

# GIS-Based Geoid Refinement by GNSS/Levelling Data: A Case Study of Egypt

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**ABSTRACT:** *As an alternative to the extensive process of entirely re-developing a geoid model when new geodetic datasets are available, this paper proposes a simple, efficient, and fast approach for geoid refinement within a Geographic Information Systems (GIS) environment. It investigates several mathematical approaches for incorporating Global Navigation Satellite Systems (GNSS)/Levelling datasets. Such methods include 2-parameters, 4-parameters, and 7-parameters regression, Inverse Weighted Distance (IDW) method, and the krigging geostatistical method. Based on the available data, the recent SRI 2021 national geoid model has been refined using the five approaches with 220 new GNSS/Levelling data points. Based on available data and attained results, it has been realized that all investigated methods generate roughly the same accuracy level, and an improvement of almost 10% has been achieved. Such a small level of enhancement might be accredited to the non-homogeneous spatial distribution of the utilized datasets over the country. The final developed geoid model, named SRI 2022, has overall accuracy equals  $\pm 0.14$  m. It is recommended that in order to achieve a 1-5 centimeter accuracy of a geoid model in Egypt, updating/establishing of both GNSS and Levelling networks, with a good homogenous spatial distribution, is a must.*

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## I. INTRODUCTION

Geoid modelling comprises an essential duty for geodesists worldwide, particularly with the rapid growth of utilizing the Global Navigation Satellite Systems (GNSS) technology. A geoid model plays the most significant role in converting the GNSS-based ellipsoidal heights to orthometric heights or elevations related to the Mean Sea Level (MSL) datum usually utilized in surveying, mapping, and civil engineering. Several national or regional geoid models have been investigated recently in quite a few countries such as Indonesia [1], Chile and Spain [2], and Vietnam [3]. Furthermore, other geoid models have been developed on a local basis within a country, such as the west desert in Egypt [4], and Jeddah city in Saudi Arabia [5]. The accuracy of geoid models differs significantly from one region to another based on the precision, number, and spatial distribution of the utilized datasets. A level of 1-cm has been achieved for a geoid model in Colorado state in the United States of America [6] and even 5-mm accuracy has been reported in Estonia [7]. On a national basis, many geoid models have been developed in Egypt even nationally such as Saadon et al. [8] or locally such as Elshewy et al. [9].

Refinement of a geoid model, a global or a national one, is a procedure that took place after its original creation by incorporating more new geodetic datasets. Such a process has been investigated by several researchers in the last decade. For example, Al-Kherayef et al. [10] have examined the addition of new observed GNSS/Levelling datasets to the Saudi geoid named KSA-Geoid17. Also, Pasuya et al. [11] have analyzed the refinement of the gravimetric geoid of Malaysia by incorporating terrestrial, marine, and airborne gravity datasets. Similarly, Wang et al. [12] proposed the enlargement of the Chinese geoid model by adding satellite altimetry levelling datasets. The mathematical and statistical methods of such geoid refinement comprise an essential research topic where several approaches have been proposed. Such models include, among others, the moving least squares approach [13], polynomial regression and Artificial Neural Network (ANN) approach [14], the 4-parameter removal [15], finite elements based bivariate [16], and minimum curvature surface [17].

Traditionally as far as new GNSS/Levelling datasets are available, a new geoid national model is developed using one of the geoid modelling packages such as the GravSoft scientific program. Instead, this paper proposes a simple and fast approach for refinement of an existing geoid model. Thus, based on one of the most-recent geoid models of Egypt, the current research examined the incorporation of more GNSS/Levelling datasets to improve its accuracy on a national scale. Such refinement procedure is carried out within a Geographic Information Systems (GIS) environment where several mathematical and statistical methods are applied and compared.

