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A Quantum Finance Model for Technical Analysis in the Stock Market

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Abstract: This paper, is a contribution to quantum finance theory. The time-dependent Schrodinger wave equation for the harmonic oscillator was used to model the movement of stocks in a daily price-limited stock market. Using the Nigeria Stock Exchange(NSE) as a case study, the "price wave" function was developed. From this, given any quoted stock, the rate of return and the investment risk measure (standard deviation)of the corresponding stock can be computed in a continuous manner. This is an improvement over earlier computational methodsuch as arithmetic and logarithmic rate of returnwhich are discrete and do not provide means for the computation of standard deviation indicator.

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I. INTRODUCTION

Stocks are risky financial assets because their prices are subjected to unpredictable behaviours. This may be attributable to what makes stocks trading attractive to aggressive or risk appetite investors who seek to gain from the price fluctuations by pursuing the old advice to "buy low and sell high". Most stock exchanges all over the world, have indexes that represent some sort of average behaviour of the contemporary stock market.

Although stock prices may vary in a rather unpredictable way,this indicates that they can be described in a stochastic manner. With the knowledge of quantum mechanics, we shall treat a single stock in the equity market as a quantum harmonic oscillator, where the stock price rate of return is presumed to oscillate and damp in a quantum harmonic oscillator potential well. The undeterministic prices of stocks in the market presents its probabilistic behaviour while its fluctuation in the market in turn presents its quantum wave property.In this study, we employed the time-dependent Schrodinger wave equation for the harmonic oscillator to construct the "price wave function" in a daily price-limited stock market – the Nigerian Stock Exchange was used as a case study.

In the numerical interpretation of the harmonic wave function, the stock price rate of return is described by a wave function $\Psi(x,t)$, and $|\Psi(x,t)|^2$ is the probability density function of the stock price at time, t from which we can estimate the expected value of the rate of return and other volatility indicators to aid technical analysis in stock markets.

1.2 Quantum Harmonic Oscillator

The harmonic oscillator, from both the quantum mechanical and classical perspectives, provides set of general solutions which contribute to the modelling and understanding of many oscillatory systems(Cresser, 2005). The mathematical form of the harmonic wave motion may be represented in terms of either harmonic sine, cosine function or exponential function:

$$\Psi(x,t) = A\sin(kx - \omega t), \quad \Psi(x,t) = A\cos(kx - \omega t) \quad \text{or} \quad Ae^{i(kx-\omega t)} \tag{1}$$

Where $k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v}$ and $\omega = \frac{2\pi}{T}$ and A is constant; and ω is the angular frequency. Thus, the velocity of the wave is given by $\frac{\omega}{k}$.

By using Heisenberg's uncertainty principle in the form $\Delta x \Delta p \approx \hbar$, it is also possible to estimate the lowest possible energy level (ground state) of a simple harmonic oscillator. The total energy for an undamped oscillator is the sum of its kinetic energy and potential energy given by:

$$E = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \tag{2}$$

Thus, from the total energy of the undamped oscillator given in equation (2) above, the simple harmonic oscillator potential can be given as, $V(x) = \frac{1}{2}m\omega^2x^2$, where m is the mass of the oscillator and ω is its natural frequency of oscillation.

1.3 The Schrodinger Equation

The Schrodinger equation, often called the Schrodinger wave equation is the fundamental equation of Physics for describing quantum mechanical behaviour (Griffiths, 2004). There are basically two variants of Schrodinger's equation – time-dependent Schrodinger and the time-independent Schrodinger equation. When the Schrodinger equation for the harmonic oscillator is solved, we can determine what the wave function of the particle will look like at a future time, therefore weareable to determine the distribution of the particle's position, momentum, and other properties that may be required.

The one-dimensional time-dependent Schrodinger equation is given as:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{dx^2}+V(x)\Psi(x,t)=i\hbar\frac{\partial\Psi(x,t)}{\partial t}, \quad (3)$$

where, $\hbar^2 \approx 6.6260693\times 10^{-34} J \times sec$ or $6.6260693\times10^{-34} m^2$ kg / s is the Plank's constant, m is the mass of particle, $\Psi(x,t)$ is the wave function, $V(x)=\frac{1}{2}m\omega^2x^2$ is the potential energy of particle, and E is the total energy of particle. On the other hand, the time-independent Schrodinger equation is given as;

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{dx^2}+V(x)\Psi(x,t)=E\Psi(x,t) \quad (4)$$

1.4 The Random Nature of Stock Prices

Stock or equity represents a small portion of a firm's valuation; hence the stock price should somehow reflect the overall net worth of the firm. However, the present value of a firm depends not only on the firm's current position but also on its future growth and performance. It is therefore obvious that the future price of a stock will always be subjected to a certain degree of uncertainty. This is demonstrated by the typical erratic behaviour that stock prices show as contained in the All-Share index in Fig.1, below.

