Heat transfer in porous medium has several applications in the situations viz: nuclear waste disposal, geothermal energy extraction, fossil fuels detection, regenerator bed etc. Understanding the development of hydrodynamic and thermal boundary layer along with the heat transfer characteristics is the basic requirement to further investigate the problem extensively and more exhaustively. In many chemical processing industries generally slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. The slurry thus formed inside the reactor vessel often acts as a porous boundary for the next cycle of chemical processing. Flow in a porous medium can be considered as an ordered flow in a disordered geometry. The transport process of fluid through a porous medium involves two substances, the fluid and the porous matrix, and therefore it will be characterized by specific properties of these two substances. A porous medium usually consists of a large number of interconnected pores each of which is saturated with the fluid. The exact form of the structure is highly complicated and differs from one medium to other medium. A porous medium may be either an aggregate of a large number of particles such as sand or gravel or solid containing many capillaries as seen in a porous rock (laterite stone). When the fluid percolates through a porous material, because of the complexity of microscopic flow in the pores, the actual path of an individual fluid particle cannot be followed analytically. In all such cases, one has to consider the gross effect of the phenomena represented by a macroscopic view applied to the masses of fluid, large compared to the dimensions of the pore structure of the medium. The process can be described in terms of equilibrium of forces. The driving force necessary to move a specific volume of fluid at a certain speed through a porous medium is in equilibrium with the resistance force generated by internal friction between the fluid and the pore structure. Such a resistive force is characterized by Darcy's [1] semi - empirical law. The simplest model for flow through a porous medium is the one dimensional model derived by Darcy. From such empirical evidence, Darcy's law indicates that, for an incompressible fluid flowing through a channel filled with a fixed uniform and isotropic porous matrix, the flow speed varies linearly with longitudinal pressure variation. Fluids possessing visco elasticity had become important industrially. Specifically in polymer processing applications as well as in chemical industry, one deals with flow of visco elastic fluids. A classical example of such a liquid is poly iso - butylene. With the development of general constitutive equations for visco elastic fluids, it has been a point of great concern for Non-Newtonian fluids. All such proposed constitutive equations should in principle lead to the definition of flow properties that need to be measured to define the rheology and also to the development of the equivalent Navier Stokes equations for the solution of all possible boundary values along with initial value

problems that arises in several situations. Some of the analytical methods for complex flows of visco elastic fluids generally predict the nature of flow field and gives rise to more or less accurate solution though not a perfect solution. In all such situations, the methodology that is applied must be evolved and considered appropriately. It is pertinent to be quite specific about the experimental conditions applicable to the relevant phenomena. The inertia effects become all the more so important that in a sparsely packed porous medium and hence their effect on free convection problems needs to be investigated. The aim of the present investigation is, therefore, to study systematically the effect of inertial terms on combined free and forced convective heat transfer past a semi-infinite inclined plate embedded in a saturated porous medium with variable permeability, porosity and thermal conductivity under the influence of gravity. The results obtained under limiting conditions agree well with the existing ones and thus verify the accuracy of the method used. There is a fast growing belief that the many provocative experimental phenomena and dilemmas now have become a realistic possibility of being explained theoretically by Non-Newtonian Fluid Mechanics. An attempt is being made in this paper to illustrate such an optimistic thought in advocating visco elastic effect that occurs in several industrial and biological applications. The phenomenon of free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. Free convective flow past vertical plate has been studied extensively and more intensively by Ostrach [2], [3], and [4]. The transient free convection from a vertical flat plate has been examined by Siegel [5]. Although this problem is important in polymer processing applications Benenati and Brosilow [6] have shown that the permeability of a porous medium varies due to the variation of porosity from the wall to the interior of the porous medium. The problem of the exact solutions of two dimensional flows of a second order incompressible fluid has been examined by Pattabhi Ramacharyulu [7] by considering rigid boundaries while Kaloni [8] examined the fluctuating flow of a visco elastic fluid past an infinite porous plate subject to uniform suction. Thereafter, Merkin [9] investigated the mixed convection boundary layer flow on a semi-infinite vertical flat plate when the buoyancy forces aid and oppose the development of the boundary layer. In this study two series solutions were obtained, one of which is valid near the leading edge and the other is valid asymptotically. In the regions where the series solutions are not valid, numerical solutions were obtained by Lloyd and Sparrow [10]. The natural convection flows adjacent to both vertical and horizontal surfaces, which result from the combined buoyancy effects of thermal and mass diffusion, was first investigated by Gebhart and Pera [11] and Pera and Gebhart [12] while, Soundalgekar [13] investigated the situation of unsteady free convective flows wherein the effects of viscous dissipation on the flow past an infinite vertical porous plate was highlighted. In the course of analysis, it was assumed that the plate temperature oscillates in such a way that its amplitude is small. Oosthuizen and Hart [14] and Wilks [15] have carried out a numerical study of the combined forced and free convection flow over a vertical plate. Later, a linear analysis of the compressible boundary layer flow over a wall was presented by Lekoudis [16] et al while, Shankar and Sinha [17] studied the problem of Rayleigh for a wavy wall. Subsequently, Lessen and Gangwani [18] examined the effect of small amplitude wall waviness on the stability of the laminar boundary layer while, the free convective heat transfer on a vertical semi-infinite plate has been investigated by Berezovsky et al. [19]. The unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer was analysed by Soundalgekar et al [20] while for the same situation when the suction is variable was examined by Soundalgekar and Wavre [21]. The problem of free convection heat transfer from a vertical plate embedded in a fluid saturated porous medium is studied by Cheng and Minkowycz [22], who have obtained the similarity solutions for the problem considered. Murthy et al [23] had examined the dispersion effects due to a heated vertical flat plate. Subsequently, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a flat wall was examined by Vajravelu and Shastri [24]. Cheng [25] has provided an extensive review of early works on free convection in porous media while, Mucoglu and Chen [26] had examined the mixed convection flow over an inclined surface for both the assisting and the opposing buoyancy forces. The linearity between speed and pressure variation breaks down for large enough flow speed was presented by Mac Donald et al. [27]. Later, Chen et al [28] studied the combined effect of buoyancy forces from thermal and mass diffusion on forced convection. Merkin and Mahmood [29] have obtained the similarity solution of the mixed convection flow over a vertical plate for the constant heat flux case, while Tsuruno and Iguchi [30] have investigated the effects of the surface mass transfer on the mixed convection flow on a permeable vertical surface. Plumb and Huenefeld [31] have investigated non-Darcy natural convection from vertical isothermal surfaces in saturated porous media. This was emphasized later by Joseph et al [32] who stressed force modeled by the Frochheimer acts in a direction opposite to the velocity vector. A numerical and experimental investigation of the effects of the presence of a solid boundary and initial forces on mass transfer in porous media was presented by Vafai and Tien [33]. The laminar free convection from a vertical plate has been studied by Martynenko et al. [34]. In all their papers, the plate was assumed to be maintained at a constant temperature, which is also the temperature of the surrounding stationary fluid. Bejan and Poulikakos [35] have used Forchheimer's model to study vertical boundary layer natural convection in a porous medium. The steady