II. METHODOLOGY

Several mathematical and statistical models have been proposed for interpolating scatter data points and constructing a 3D spatial surface in various geodetic tasks. Of then, the regression method is expensively utilized in different mathematical forms. The simple linear equation formula [18] for geoidal errors ( $\Delta N$ ) in terms of latitude  $\phi$  and longitude  $\lambda$  is given by :

$$\Delta N 1 = a_0 + a_1 \cos \phi + a_2 \cos \lambda + \varepsilon \tag{1}$$

The 4-parameters and 7-parameters regression models used in several geodetic applications [15] could be written as:

$$\Delta N 2 = a_0 + a_1 \cos \phi \cos \lambda + a_2 \cos \phi \sin \lambda + a_3 \sin \phi + \varepsilon \tag{2}$$

$$\Delta N 3 = a_0 + a_1 \cos \phi \cos \lambda + a_2 \cos \phi \sin \lambda + a_3 \sin \phi + a_4 (\cos \phi \sin \phi \cos \lambda) / W + a_5 (\cos \phi \sin \phi \sin \lambda) / W + a_6 (\sin^2 \phi) / W + \varepsilon \tag{3}$$

where,

$a_0, a_1, a_2, a_3, a_4, a_5,$  and  $a_6$  are the unknowns to be estimated,

$$W = \sqrt{1 - e^2 \sin^2 \phi} \tag{4}$$

$e^2$  is the second eccentricity of the WGS84 ellipsoid, and  $\varepsilon$  represents the residuals or errors of the regression process.

Each observation point gives one observation equation (of 1 or 3) and the least-squares adjustment method will be applied to solve all equations simultaneously to produce unique estimates of the unknowns. Within a GIS environment, there exist quite a few models for converting scatter data points into a grid or a 3D surface. Such methods include for example the kriging, spline, trend, natural neighbor, and the Inverse Distance Weighted (IDW) technique. The IDW is a mathematical deterministic method that computes the value of an unknown point by averaging the values of neighboring known points. However, IDW takes into account the distances to each known point so that the weights are inversely proportional to distances. The basic formula of the IDW method is [19]:

$$\Delta N 4_{i,j} = \frac{\sum_{k=1}^n \Delta N_k \cdot d_k^{-r}}{\sum_{k=1}^n d_k^{-r}} \tag{5}$$

where

$\Delta N_{4i,j}$  is the geoidal error of the unknown points,

$\Delta N_k$  is the geoidal error of the known points,

$dk$  represents the distance between a known and an unknown point, and

$r$  is a power weighting function, usually in practice equals 2.

Moreover, the kriging is a geostatistical analysis that takes into account the spatial distribution of the sample points to explain the variations in the 3D surface. The general form of the kriging interpolator is (ibid):

$$\hat{Z}(S_o) = \sum_{i=1}^n \lambda_i Z(s_i) \tag{6}$$

where,

$Z(s_i)$  is the measured quantity at the  $i$ th location,

$\lambda_i$  represents an unknown weight for point  $i$ ,

$s_o$  is the prediction location, and

$n$  equals the number of measurements.

Accordingly, each equation (Eq. 1 to 6) will be utilized in a GIS environment to model spatially the geoidal errors and construct a 3D corrector surfaces. Each corrector surface will be added to the original geoid model to attain a modified or enhanced version:

$$Enh\_geoid_i = SRI_{2021} + Corrector_i \tag{7}$$

Finally, the accuracy of all enhanced geoid models will be externally evaluated, in terms of standard deviations, over the known checkpoints. The overall processing steps are depicted in Fig. 1.