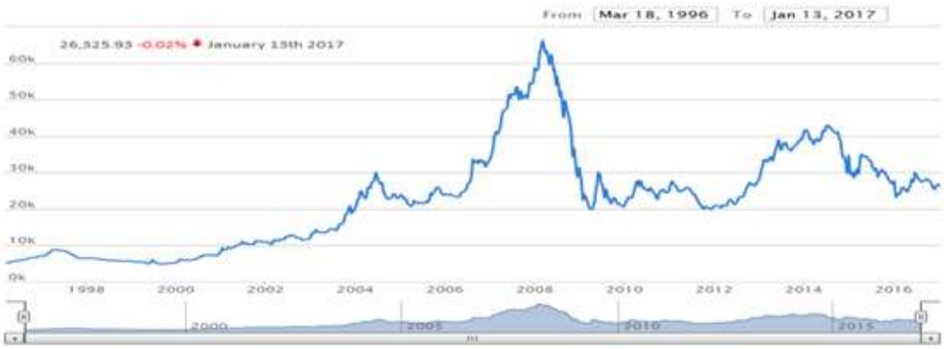

Fig. 1: The NSE All-Share Index (NSE, 2017).

Although stock prices may vary in a rather unpredictable way, the random nature of stock market prices has been studied since the early work of Bachelier (Barrett & Wright, 1974), who suggested that the price movements may be described by what is now known as Brownian motion. The first theory of Brownian motion was developed to model financial asset prices.

In the Nigerian Stock Exchange (NSE) Rule Book, Rule 15.29 (NSE, 2015)states that securities shall trade in price increments of one (1) kobo (10^{-2} Naira) and the price movements and price limits on any given trading day shall be $\pm10\%$. In other words, the rate of return in any trading day cannot exceed $\pm10\%$ in relation to the previous day's closing price. Thus, the fluctuation of the stock price takes place between the price band in a one-dimensional harmonic oscillator motion.

II. THE MODEL

In order to describe the quantum nature of the stock, we build the stock model on the basic hypotheses or principles of quantum mechanics, assuming the individual stock as a quantum system. The mathematical form for the harmonic wave motion is given as $\Psi(x,t)$ and this wave function, contains the probabilistic properties of the behaviour of the stock price. Due to the wave property of the stock price, the wave function $\Psi(x,t)$ thus represents the price distribution, where x denotes the stock price and t is the time. The state of the stock price before trading can be considered as a wave packet or a distribution (Piotrowski & Sladkowski, 2003), which in other wordsinfers the superposition of its various possible states with distinct prices.

III. SETTING UP THE MODEL

We shall consider the one-dimensional time-dependent Schrodinger equation as given in equation (3);

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{dx^2}+\frac{1}{2}m\omega^2x^2\psi(x,t)=i\hbar\frac{\partial\psi(x,t)}{\partial t}, \quad (5)$$

describing the "price" particle in a quantum harmonic oscillator motion with mass, m in the interval $0<x<L$ and $t>0$ under the influence of the potential, $V(x)$.

Subject to the following boundary and initial conditions respectively:

1. $\psi(0,t)=\psi(L,t)=0, \quad 0<x<L, \quad t>0$ (6)
2. $\psi(x,0)=f(x)=\sqrt{\frac{2}{5L}}\sin\left(\frac{n\pi x}{L}\right)$ (7)

Where, $|\psi(x,t)|^2$ is the probability density function of the wave function which needs to be normalized to unity, and

Solving analytically by separation of variables method, we obtain the wave function $\psi(x,t)$ in implicit form and the function is used to evaluate the values of the properties required. Assume a solution in the form:

$$\psi(x,t)=X(x)T(t) \quad (8)$$

Thus, the general solution for equation (5) is;

$$X(x)=C_1Cos\left(\frac{\lambda}{\hbar}\right)x+C_2Sin\left(\frac{\lambda}{\hbar}\right)x \quad (9)$$

where C_1 and C_2 are arbitrary constants.