flow of an incompressible second grade fluid past an infinite porous plate subject to suction or blowing was investigated by Rajagopal and Gupta [36]. Chandrasekhar and Namboodiri [37] have shown the effectiveness of variable permeability of the porous medium on velocity distribution and heat transfer. Hong et al [38] have studied analytically the non-Darcian effects on a vertical plate natural convection in porous media. They used a combination of Rayleigh and Darcy numbers to describe the inertia and boundary terms and obtained similar solutions. They found that these effects decrease the velocity and reduce the heat transfer rate. Lai and Kulacki [39] have used both Darcy and non-Darcy models to study mixed convection from horizontal and vertical surfaces embedded in saturated porous media. Nakayama and Koyama [40] have obtained the similarity solution for the problem of free convection in the boundary layer adjacent to a vertical plate immersed in a thermally stratified porous medium. Ramachandran et al [41] have studied the mixed convection flow over vertical and inclined surfaces, theoretically as well as experimentally. Kumari et al [42] have investigated the non-Darcian effects on forced convection heat transfer over a flat plate in a highly porous medium. Subsequently, Ramanaiah and Malarvizhi [43] investigated the free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient. Wickern [44] has examined the influence of the inclination angle of the plate and the Prandtl number on the mixed convection flow over an inclined plate. Thereafter, Das and Ahmed [45] had studied the effects of thermal dispersion and dissipation effects on non-Darcy mixed convection problems and established the trend of heat transfer rate convection from a vertical plate in porous medium and investigated the flow and temperature fields. Hsieh et al [46] have obtained a non-similar solution for combined convection from vertical plates in porous media with variable surface temperatures or heat flux. Later, Knupp and Lage [47] analyzed the theoretical generalization to the tensor permeability case (anisotropic medium) of the empirically obtained Frochheimer extended Darcy unidirectional flow model. Hung and Chen [48] have studied non-Darcy free convection in a thermally stratified fluid saturated porous medium along a vertical plate with variable heat flux. It follows that, in multidimensional flow, the momentum equations for each velocity component derived by using the Frochheimer extended Darcy equation is atleast speculative while Patidar and Purohit [49] studied the free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls. Vighnesam and Soundalgekar [50] investigated the combined free and forced convection flow of water from a vertical plate with variable temperature. The transient free convection flow past an infinite vertical plate with periodic temperature variation was studied by Das et al [51]. Later, Kuznetsov [52] had investigated the effect of transverse thermal dispersion on forced convection in porous media and identified the situations favorable to heat transfer under dispersion effects. Thereafter, Mohammadien and El-Amin [53] studied the dispersion and radiation effects in fluid saturated porous medium on heat transfer rate for both Darcy and non-Darcy medium. In recent times, Hossain et al. [54] studied the influence of fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate. An explicit analytical technique namely homotopy analysis to solve the non-Darcy natural convection over a horizontal plate with surface mass flux and thermal dispersion was studied by Wang et al [55]. The wide range of its technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle on the surface acquires finite tangential velocity and hence "slips" along the surface. The flow regime is called the slip-flow regime and this effect cannot be neglected. Under the assumptions made by Sharma and Chaudhary [56] and Sharma and Sharma [57] have also discussed the free convection flow past a vertical plate in slip-flow regime. Several applications were cited in their papers that occur in several engineering applications wherein heat and mass transfer occurs at high degree of temperature differences. Subsequently, Taneja and Jain [58] had examined the problem of MHD flow with slip effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate. Recently, Ramana Murthy and Kulkarni [59] examined the problem of elastic-viscous fluid of second order type by causing disturbances in the liquid which was initially at rest and the bounding surface was subjected to sinusoidal oscillations. In view of the above studies, in this paper, we have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time.

## II. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

We consider the laminar flow of a Newtonian fluid in the gap between two infinitely-wide plane parallel walls. The flow is assumed to be parallel such that  $U$  has the only non-vanishing component along the  $X$ -axis. The axis orthogonal to the walls, the gravitational acceleration ' $g$ ' and the  $X$ -axis lie on the same plane. The latter condition ensures that the flow can be considered as two-dimensional, i.e. both the velocity field and the temperature field depend only on two spatial coordinates. A uniform magnetic field  $B_0$  is applied in the transverse direction to the flow. The electrical conductivity of the fluid is assumed to be small so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The external electric field is zero and the electric field due to polarization of charges is also

negligible. Heat due to Joule dissipation is neglected. The system under consideration is sketched in Fig. 1, where the chosen coordinate axes  $(X, Y)$  are drawn. Let us assume that the wall at  $Y = -L$  is kept isothermal with a constant temperature  $T_1$ , while the wall at  $Y = L$  is subjected to an oscillating temperature

$$T(X, L, t) = T_2 + \Delta T \cos(\omega t). \tag{1}$$

Moreover, heat flow is assumed to occur only in the transverse direction, so that  $\partial T / \partial X = 0$ . The latter assumption is conceivable since each wall is kept at a uniform temperature. The Boussinesq approximation is invoked, so that  $U$  is a solenoidal field and, as a consequence,  $\partial U / \partial X = 0$ . A steady mass flow rate is prescribed; therefore the average velocity in a channel section, defined as

$$U_0 = \frac{1}{2L} \int_{-L}^L U \, dy \tag{2}$$

is time independent.