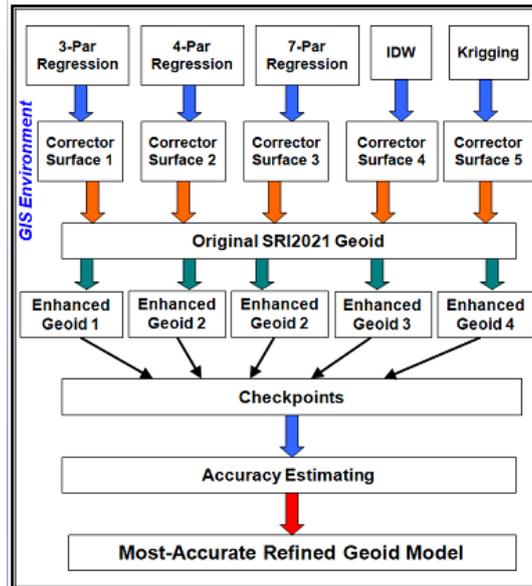


Fig. 1: Workflow of the Processing Strategy

### III. AVAILABLE DATA

The basic piece of information utilized in this research is the SRI2021 geoid model developed by Al-Krargy and Dawod [20]. This research has investigated the utilization of several Global Geopotential Models (GGMs) and Global Digital Elevation Models (GDEMs) to find out their optimum combination in geoid modelling of Egypt. Additionally, it utilized 247 terrestrial gravity stations to develop a gravimetric geoid then fit it to 1100 GNSS/Leveling points. The resulting optimum national geoid model, SRI2021, has an accuracy level of  $\pm 0.151$  m when judged over 100 GNSS/Levelling checkpoints (Fig. 2). In addition, a total of 245 GNSS/Levelling points have been collected from the projects carried out by the Survey Research Institute (SRI) in the last few years. Mostly, they cover the shorelines of the Red Sea, Suez Gulf, and Aqaba Gulf. Herein, those available points have been divided into two groups: 223 stations used in the processing stage, and 22 stations kept as checkpoints to judge the attained results (Fig. 3).

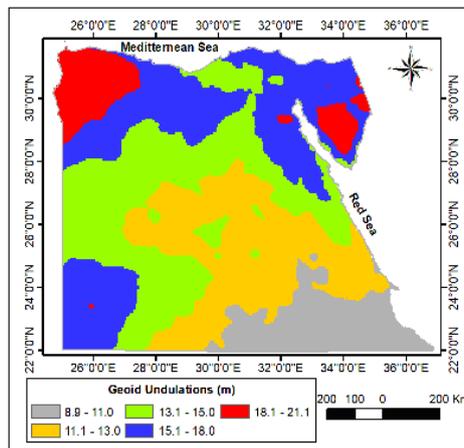


Fig. 2: The SRI2021 Geoid Model of Egypt (after Al-Krargy and Dawod 2021)

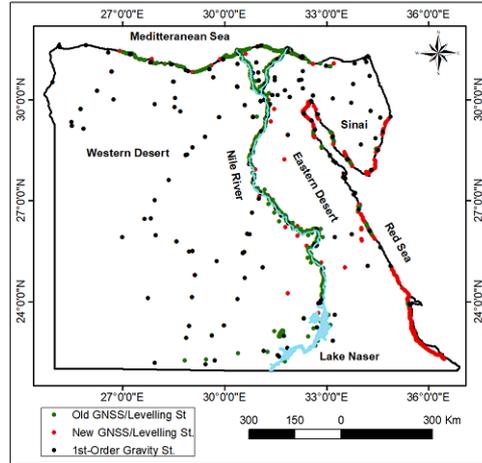


Fig 3: Available GNSS/Levelling Stations

#### IV. PROCESSING AND RESULTS

The first step has been performed using the known geoid undulations,  $N$ , at the utilized 223 GNSS/Levelling stations and comparing them to the corresponding values of the SRI2021 geoid to compute their residuals  $\Delta N$ . Next, the different investigated regression equations (Eq. 1, 2, and 3) have been solved. The attained regression models are:

$$\Delta N 1 = -2.3288 + 1.0852 \cos \phi + 2.0604 \cos \lambda \tag{8}$$

$$\Delta N 2 = -8.8548 + 7.8536 \cos \phi \cos \lambda + 3.5186 \cos \phi \sin \lambda + 3.6557 \sin \phi \tag{9}$$