Applying the boundary conditions, it follows that, since $C_2\neq 0$, for non-trivial solutions, this gives,$\lambda=\frac{n\pi\hbar}{L}, n=1,2,...$,which are called the eigenvalues of the problem.

Thus, the corresponding eigenfunctions are,

$$X_n(x)=C_{2n}Sin\left(\frac{n\pi x}{L}\right), \quad n=1,2,... \quad (10)$$

Next, we have,

$$T_n(t)=C_{3n}e^{\alpha t}, \; n=1,2,... \quad (11)$$

where $\alpha=\frac{m^2\omega^2x^2+\lambda^2}{i2m\hbar}$.But $\lambda=\frac{n\pi\hbar}{L}, \Rightarrow \lambda^2=\frac{n^2\pi^2\hbar^2}{L^2}$. i.e.,$\alpha=\frac{m^2\omega^2x^2L^2+n^2\pi^2\hbar^2}{i2m\hbar L^2}$

Employing equations (10) and equation (11) into (8) yields,

$$\psi_n(x,t)=X_n(x)T_n(t)=A_ne^{\alpha t}Sin\left(\frac{n\pi x}{L}\right), \quad n=1,2,... \quad (12)$$

where $A_n=C_{2n}C_{3n}$.By superimposition principle, we have,

$$\psi(x,t)=\sum_{n=1}^{\infty}A_ne^{\alpha t}Sin\left(\frac{n\pi x}{L}\right) \quad (13)$$

Hence, the required particular solution of the problem in (12), is given by;

$$\psi_n(x,t)=\sum_{n=1}^{\infty}A_n\,e^{-\frac{i}{\hbar}E_nt}sin\left(\frac{n\pi x}{L}\right) \quad (14)$$

which is the required wave function. Where$E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}$, is the allowed energy.

Next, in order to obtain the constant, A_n, we apply the initial condition, $\psi(x,0)=f(x)$ which gives;

$$\psi(x,0)=\sum_{n=1}^{\infty}A_n\,e^{-\frac{i}{\hbar}E_n(0)}sin\left(\frac{n\pi x}{L}\right)=f(x) \quad (15)$$

i.e.,$f(x)=\sum_{n=1}^{\infty}A_n\,sin\left(\frac{n\pi x}{L}\right)$, which is a Fourier sine series for $f(x)$.

Thus, $A_n=\sqrt{\frac{2}{5L}}=A, say.$

It follows that the required particular solution of the problem is therefore given by,

$$\psi(x,t)_n=\sum_{n=1}^{\infty}\sqrt{\frac{2}{5L}}e^{-\frac{i}{\hbar}E_nt}sin\left(\frac{n\pi x}{L}\right) \quad (16)$$

which is the required wave function and eq. (16) is the required Quantum Model for the price wave function.

Thus, if the stock price rate of return is described by the wave function $\psi(r,t)$ of equation (16), then the probability density function of the stock price rate of return at time t, is given as;

$$P_n(t)=\int_0^L|\psi(r,t)_n|^2dr, \quad (17)$$

which describes the probability of the stock price rate of return between 0 and L at time, t.

In order to ensure that the time component is retained, we have,

$$\int_0^L|\psi(r,t)|^2dr=\int_0^L\left|A_ne^{-\frac{i}{\hbar}E_nt}sin\left(\frac{n\pi r}{L}\right)\right|^2dr$$

$$=\int_0^L\left|A_n\left[cos\left(\frac{1}{\hbar}E_nt\right)-isin\left(\frac{1}{\hbar}E_nt\right)\right]sin\left(\frac{n\pi r}{L}\right)\right|^2dr \quad (18)$$

It follows that, the real part of the Model can be written as;

$$\Re(\psi_n(r,t))=A_ncos\left(\frac{1}{\hbar}E_nt\right)sin\left(\frac{n\pi r}{L}\right)=\sqrt{\frac{2}{5L}}cos(E_nt)sin\left(\frac{n\pi r}{L}\right) \quad (19)$$

Here, we assume that the Planck constant, $\hbar = 1$(Becerril, 2008).