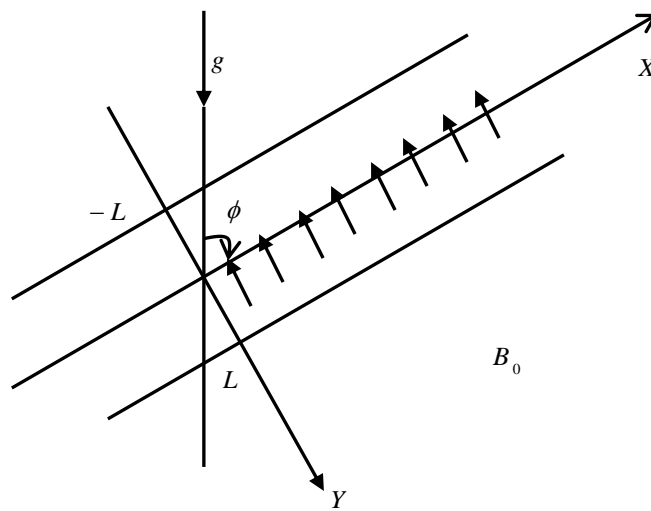


Fig.1 The Physical Model

The equation of state,  $\rho = \rho(T)$  considered as linear,

$$\rho = \rho_0 [ 1 - \beta (T - T_0) ] \tag{3}$$

Where  $T_0$  is an average temperature both with respect to the interval  $-L \leq Y \leq L$  and to the period  $0 \leq t \leq \frac{2\pi}{\omega}$ , namely

$$T_0 = \frac{\omega}{4\pi L} \int_0^{\frac{2\pi}{\omega}} dt \int_{-L}^L T \, dY \tag{4}$$

Since,  $\partial T / \partial X = 0$  the reference temperature  $T_0$  is a constant. According to the Boussinesq approximation, the momentum balance equation yields, along the  $X$  and  $Y$  axes,

$$\rho_0 \frac{\partial U}{\partial t} = \rho_0 g \beta (T - T_0) \cos \phi - \frac{\partial P}{\partial X} + \mu \frac{\partial^2 U}{\partial Y^2} - \sigma B_0^2 U \tag{5}$$

$$\rho_0 g \beta (T - T_0) \sin \phi + \frac{\partial P}{\partial Y} = 0. \tag{6}$$

Where  $P = p + \rho_0 g (X \cos \phi - Y \sin \phi)$ .

Differentiating equations (5) and (6) on both sides with respect to  $X$ , we obtain,

$$\frac{\partial^2 P}{\partial X^2} = 0. \tag{7}$$

$$\frac{\partial^2 P}{\partial X \partial Y} = 0 \tag{8}$$

It is easily verified that Equations (7) and (8) imply the existence of two functions  $A(Y, t)$  and  $B(t)$  such that

$$P(X, Y, t) = A(Y, t) - B(t) X. \tag{9}$$

The energy balance equation is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial Y^2}. \tag{10}$$

Let us define the dimensionless quantities

$$\theta = \frac{T - T_0}{\Delta T}, u = \frac{U}{U_0}, y = \frac{Y}{D}, \eta = \omega t, \lambda = \frac{D^2 B}{\mu U_0}, \Omega = \frac{\omega D^2}{\nu}, \text{Pr} = \frac{\nu}{\alpha}, \text{Re} = \frac{U_0 D}{\nu},$$

$$Gr = \frac{g \beta \Delta T D^3 \cos \varphi}{\nu^2}, \chi = \frac{T_2 - T_1}{\Delta T}, \xi = \frac{T_1 - T_0}{\Delta T}, M^2 = \frac{\sigma B_0^2 D^2}{\mu} \tag{11}$$

Where  $D = 4L$  is the hydraulic diameter. By employing Equations (9) and (11), Equations (5) and (10) can be rewritten as

$$\Omega \frac{\partial u}{\partial \eta} = \frac{Gr}{\text{Re}} \theta + \lambda + \frac{\partial^2 u}{\partial y^2} - M^2 u \tag{12}$$

$$\frac{\partial \theta}{\partial \eta} = \frac{1}{\Omega \text{Pr}} \frac{\partial^2 \theta}{\partial y^2} \tag{13}$$

The no slip condition at the walls implies that

$$u(-1/4, \eta) = 0 = u(1/4, \eta) \tag{14}$$

While the dimensionless thermal boundary conditions are

$$\theta\left(\frac{-1}{4}, \eta\right) = \xi \tag{15}$$

$$\theta\left(\frac{1}{4}, \eta\right) = \xi + \chi + \cos \eta \tag{16}$$

Equations (2) and (4) imply the following constraints on the functions  $u(y, \eta)$  and  $\theta(y, \eta)$  :

$$\int_{-1/4}^{1/4} u(y, \eta) dy = \frac{1}{2} \tag{17}$$

$$\int_0^{2\pi} d\eta \int_{-1/4}^{1/4} \theta(y, \eta) dy = 0 \tag{18}$$

Obviously, Equation (17) yields the further constraint

$$\int_{-1/4}^{1/4} \frac{\partial u(y, \eta)}{\partial \eta} dy = 0 \tag{19}$$

On account of Equation (19), an integration of Equation (12) with respect to  $y$  in the range  $[-1/4, 1/4]$  yields

$$\frac{\partial u}{\partial y} \Big|_{y=-1/4} - \frac{\partial u}{\partial y} \Big|_{y=1/4} = \frac{\lambda}{2} + \frac{Gr}{\text{Re}} \int_{-1/4}^{1/4} \theta(y, \eta) dy \tag{20}$$

The friction factors  $f_1$  and  $f_2$  at the walls  $Y = -L$  and  $Y = L$  respectively are defined as

$$f_1 = \frac{2\nu}{U_0^2} \frac{\partial U}{\partial Y} \Big|_{Y=-L} = \frac{2}{\text{Re}} \frac{\partial u}{\partial y} \Big|_{y=-1/4} \quad f_2 = \frac{-2\nu}{U_0^2} \frac{\partial U}{\partial Y} \Big|_{Y=L} = \frac{-2}{\text{Re}} \frac{\partial u}{\partial y} \Big|_{y=1/4} \quad (21)$$

On account of Equation (3.20), the friction factors and the parameter  $\lambda$  are related as follows:

$$(f_1 + f_2) \text{Re} = \lambda + 2 \frac{Gr}{\text{Re}} \int_{-1/4}^{1/4} \theta(y, \eta) dy. \quad (22)$$