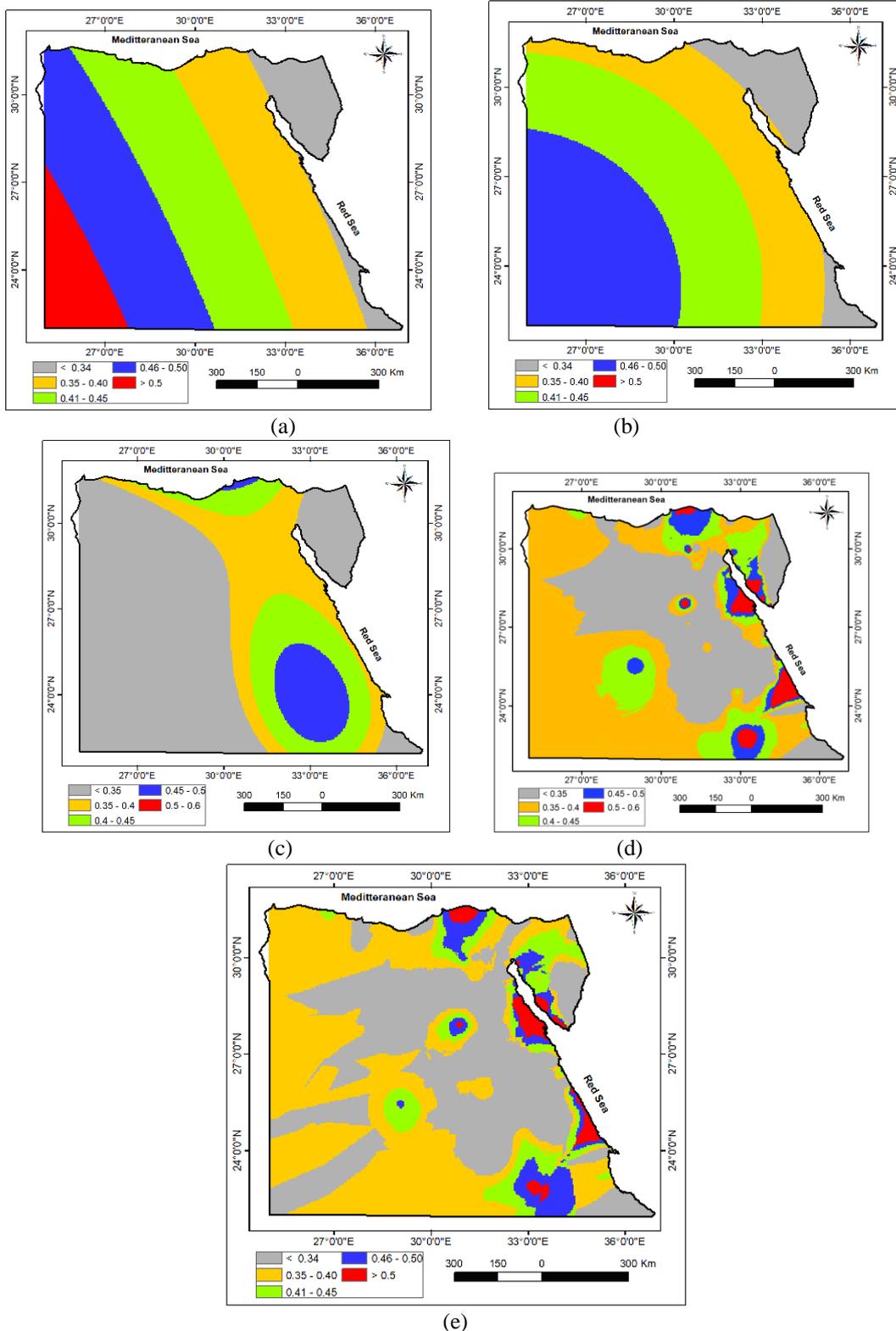
$$\Delta N 3 = -430.7820 + 326.7796 \cos \phi \cos \lambda + 232.1219 \cos \phi \sin \lambda + 8267.1712 \sin \phi - 501.4360 (\cos \phi \sin \phi \cos \lambda) / W - 374.0123 (\cos \phi \sin \phi \sin \lambda) / W - 232.8917 (\sin^2 \phi) / W \tag{10}$$

With the coefficient of determination,  $R^2$ , equals 0.029, 0.033, and 0.147 respectively.