3.1 The Probability Distributions of Stock Rate of Return

The probability distributions which is dependent on time, t corresponding to the wave function obtained above in equation (19) are;

$$P_n(r,t) = |\psi(r,t)_n|^2 = \begin{cases} \left|\sqrt{\frac{2}{5L}}cos(E_n t)sin\left(\frac{n\pi r}{L}\right)\right|^2; & 0 < r < L \\ 0; & r < 0, r > L \end{cases} \quad (20)$$

Which is the probability density function. Hence, when $n = 1$, i.e., at the ground state of the harmonic oscillator, we have;

$$P_1(r,t) = |\psi(r,t)_1|^2 = \left|\sqrt{\frac{2}{5L}}cos(E_1 t)sin\left(\frac{\pi r}{L}\right)\right|^2 \quad (21)$$

3.2 The Average Rate of Return and Uncertainties of Stock Using the Model

We want to use the model to estimate the expectation values and the uncertainty of "price waves" using the wave function which provides a metric to assess the volatility and relative risk to potential investors. The table below displays the trading activities as at 03/07/2017 on the floor of the Nigerian Stock Exchange of three securities selected from among the lowest to the highest capitalisation.

STOCKS	PCLOSE	OPEN	HIGH	LOW	CLOSE	CHANGE	% CHANGE	DEALS	VOLUME	VALUE
AIICO	0.60	0.60	0.62	0.59	0.62	0.02	0.0333	25	1,538,420	933,198
NB	161.00	159.99	161.05	159.99	161.05	0.05	0.0003	91	420,750	67,479,195
MOBIL	250.02	237.53	237.53	237.53	237.53	-12.49	-0.0500	14	24,385	5,795,468

Table 1: Sample of 3 equities with daily trading data for 03/07/2017 on the floor of the NSE(The Guardian, 2017).

STOCKS	PCLOSE	OPEN	HIGH	LOW	CLOSE	CHANGE	% CHANGE	DEALS	VOLUME	VALUE
ACCESS	10.10	10.07	10.14	10.01	10.01	-0.09	-0.89	156.00	4,988,656	50,329,342.73
DSUGAR	15.50	15.83	15.66	15.21	15.46	-0.04	-0.26	67.00	1,149,138.00	17,766,228.42
GTB	42.03	41.39	42.10	41.39	42.10	0.07	0.17	207.00	5,278,500.00	221,820,122.50
UBA	9.75	9.80	9.84	9.76	9.80	0.05	0.51	133.00	6,726,404.00	65,875,525.02
ZENITH	25.80	25.65	25.65	25.51	25.58	-0.22	-0.85	246.00	6,899,958.00	176,515,085.20

Table 2: Sample of 4 equities with daily trading data for 08/11/2017 on the floor of the NSE(Capital Assets, 2017).

The summary of results from the table 2, is displayed in the table below:

STOCK	PCLOSE	CLOSE	RATE OF RETURN (r)			ERROR	INV. RISK
			ARITHMETIC	LOGARITHMIC	MODEL	|LOGARITHMIC – MODEL|	%
AIICO	0.60	0.62	0.0333	0.0328	0.0317	0.0011	2.83
NB	161.00	161.05	0.0003	0.0003	0.0003	0.0000	0.02
MOBIL	250.02	237.53	-0.0500	-0.0512	-0.0498	0.0014	3.29
ACCESS	10.10	10.07	-0.0030	-0.0030	-0.0029	0.0001	0.21
DSUGAR	15.50	15.83	0.0213	0.0211	0.0217	0.0006	1.66
GTB	42.03	41.39	-0.0152	-0.0153	-0.0145	0.0008	1.02
UBA	9.75	9.80	0.0051	0.0051	0.0051	0.0000	0.37
ZENITH	25.80	25.65	-0.0058	-0.0058	-0.0056	0.0.0002	0.40

Table 3: Summary results of expected rate of return and investment risk (standard deviation) from tables 1&2.

From the table 3, below are the interpretation of the table header (key):
STOCK: List of three equities traded on the floor of the NSE on 03/07/2017; PCLOSE: The closing price of stock on the previous trading day of 02/07/2017; CLOSE: The closing price of stock on the floor of the NSE on 03/07/2017; RATE OF RETURN (r): The returns – profit or loss on the stock on the floor of the NSE on 03/07/2017; ARITHMETIC: Computed arithmetic rate of return of stock; LOGARITHMIC: Logarithmic rate of return stock; MODEL: Rate of return of stock obtained using the Model; $\text{ERROR} = \text{abs}\ (\text{LOGARITHMIC} - \text{MODEL})$;INV. RISK: Investment risk or standard deviation of stock obtained using the Model.