**Analytical solution for steady periodic regime:**

Since Equations (12) – (18) are linear, one can define the complex valued functions namely the velocity  $u^*(y, \eta)$ , temperature  $\theta^*(y, \eta)$  and friction factor  $\lambda^*(\eta)$  which fulfill the equations

$$\Omega \frac{\partial u^*}{\partial \eta} = \frac{Gr}{\text{Re}} \theta^* + \lambda^* + \frac{\partial^2 u^*}{\partial y^2} - M^2 u^* \quad (23)$$

$$\frac{\partial \theta^*}{\partial \eta} = \frac{1}{\Omega \text{Pr}} \frac{\partial^2 \theta^*}{\partial y^2} \quad (24)$$

$$u^*(-1/4, \eta) = \xi \quad (25)$$

$$\theta^*(1/4, \eta) = \xi + \chi + e^{i\eta} \quad (26)$$

$$\int_{-1/4}^{1/4} u^*(y, \eta) dy = \frac{1}{2}, \quad \int_0^{2\pi} d\eta \int_{-1/4}^{1/4} \theta^*(y, \eta) dy = 0, \quad (27)$$

and are such that  $u = \text{Re}(u^*)$ ,  $\theta = \text{Re}(\theta^*)$ ,  $\lambda = \text{Re}(\lambda^*)$ .

In steady-periodic regime, a solution of Equations (27) can be written in the form

$$\begin{aligned} u^*(y, \eta) &= u_a(y) + \frac{Gr}{\text{Re}} u_b(y) e^{i\eta} \\ \theta^*(y, \eta) &= \theta_a(y) + \theta_b(y) e^{i\eta} \\ \lambda^*(\eta) &= \lambda_a + \frac{Gr}{\text{Re}} \lambda_b e^{i\eta}. \end{aligned} \quad (28)$$

Due to the linearity of Equations (27), if we substitute Equations (28) into Equations (27) we obtain two distinct boundary value problems. The first is given by

$$\frac{d^2 u_a}{dy^2} + \frac{Gr}{\text{Re}} \theta_a + \lambda_a - M^2 u_a = 0 \quad (29)$$

$$\frac{d^2 \theta_a}{dy^2} = 0 \quad (30)$$

$$u_a(-1/4) = 0 = u_a(1/4) \quad (31)$$

$$\theta_a(-1/4) = \xi, \quad \theta_a(1/4) = \xi + \chi$$

$$\int_{-1/4}^{1/4} u_a(y) dy = \frac{1}{2}, \quad \int_{-1/4}^{1/4} \theta_a(y) dy = 0 \quad (32)$$

While the second one is given by

$$\frac{d^2 u_b}{dy^2} - i\Omega u_b + \lambda_b - M^2 u_b = 0 \quad (33)$$

$$\frac{d^2 \theta_b}{dy^2} - i\Omega \text{Pr} \theta_b = 0 \quad (34)$$

$$u_b(-1/4) = 0 = u_b(1/4), \tag{35}$$

$$\theta_b(-1/4) = 0, \theta_b(1/4) = 1$$

$$\int_{-1/4}^{1/4} u_b^*(y) dy = 0, \theta_a(y) = 2 \chi y \tag{36}$$

Solving the Equations making use of corresponding boundary conditions, we get

$$u_a = - \frac{\lambda a}{M^2 \cosh(\frac{m}{4})} \cosh My - \frac{Gr \chi \sinh my}{2 Re M^2 \sinh(\frac{m}{4})} + \frac{2Gr \chi}{Re M^2} y + \frac{\lambda a}{M^2} \tag{37}$$

Where,  $\lambda_a = - \frac{M^2}{(4 \tanh(\frac{M}{4}) - \frac{1}{M^2})}$ .

and  $\theta_b(y) = \frac{\cosh(\sigma y)}{2 \cosh(\sigma / 4)} + \frac{\sinh(\sigma y)}{2 \sinh(\sigma / 4)}$  (38)

$$u_b(y) = \frac{\cosh NY}{2 \cosh(N / 4)} \left[ \frac{1}{\sigma^2 - N^2} - \frac{2\lambda_b}{N^2} \right] + \frac{\sinh Ny}{2 \sinh(N / 4)(\sigma^2 - N^2)} - \frac{1}{2(\sigma^2 - N^2)} \left\{ \frac{\cosh(\sigma y)}{\cosh(\sigma / 4)} + \frac{\sinh(\sigma y)}{\sinh(\sigma / 4)} \right\} + \frac{\lambda b}{N^2} \tag{39}$$

where  $\lambda_b = \frac{N^2}{(\sigma^2 - N^2)}$ .

By employing Equation (11), the heat flux per unit area can be written as

$$q = k \frac{\partial T}{\partial Y} = \frac{k \Delta T}{D} \frac{\partial \theta}{\partial y} \tag{40}$$

In analogy with the literature (10) – (13), we will define a dimensionless heat flux per unit area, called Nusselt number, as follows:

$$Nu = \frac{qD}{k \Delta T} = \frac{\partial \theta}{\partial y} = Re \left( \frac{\partial \theta^*}{\partial y} \right) = Re(Nu^*), \tag{41}$$

Where  $Nu^*$  is defined as

$$Nu^* = \frac{\partial \theta_a}{\partial y} + \frac{\partial \theta_b}{\partial y} e^{in} = Nu_a + Nu_b e^{in} \tag{42}$$

