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A Simple Method for Obtaining All the Triplets

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Abstract: *A triplet is defined as a combination of three integer numbers {x,y,z} which satisfy the Pythagoras theorem. A systematic method for obtaining all possible triplets when x or y is specified is presented. It is shown that each prime number, except 1 and 2, has a unique triplet. A compound number has multiple triplets depending upon the number of real factors. A triplet which will have the maximum value of z is derived. A few illustrative examples are included.*

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I. INTRODUCTION

Pythagoras theorem [1] gives a relation among three numbers *x, y* and *z*

$$x^2 + y^2 = z^2. \qquad (1)$$

Thus, if *x* and *y* are given, *z* can be obtained. Both *x* and *y* are interchangeable.

An ancient Tamil Mathematician/poet Pothayanar [2] gave the following quatrain articulating the method of finding *z*, when *x* and *y* are given, without the need to find the square and the square root; using *x* and *y*, and simple fraction [2].

ஓடும் நீளம் தனை ஒரேஎட்டுக்

கூறு ஆக்கி கூறிலே ஒன்றைத்

தள்ளி குன்றத்தில் பாதியாய்ச் சேர்த்தால்

வருவது கர்ணம் தானே. - போதையனார்

The English translation of the quatrain is the following. Divide *x* into eight equal parts. Delete one portion and add the remaining to half of *y*. Thus, the *z* is obtained as

$$z = \frac{7}{8}x + \frac{1}{2}y. \qquad (2)$$

It gives accurate result only when the following condition is satisfied [3]

$$x = \frac{12}{5}y \text{ or } \frac{4}{3}y. \qquad (3)$$

For a given value of *y,* there are only two possible values of *x* and therefore, two values of *z* from Equation (2).
From Equation (2), for a given value of *y*, for every value of *x* there is a corresponding value of *z*. For example, if *y* = 3, then from Equation (2), *x* = 4 and 7.2. Then *z* = 5 and 7.8 which are the correct values as given by Equation (1).
As another example, if *y* = 9, then *x* = 12 and 23.4 and *z* = 15 and 25.071. However, there are several possible values of *x* and *z*, one of them is *x* = 40 and *z* = 41.
In Equation (1), *x* and *y* can be interchanged, but in Equation (2) they cannot be.
Inequality *x* > *y* holds while using Equation (2). No such restriction on *x* and *y* while using Equation (1).
In Equation (1), *x, y* and *z* need not be integers. However, if they are integers, then it is called a *triplet* (*x,y,z*) [1].
In this short note, a method is proposed to find all possible triplets when only *x* or *y* is given.

II. METHOD

From Equation (1)

$$x^2 = z^2 - y^2 = (z-y)(z+y) = x_1 x_2. \qquad (4)$$

where x_1 and x_2 are two real factors of x^2.
Identify from Equation (4)

$$x_1 = (z-y),\ x_2 = (z+y). \qquad (5)$$

Obviously,

$z > y$ and $x_2 > x_1$. (6)

From Equation (5)

$$z = \frac{x_2 + x_1}{2}, \quad (7)$$

$$y = \frac{x_2 - x_1}{2} = z - x_1. \quad (8)$$

Thus, the triplet is

$$\{x, y, z\} = x, \frac{x_2 - x_1}{2}, \frac{x_2 + x_1}{2}. \quad (9)$$

From Equations (7) and (8) Note that

x_1, x_2 have to be either both even or odd, but not equal. (10)

2.1 Procedure

1. Find the integer factors of x^2 and prepare all the possible combinations of two $\{x_1, x_2\}$, including 1.
2. Choose the valid combinations of $\{x_1, x_2\}$ starting with $\{x, 1\}$ until $\{x_1 = x_2\}$.

(a) Discard the combination $x_1 = x_2$.

(b) Choose both x_1 and x_2 odd/even, if x is odd/even.

3. Find $\{y,z\}$ using Equations (8) and (7), respectively.
4. Find all the triplets $\{x,y,z\}$ using Equation (9).

2.2 Some important triplets

(1) *x is a prime number x_p*

In this case, possible factors are 1 and x_p^2. Then

$\{x_1, x_2\} = \{1, x_p^2\}, \{x_p, x_p\}$

Discard $\{x_p, x_p\}$. Since x_p is a prime number, it is an odd number, except when it is 2. Hence x_p^2 is an odd number. Therefore, valid combination is $\{x_1, x_2\} = \{1, x_p^2\}$. Thus, Equations (7) and (8) will give unique value for $\{y,z\}$

$$z = \frac{{x_p}^2 + 1}{2}, \quad (11)$$

$$y = \frac{{x_p}^2 - 1}{2}. \quad (12)$$

Finally, the unique triplet, using Equation (9), is

$$\{x, y, z\} = \left\{x_p, \frac{x_p - 1}{2}, \frac{x_p + 1}{2}\right\}. \quad (13)$$

When $x_p = 1$, y will be 0; not permissible for a triplet.