Next, a rectangular 2-km x 2-km grid has been created covering the entire territory of Egypt. Using the Arc GIS 10.8 package, the results of Eq. 8, 9, and 10 have been estimated at each corner of that grid. The attained surfaces represent the three correction surfaces for these regression models. Next, the IDW and Krigging tools have been applied to model the geoidal errors of SRI2021 using the IDW and Krigging methods (Eq. 5 and 6). Accordingly, five correction surfaces have been developed for the five investigated refinement models (Fig. 4). The statistics of those correction models are presented in Table 1. Even though Fig. 4 shows considerable differences in the spatial distribution of geoid errors for the utilized models, Table 1 proves that there are no statistical variations in the overall performance of all models over Egypt. Thus, it can be said that the smaller values of standard deviations characterize the precision of the investigated models not their accuracy of geoid enhancement.

Table 1: Statistics of Correction Surfaces (m)

Model	Correction Surfaces			
	Minimum	Maximum	Average	Standard Deviation
2-Parameters Regression	0.303	0.545	0.426	± 0.054
4-Parameters Regression	0.303	0.544	0.426	± 0.055
7-Parameters Regression	0.301	0.545	0.425	± 0.055
IDW	0.302	0.545	0.426	± 0.055
Krigging	0.302	0.544	0.425	± 0.054



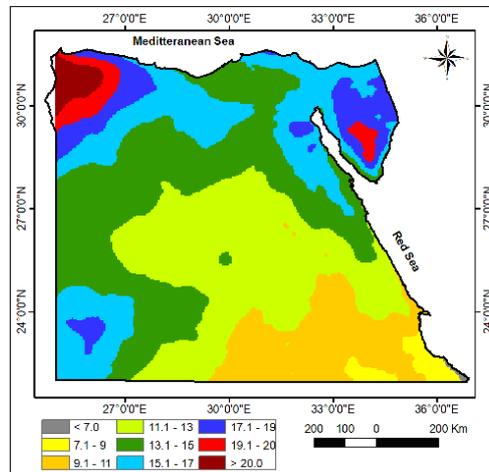
**Fig. 4: The Attained Correction Surfaces**

Next, Equation 7 has been applied to develop five new geoid models by adding each corrector surface to the original SRI 2021 geoid. In order to judge the accuracy of such geoid models, they have been compared over the known 22 checkpoints. Table 2 presents the accomplished findings. First, it can be realized from this table that all investigated methods produce almost the same accuracy level. Second, a comparison between the

new refinement geoid models and the original SRI 2021 one reveals that an improvement of almost 10% has been achieved. Third, the 4-parametr regression approach relatively provide the best improvement level. This model is called SRI 2022 geoid whose external overall accuracy equals  $\pm 0.136$  m and produced 9.3% improvements. This level of accuracy is generally compatible with those of other recent geoid modelling in Egypt (e.g. Saadon et al. 2021). Finally, based on the achieved findings, it can be concluded that the proposed GIS-based approach of geoid refinement is straightforward, efficient, and rapid. It should be carried out as long as new GNSS/Levelling datasets are available.