Finally, Equations (37) and (39) allow one to express the quantities  $f_1 Re$  and  $f_2 Re$  as

$$f_1 Re = 2 Re \left( \frac{\partial u_a}{\partial y} \Big|_{y=-1/4} + \frac{Gr}{Re} \frac{\partial u_b}{\partial y} \Big|_{y=-1/4} e^{in} \right) = Re(f_{1a} Re + \frac{Gr}{Re} f_{1b} Re e^{in}) \tag{43}$$

$$f_2 Re = -2 Re \left( \frac{\partial u_a}{\partial y} \Big|_{y=1/4} + \frac{Gr}{Re} \frac{\partial u_b}{\partial y} \Big|_{y=1/4} e^{in} \right) = Re(f_{2a} Re + \frac{Gr}{Re} f_{2b} Re e^{in}) \tag{44}$$

where  $f_{1a} Re, f_{1b} Re, f_{2a} Re$  and  $f_{2b} Re$  are given by

$$f_{1a} Re = 2 \frac{\partial u_a}{\partial y} \Big|_{y=-1/4}, \quad f_{1b} Re = 2 \frac{\partial u_b}{\partial y} \Big|_{y=-1/4}, \tag{45}$$

$$f_{2a} Re = -2 \frac{\partial u_a}{\partial y} \Big|_{y=1/4}, \quad f_{2b} Re = -2 \frac{\partial u_b}{\partial y} \Big|_{y=1/4}, \tag{46}$$

$$f_{1a} Re = - \frac{2\lambda_a}{M} \tanh(M / 4) - \frac{Gr \chi}{Re M} C Oth(M / 4) - 4 \frac{Gr \chi}{Re M^2}$$

$$f_{2a} \text{Re} = -\frac{2\lambda_a}{M} \tanh(M/4) + \frac{Gr \chi}{\text{Re} M} \text{Coth}(M/4) - 4 \frac{Gr \chi}{\text{Re} M^2}$$

$$f_{1b} \text{Re} = -N \tanh(N/4) \left( \frac{1}{(\sigma^2 - N^2)} - \frac{2\lambda_b}{N^2} \right) + \frac{N \coth(N/4)}{\sigma^2 - N^2} - \frac{\sigma}{\sigma^2 - N^2} \{-\tanh(\sigma/4) + \coth(\sigma/4)\}$$

$$f_{2b} \text{Re} = -N \tanh(N/4) \left( \frac{1}{(\sigma^2 - N^2)} - \frac{2\lambda_b}{N^2} \right) - \frac{N \coth(N/4)}{\sigma^2 - N^2} + \frac{\sigma}{\sigma^2 - N^2} \{\tanh(\sigma/4) + \coth(\sigma/4)\}$$

### III. RESULTS AND DISCUSSIONS

We have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time. The velocity field, temperature field, the pressure drop the friction factor and the Nusselt number obtained so far are discussed through graphically for various values of the physical parameters namely the Prandtl number  $Pr$ , angular frequency  $\Omega$ , Hartmann number  $M$ .

Figs. (2 – 3) represents the effects of Prandtl number  $Pr$ , angular frequency  $\Omega$  and Hartmann number  $M$  on the velocity  $|u_b|$  (amplitude of the velocity oscillations). Fig.(2) shows the effect of  $Pr$  on the  $|u_b|$  with  $r = 10$  and  $M = 1$ . It is found that, for  $Pr = 0.7$ ,  $|u_b|$  is exactly symmetric with respect to the midplane of the channel on the other hand, for  $Pr = 7$  and  $Pr = 100$ , the  $|u_b|$  is not symmetric with respect to the midplane of the channel. Fig.(3) depicts the effect of  $\Omega$  on  $|u_b|$  with  $Pr = 7$  and  $M = 1$ . It is observed that, for  $\Omega = 1$ ,  $|u_b|$  is symmetric with respect to the midplane of the channel, whereas for  $\Omega = 10$  and  $\Omega = 100$ , the  $|u_b|$  is not symmetric with respect to the midplane of the channel. Fig.(4) presents the effect of Hartmann number on  $|u_b|$  with  $Pr = 7, \Omega = 10$ . It is observed that,  $|u_b|$  decreases with increasing  $M$  at any position except at the else, where velocity oscillations cannot occur.

Fig.(5) shows the effect of  $Pr$  on the  $|\lambda_b|$  as a function of  $\Omega$  with  $M = 1$ . It is observed that, the friction factor  $|\lambda_b|$  decreases with increase of  $\Omega$ . Further it is observed, the  $|\lambda_b|$  decreases with an increase in  $Pr$ . The effect of Hartmann number  $M$  on  $|\lambda_b|$  as a function of  $\Omega$  with  $Pr = 0.7$  is shown in Fig.(6). It is found that, the  $|\lambda_b|$  decreases with an increase in  $M$ . Fig.6(a) represents enlargement of Fig.(6).

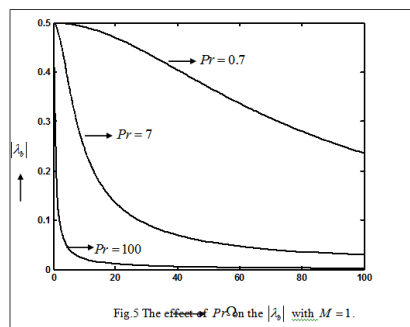
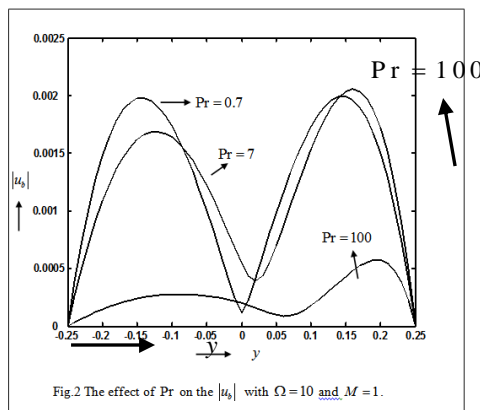
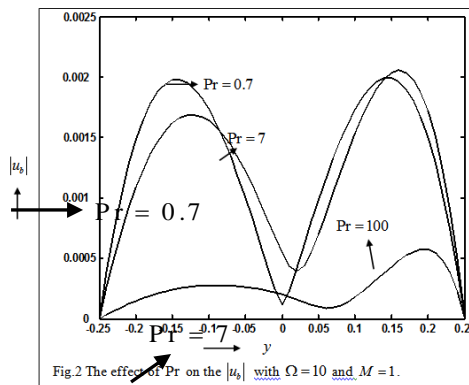
Fig.(7). shows the effect of  $Pr$  on the amplitude of oscillations of  $|f_{1b} \text{Re}|$  with  $M = 1$ . It is observed that, the  $|f_{1b} \text{Re}|$  decreases with increasing  $Pr$ . Fig.(8) depicts the effect of  $M$  on the  $|f_{1b} \text{Re}|$  with  $Pr = 7$ . It is found that, the  $|f_{1b} \text{Re}|$  decreases with an increase in  $M$ . Fig.(9) presents the effect of  $Pr$  on the amplitude of the  $|f_{2b} \text{Re}|$  (resonance frequency for the oscillations of the friction factor at the wall  $Y = L$ ) with  $M = 1$ . It is observed that, with increase in the  $Pr$ , decreases the  $|f_{2b} \text{Re}|$ . The effect of  $M$  on the  $|f_{2b} \text{Re}|$  with  $Pr = 7$  is shown in Fig.(10). It was found that, the  $|f_{2b} \text{Re}|$  decreases with increasing  $M$ .