When $x_p = 2$, both z and y will be fractions which is not admissible for a triplet. Thus, there exist no triplets for 1 and 2.

(2) *x is multiplied by n.*

The procedure will lead to the triplet

$\{x, y, z\} = \{nx, ny, nz\}$. (14)

It means that all the x,y,z are multiplied by n. The triplet with $n = 1$ will be called a *basic triplet*.

(3) *x is a compound number*

This is illustrated with the following 2 examples.

Example 1: Let $x = 36$ (even).

Since $x^2 = 1296 = 1 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$,

pairs of factors are

{1,1296}, {2, 648}, {3,432}, {4,324}, {6,216},
{9,144}, {12,108}, {16,81}, {18,72},
{24,54}, {27,48}, {36,36}.

However, required pairs of even factors, excluding the pair of equal factors, are

$\{x_1, x_2\}$ = {2,648}, {4,324}, {8,162}, {12,108},
{18,72}, {24,54}.

Using Equations (7) and (8)

$\{y,z\}$ =\{323,325\},\{160,164\},\{77,135\},\{105,111\}, \{60,248\},\{27,45\}
Corresponding triplets, from Equation (9), are
$\{x,y,z\}$ =\{36,323,325\},\{36,160,164\}, \{36,77,135\},\{36,105,111\},\{36,60,48\},
\{36,27,45\}

Example 2: Let $x = 27$ (odd).
Since $x^2 = 729 = 1 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$,
the pairs of factors are
$\{1,729\}, \{3,243\}, \{9,81\}, \{27,27\}$
However, required pairs of both odd factors, except two equal factors, are
$\{x_1, x_2\} = \{1,729\}, \{3,243\}, \{9,81\}$
Using Equations (7) and (8)
$\{y,z\}$ = \{364,365\}\{120,123\}, \{36,45\}
Corresponding triplets, using Equation (9), are
$\{x,y,z\}$ = \{27,364,365\},\{27,120,123,\},\{27,36,45\}.

III. TRIPLETS WITH THE HIGHEST VALUE OF z

If x is odd, then x^2 is also odd. Therefore, the choice for maximum z is x_1,x_2 = $(1,x^2)$. The triplet is, using Equation (9),

$$\left\{x, \frac{x^2-1}{2}, \frac{x^2+1}{2}\right\}, x \text{ odd.} \qquad (15)$$

If x is even, then x^2 is also even. Therefore, the choice for maximum z is x_1, x_2 = $(2, x^2/2)$. The triplet is, using Equation (9),

$$\left\{x, \frac{\frac{x^2}{2}-2}{2}, \frac{\frac{x^2}{2}+2}{2}\right\}, x \text{ even.} \qquad (16)$$

Example 3: Let $x = 45$ (odd).
From Equation (15)

$$\left\{x, \frac{x^2-1}{2}, \frac{x^2+1}{2}\right\},$$

$$\left\{45, \frac{45^2-1}{2}, \frac{45^2+1}{2}\right\} = \{45, 912, 913\}$$

Thus, highest value of $z = 913$

Example 4: Let $x = 100$ (even)
From Equation (16)

$$\left\{x, \frac{\frac{x^2}{2}-2}{2}, \frac{\frac{x^2}{2}+2}{2}\right\},$$

$$\left\{100, \frac{\frac{100^2}{2}-2}{2}, \frac{\frac{100^2}{2}+2}{2}\right\}.$$

$= \{100, 2499, 2501\}$.
Thus, highest value of $z = 2501$.

IV. CONCLUSION

A triplet is defined as $\{x,y,z\}$ where x,y,z are integers and satisfy the relation $x^2 + y^2 = z^2$. A simple method for obtaining all the triplets when either x or y is specified is presented. There is no triplet for x equal to 1 and 2. Each of the remaining prime numbers has a unique triplet. A compound x has multiple triplets depending upon the number of real factors of x^2. The triplets with the highest values of z are derived.

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