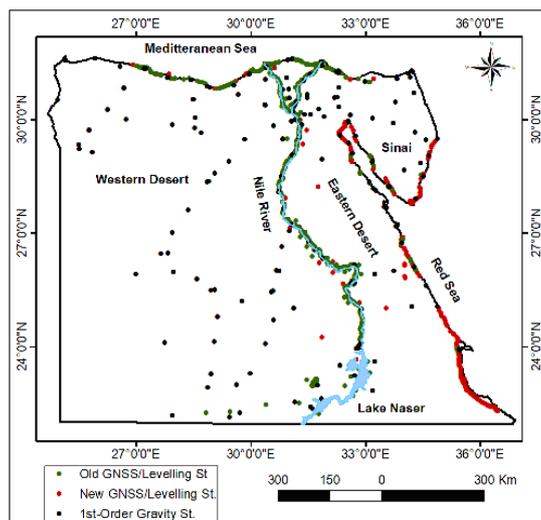
**Table 2: Statistics of Undulations of Different Geoid Models Over Checkpoints (m)**

Geoid Model	Geoid Undulation				
	Minimum	Maximum	Average	Standard Deviation	Improvement %
SRI 2021 Geoid	7.158	16.799	12.753	$\pm 3.049$	NA
Geoid of 2-Parametr Regression	7.046	16.866	12.768	$\pm 2.768$	9.2 %
Geoid of 4-Parametr Regression	7.066	16.860	12.766	$\pm 2.766$	9.3 %
Geoid of 7-Parametr Regression	6.934	16.316	12.518	$\pm 2.773$	9.1 %
Geoid of IDW	6.966	16.240	12.517	$\pm 2.774$	9.0 %
Geoid of Krigging	6.966	16.234	12.515	$\pm 2.779$	8.9 %



**Fig. 5: The SRI 2022 Egyptian Geoid**

Furthermore, the attained findings reveal that adding almost two hundred of GNSS/Levelling points enhance the accuracy of the existing national geoid of Egypt by only 10% approximately. This might be attributed to the spatial distribution of the utilized datasets which is not homogenous over the country (Fig. 3). Such a situation exists in all trials to develop an accurate Egyptian geoid. Fig. 6 depicts the distribution of the most available geodetic datasets accessed by researchers and academia in the geodetic community. That figure clearly shows that significant spatial gaps are presented in the Eastern desert, the Western desert, and the Sinai peninsula. As far the authors' concern, there exist other datasets owned by other authorities such as the geological survey authority (terrestrial gravity data) and the nuclear energy authority (airborne gravity data). It is highly recommended that all datasets should be collected, analyzed, and utilized in developing a precise national geoid of Egypt.



**Fig. 6: The Currently Available Geodetic Datasets for Geoid Modelling**

## V. CONCLUSIONS

The current research proposes a simple and fast approach for the refinement of an existing national or regional geoid model. Based on one of the most-recent geoid models of Egypt, the current research examined the incorporation of more GNSS/Levelling datasets to improve its accuracy on a national scale. Several mathematical and statistical methods have been investigated and judged. Such models include 2-parameters, 4-parameters, 7-parametres regression models, the inverse weighted distance, and the krigging approach. All those models have been applied, along with two hundred GNSS/Levelling stations, within a GIS environment for efficient and simple refinement of the geoid in Egypt.

The accomplished results indicated that all investigated methods generate roughly the same accuracy level. Additionally, an improvement of almost 10% has been achieved based on the available datasets. Such a small level of enhancement might be accredited to the non-homogenous spatial distribution of the utilized datasets over the country. The final developed geoid model, SRI 2022, has an overall accuracy equals  $\pm 0.136$  m. Based on the achieved findings, it can be concluded that the anticipated GIS-based technique of geoid refinement should be carried out as long as new GNSS/Levelling datasets are available.

Few recommendations could be drawn based on the results of the current study, as:

1. All available geodetic datasets should be collected from all local organizations, analyzed, and utilized in developing a precise national geoid of Egypt.
2. Achieving a few-centimeter accuracy of a geoid model in Egypt requires the updating/establishment of both GNSS and Levelling networks with a good homogenous distribution over the country.
3. A Geodetic Data Infrastructures (GDI) should be developed and is accessible to professionals as a component of the undergoing Spatial Data Infrastructures (SDP) huge project

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