The modules of  $Nu_b$ , the steady-temperature wall ( $Y = -0.25$ ), at  $Y = -0.05$ , and at the midplane of the channel ( $Y = 0$ ) is plotted versus  $\Omega Pr$  in Fig.(11), in the range  $0 \leq \Omega Pr \leq 400$ . As is illustrated by this figure, the amplitude of the oscillations of the Nusselt number is a decreasing function of  $\Omega Pr$  in the whole interval  $-0.25 \leq y \leq 0$ . On the contrary, is the open interval  $0 \leq y \leq 0.25$ , for every value of  $y$  there exists a value of  $\Omega Pr$  which maximizes the modules of  $Nu_b$  i.e. there exists a reasonable frequency for the fluctuation of the Nusselt number which is proportional to the inverse of  $Pr$ . In this interval, the value of  $\Omega Pr$  which maximizes the modules of  $Nu_b$  is an increasing function of  $y$ . Finally, at the right wall, the modules of  $Nu_b$  are an increasing function of  $\Omega Pr$ , and no resonance occurs. These phenomena are illustrated in fig.(12) & (13). In Fig.(12), plots of the modulus of  $Nu_b$  versus  $\Omega Pr$  are reported in the range  $0 \leq \Omega Pr \leq 1200$ ,

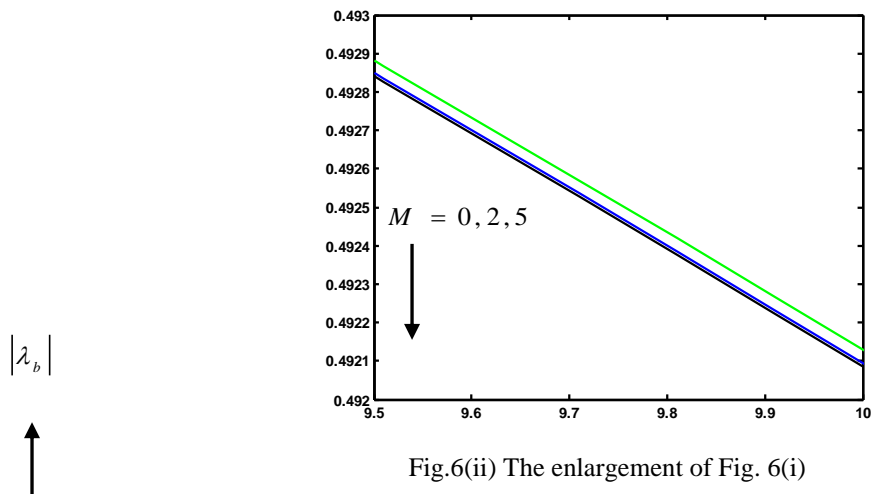
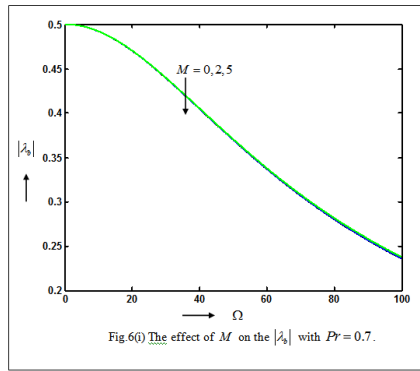


for  $y = 0.1, 0.17$  &  $0.2$ . For the plots reported in Fig.(12) the resonance frequencies correspond to  $\Omega Pr = 93.61, 312.7$  and  $800.0$ . In Fig.(13), plots of the modules of  $Nu_b$  versus  $\Omega Pr$  are reported in the range  $0 \leq \Omega Pr \leq 40,000$ , for  $y = 0.22, 0.24$  &  $0.25$  (right wall). The first plot presents a resonance frequency for  $\Omega Pr = 2222$ , the second presents a resonance frequency for  $\Omega Pr = 20,000$ , while the third presents no resonance.

Fig.(14) shows the effect of  $\sigma Pr$  as the amplitude of temperature oscillations  $|\theta_b|$ . The figure shows that, for  $\Omega Pr \leq 10$ , the amplitude of the temperature oscillations looks like a line as function of  $y$  on the other hand, when the value of  $\Omega Pr$  becomes higher and higher, the temperature oscillations land to be sensible only in a narrow region of the channel adjacent to the wall  $Y = L$ .

