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Harnessing the Abaoub-Shkheam Decomposition Method: A Novel Method for Solving Linear fractional Diffusion Equations

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ABSTRACT: *This paper applies the Abaoub – Shkheam Decomposition Method (QDM) to obtaining solutions of linear fractional diffusion equations. The fractional derivative is described in the Caputo sense. Some illustrative examples are given, revealing the effectiveness and convenience of the method.*
Keywords: *Abaoub Shkheam transform, Adomian decomposition method, Fractional derivatives.*

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I. INTRODUCTION

In mathematical physics, the analysis of the diffusion and wave equations has generated a lot of attention in the literature. Commonly employed analytical methods to these equations are highly limited, whereas numerical methods that require discretisation of the variables, on the other hand, result in rounding off errors, which needs a lot of computer memory because of the massive computations, are another disadvantage of numerical approaches [1]. Recently, analysis of both linear and nonlinear differential equations has shown success with the Adomian decomposition approach [2], which gives a solution in terms of a quickly convergent power series. Adomian decomposition method has been explored for solving Fredholm and Volterra integral equations [3,4]. The Adomian decomposition method was used by Biazar and Islam [5] to solve the wave equation. In the recent years several authors, for example, Mainardi [6,7], Schneider and Wyss [8] and El-Sayed [9] have investigated the fractional diffusion - wave equation and its special properties. Fractional diffusion and wave equations have important applications to mathematical physics. In the present paper, we use the Adomian decomposition method in combination with the Abaoub Shkheam transform to solve fractional diffusion and wave equations.

II. PRELIMINARIES

II.1. Basic Definitions

Definition II.1. [10]

A real function is said to be in the space if there exists a real number , such that .Where Clearly

Definition II.2. [10]

A function is said to be in the space

Definition II.3. [11]

The Riemann-Liouville (R-L) fractional integral of order of a function is defined as

Definition II.4. [11]

The Caputo fractional derivatives of order is defined as

where

The following are the basic properties of the operator :

1.
2.

Definition II.5. [12]

The Mittag-Leffler function with is defined by the following series representation, valid in the whole complex plane :

Definition II.6. [13]
The Abaoub Shkheam transform is defined over the set of function

by the following formula

II.2. Abaoub Shkheam transform of Caputo Fractional Derivative
Theorem. 1.
The Abaoub Shkheam transform of the Caputo fractional derivative is defined as

where
Proof
Since

Setting and applying Abaoub Shkheam transform on both sides of above equation

III. ANALYSIS OF THE METHOD

Consider the following general form of nonlinear fractional partial differential equation :

where is the nonlinear, ; denotes the Caputo fractional derivative and . For or and , represents a linear fractional diffusion equation, and , represents a linear fractional wave equation (homogeneous if and nonhomogeneous otherwise).

III.1. Linear fractional diffusion equation and Abaoub Shkheam decomposition Method
Consider the following general form of linear fractional diffusion equation with the specified initial condition:

The process starts by applying the Q-transform to both sides of Equation ().

by theorem (1) we get

Using the inverse of Abaoub Shkheam transform on both sides of Eq. () gives

The next step is using the Adomian decomposition method that represent the solution as an infinite series given by

where the components will be determined in a recursive manner. For the case, we set

For the case , we set

III.2. Illustrative Examples
We introduce some examples to provide a comprehensive overview of this method
Example. 1.
Consider the fractional diffusion equation

Taking Abaoub – Shkheam transform of equation) we have

by taking of both sides

By using the Adomian decomposition method that represent the solution as an infinite series which will determine the components in a recursive relation

The subsequent terms are

Substituting in , we obtain

Example. 2.
Consider the two-dimensional fractional wave equation

where
The following scheme is produced via the Abaoub Shkheam decomposition method:

in the first iteration we have

The subsequent terms are

Using the above terms, the solution is

IV. CONCLUSION

Fractional diffusion equations can be solved effectively and efficiently with the Abaoub–Shkheam Decomposition Method (QDM). It offers quickly convergent series solutions without the computational difficulties of numerical approaches by fusing the Abaoub–Shkheam transform with the Adomian decomposition method. The approach is a useful tool in mathematical physics, as demonstrated by the examples, which demonstrate its accuracy and versatility for both linear and nonlinear problems. This method's potential for wider applications in science and engineering is demonstrated by its ability to be adapted to more complicated systems.

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