IV. CONCLUSION

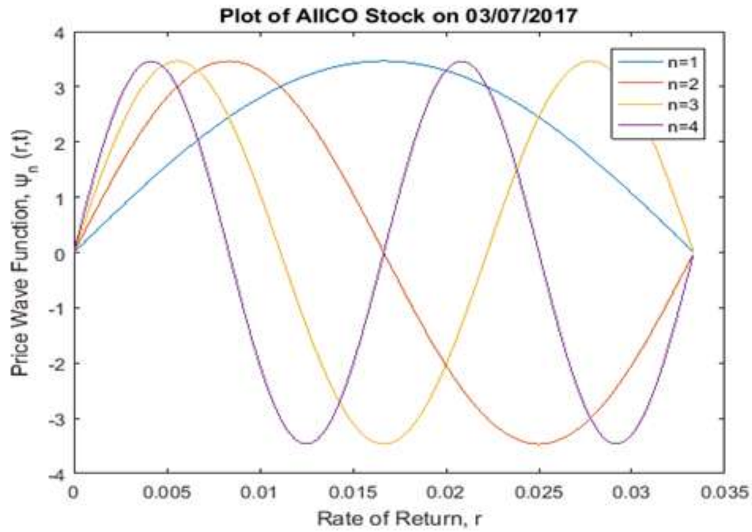

Fig. 2: The graph of price wave function, ψ_n (r,t) vs rfor AIICO stock when t=0 and n=1,2,3,4.

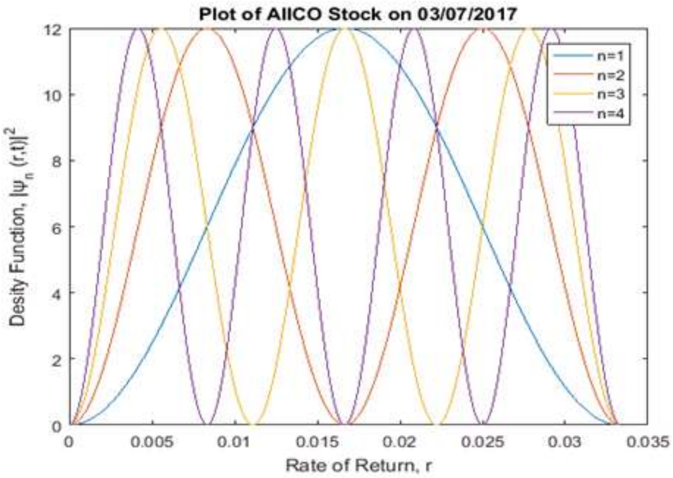

Fig. 3: The graph of probability density function,〖|ψ_n (x,t)|〗^2 vs r when t=0 and n=1,2,3,4.

4.1.1 Expected Value of Rate of Return

From table 3 above, the Expected Value for the Rate of Returns on the stocks using three methods including Arithmetic Method, Logarithmic Method and the Model are displayed. The absolute error between the Logarithmic Method and the Model is equally computed and given. It can be observed that the values obtained by the Model compared favourably with the other two methods for the three stocks under consideration and have very minimal error of less than 10^{-2}. In particular, the error for NB stock is 0.00 – this is attributable to the fact that the returns on the stock is less than 10^{-2} Naira which is the minimum price variation (MPV) contained in Rule 15.27: Unit of Trading of the NSE Rule Book as contained in section 3.4 (NSE, 2015).

The Error column on the table, is a comparison between the generally accepted Logarithmic Method for the computation of Expected Rate of Return and the value obtained using the Model. It can be observed from the table that, the errors for all the stocks are less than 10^{-2}.

Our Model therefore gives a good estimate for the daily Expected Value for the Rate of Return on Stocks.

4.1.2 Standard Deviation (Investment Risk)

From the table 3 above, the standard deviation for the eight stocks – AIICO, NB, and MOBIL, ACCESS, DSUGAR, GTB, UBA and ZENITH under consideration are 2.83%, 0.02%, 3.29%, 0.21%, 1.66%, 1.02%, 0.37% and 0.40% respectively. It follows that given the daily standard deviation of returns for the AIICO and MOBIL stocks of 2.83% and 3.29% respectively, it can be observed that MOBIL stock was a much riskier investment than the AIICO stock. In fact, MOBIL return is 1.16 times as volatile as the AIICO stock. In addition, the least risky stock from the table is ACCESS stock with SD of 0.21% while MOBIL stock is the riskiest stock with SD of 3.9%.

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