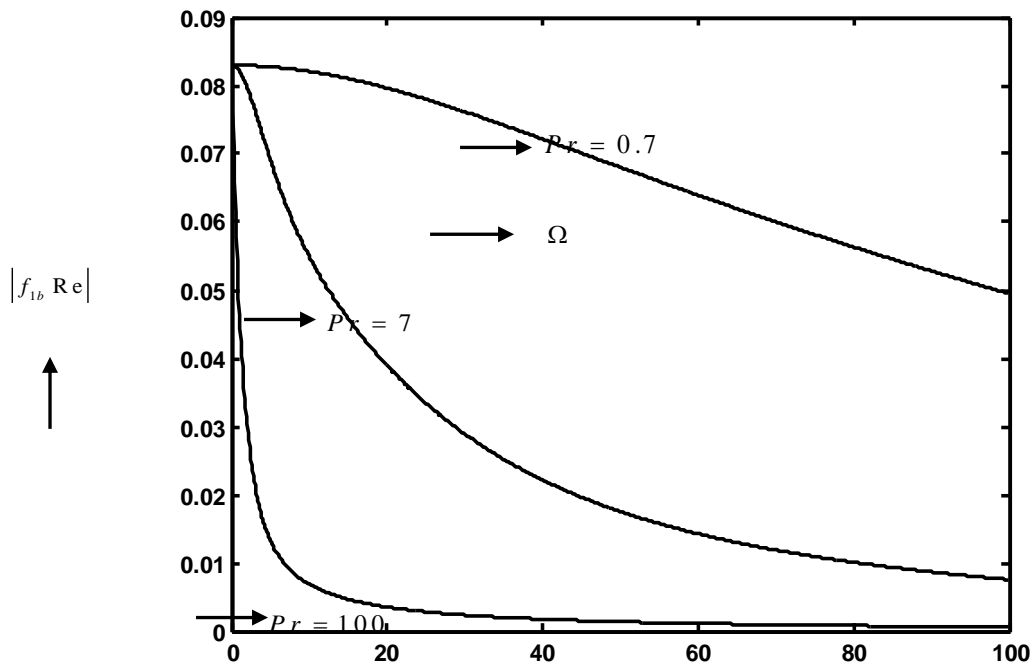


$|u_b|$   
↑



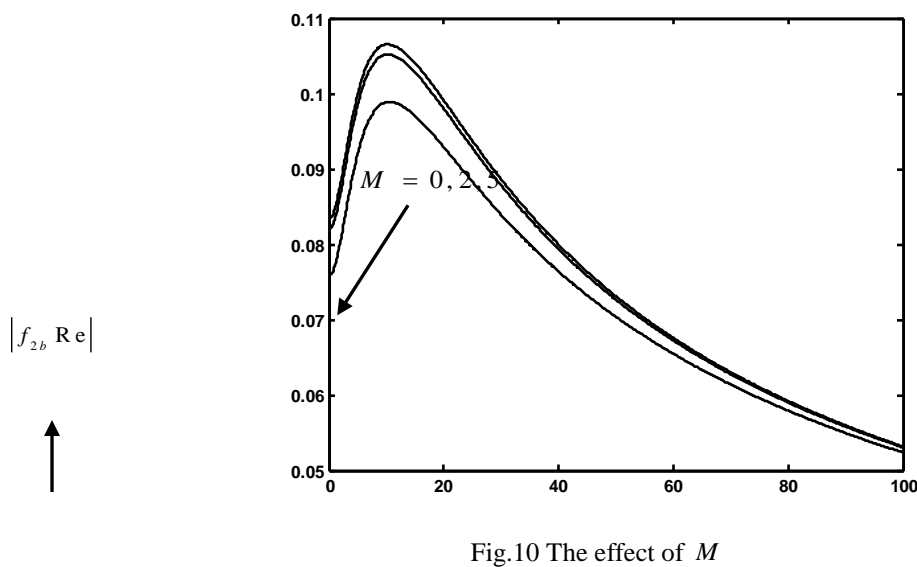
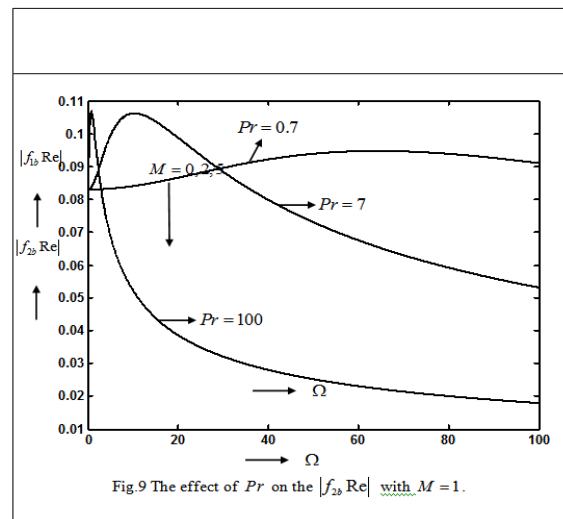
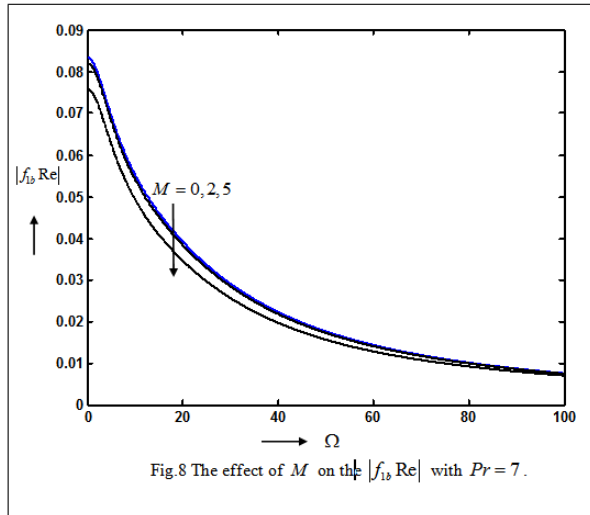
$|\lambda_b|$   
↑

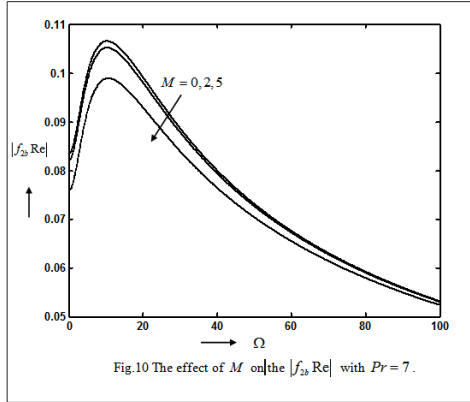
Fig.6(ii) The enlargement of Fig. 6(i)



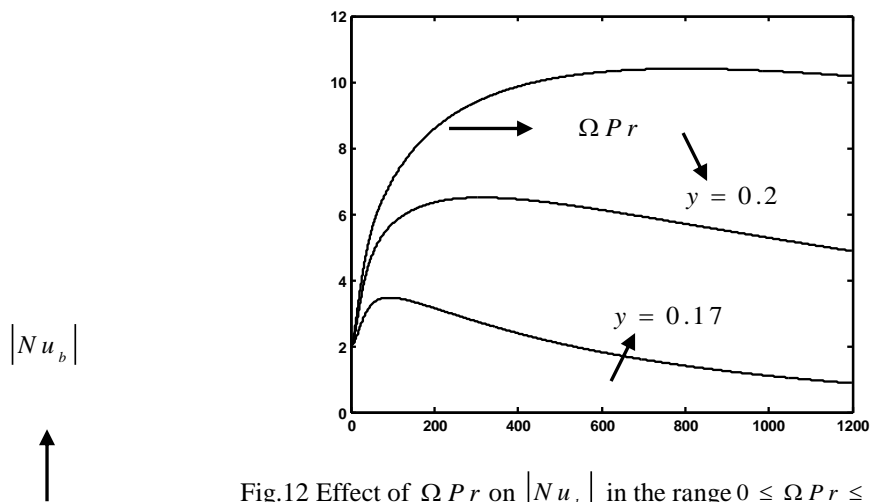
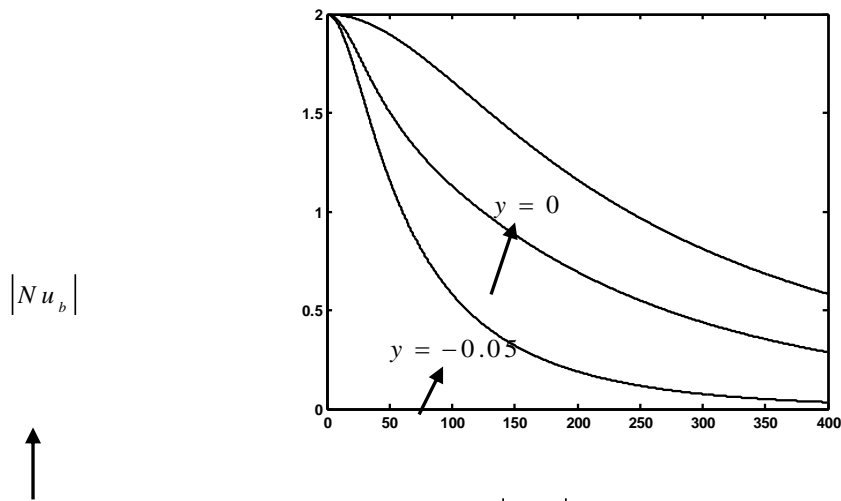
$|f_{1b} Re|$   
↑

Fig.7 The effect of  $Pr$  on the  $|f_{1b} Re|$  with  $M = 1$ .  
→  $\Omega$





the  $|f_{2b} Re|$  with  $Pr = 7$ .



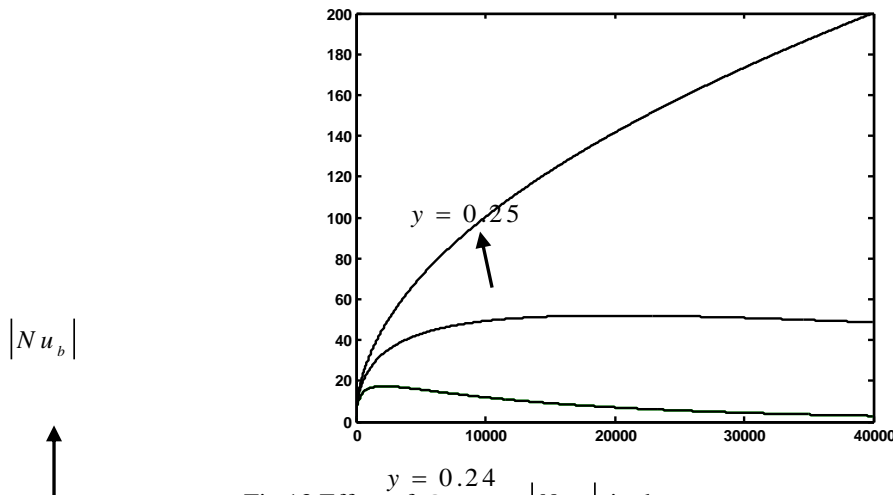


Fig.13 Effect of  $\Omega Pr$  on  $|Nu_b|$  in the range  $0 \leq \Omega Pr \leq 40000$  for  $y = 0.22, 0.24$  and  $0.25$ .

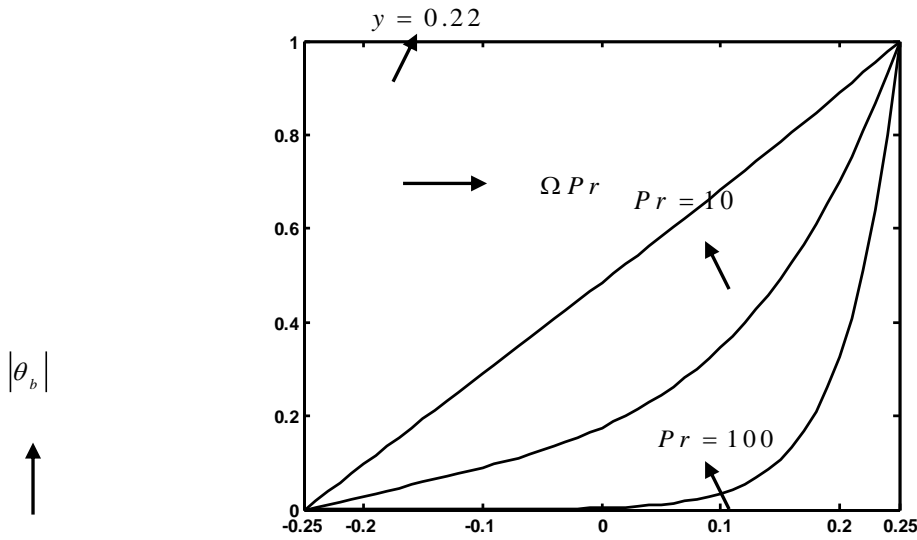


Fig.14 Effect of  $\Omega Pr$  on  $|\theta_b|$  for  $\Omega Pr = 10, 100$  and  $1000$ .

#### IV. CONCLUSIONS

→ y

We have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time. The conclusions are made as the following.

- [1]. The amplitude of the velocity oscillations is exactly symmetric with respect to the midplane of the channel for  $Pr=0.71$  on the other hand, the same is not symmetric with respect to the midplane of the channel for  $Pr=7$ .
- [2]. The friction factor  $|\lambda_b|$  decreases with increase of angular frequency  $\Omega$  and Hartmann number  $M$ .
- [3]. The resonance frequency for the oscillations of the friction factor decreases with increase in Prandtl number  $Pr$  and Hartmann number  $M$ .

- [4]. The amplitude of the temperature oscillations  $|\theta_b|$  looks like a line as fraction  $y$  on the other hand when the value of  $\Omega Pr$  becomes higher and higher the temperature oscillations land to be sensible only in a narrow of the channel adjacent to the wall.

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