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Most Popular Topology Subjects

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***Abstract**: This paper examines some of the most popular topology subjects, covering both fundamental studies and multidisciplinary applications.*

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I. Introduction

1.Topological Data Analysis (TDA) is a mathematical framework that uses ideas from algebraic topology to glean structures and insights from high-dimensional, complicated data. A flexible method for drawing insightful conclusions from complicated, high-dimensional, and noisy datasets is topological data analysis. It is especially useful in fields where conventional techniques find it difficult to capture the underlying structure because of its capacity to disclose the "shape" of data. The aforementioned applications demonstrate the wide-ranging effects of TDA in industry, engineering, and science.

Concentrating on the "shape" of data enables researchers to find topological features such as gaps, clusters, and patterns that might not be seen through the use of conventional statistical or machine learning techniques.

(i) Persistent Homology:
An instrument for examining the form of data by locating recurring topological elements (such as holes and relat ed components) across scales.extensively utilized in machine learning, neurology, and biology.
A space must first be represented as a simplicial complex to determine its persistent homology. A filtration of the simplicial complex, or a nested sequence of rising subsets, is equivalent to a distance function on the underlying space. To obtain the simplicial filtration known as Čech filtration, a popular technique is to take the sublevel filtration of the distance to a point cloud, or equivalently, the offset filtration on the point cloud, and take its nerve [1]. The Vietoris–Rips filtration is a similar construction that employs a nested sequence of Vietoris–Rips complexes [2].One of the fundamental instruments of TDA is persistent homology.
A few books and studies that thoroughly describe the theoretical features of persistent homology have already been produced due to its applicability in numerous practical sectors. The works by A. Zomorodian [3] and R. Ghrist [4], as well as the survey and book by H. Edelsbrunner and J. Harer [5,6], are noteworthy.
For computer science or mathematics students who have a strong foundation in topology and algorithms, H. Edelsbrunner and J. Harer's two volumes offer an introduction to persistent homology and, more generally, computational topology.

(ii) Mapper Algorithm: The Mapper algorithm can extract Global features from high-dimensional data [7]. It makes it possible to abstract away precise distances, angles, and even individual data points and instead describes point clouds as simplicial complexes. A simplicial complex [8], which offers a condensed, worldwide representation of the data, is the result of Mapper.

Key Applications of TDA
1. Genetics and Biology
a. Protein Structure Analysis: By examining the geometric and topological characteristics of protein structures, TDA aids scientists in comprehending interactions, folding patterns, and functional characteristics [9].
b. Gene Expression Data: TDA can uncover biological pathways and regulatory networks by locating clusters and links in high-dimensional gene expression datasets.
c. Single-Cell RNA Sequencing: TDA makes it possible to identify uncommon cell types and state transitions by revealing the underlying architecture of cell populations.

2. Neuroscience and Medical Imaging
a. Brain Connectivity: TDA is used to find strong structural and functional connectivity patterns in the topology of brain networks that are derived from fMRI or DTI data.

b. Early tumor detection and classification are made possible by persistent homology, a key component of TDA that can identify minute topological patterns in medical pictures.
c. Neurodegenerative Diseases: By monitoring alterations in the structure of brain networks over time, TDA sheds light on conditions like Parkinson's and Alzheimer's.

3. Science of Materials
a. Porous Materials: To maximize the performance of materials like zeolites and metal-organic frameworks (MOFs) for uses like gas storage and catalysis, TDA is used to examine their pore structure.
b. Microstructure Characterization: To help with material design and optimization, TDA measures the topological characteristics of microstructures in alloys, ceramics, and composites.

4. Economics and Finance
a. Market Dynamics: By examining the topology of market behavior, TDA reveals hidden patterns in financial time series data, including stock prices and currency rates [10,11].
b. Risk management: By identifying important nodes and links, persistent homology can uncover systemic problems in financial networks.
c. Portfolio Optimization: TDA aids in comprehending the connections among the assets in a portfolio, which results in improved diversification tactics.

5. Social Networks and Human Behavior Community Detection: TDA uses the topological pattern of links to find communities and subgroups within social networks [12,13].
a. Human Mobility Patterns: By examining the temporal and spatial patterns of human movement, TDA provides information about transportation and urban development.
b. Sentiment analysis: TDA increases the precision of sentiment classification models by capturing the underlying topology of sentiment distributions in text data.

6. Artificial Intelligence and Machine Learning
a. Feature Engineering: Solid topological characteristics that may be utilized as inputs for machine learning models are extracted from data by TDA [14].
b. Anomaly Detection: By spotting departures from the anticipated topological structure, persistent homology finds anomalies in datasets.
c. Explainability: TDA improves transparency and trust by offering comprehensible insights into how AI algorithms make decisions [15].

7. Climate Modeling and Environmental Science
a. Climate Data Analysis: By collecting the topological aspects of climate data, such as temperature or precipitation fields, TDA can identify patterns in the data [16].
b. Ecosystem Dynamics: To comprehend species interactions and ecosystem resilience, TDA examines the topology of ecological networks, such as food webs.
c. Pollution Monitoring: TDA supports environmental monitoring and policy-making by analyzing temporal and spatial patterns of pollutants.

8. Robotics and Autonomous Systems Path Planning: By examining the topology of configuration spaces, TDA is utilized to determine the best routes in intricate situations.
a. Sensor Networks: By examining the topological characteristics of sensor networks, TDA guarantees strong coverage and connectivity.
b. Shape Recognition: Robots can identify and engage with objects using their topological features thanks to TDA [17].

9. Signal and Image Processing
a. Image Segmentation: By locating gaps and related components in images, TDA increases segmentation accuracy.
b. Signal Denoising: By concentrating on persistent topological properties, persistent homology distinguishes significant signals from noise.
c. 3D Shape Analysis: To help with applications like computer vision and virtual reality, TDA examines the topology of 3D shapes [18].
10. Security of Cyberspace
a. Network Intrusion Detection: TDA uses communication graph topology analysis to find anomalous patterns in network traffic [19].

b. Malware Detection: By capturing the structural traits of malware, persistent homology makes detection and categorization more efficient [20].
c. Threat Intelligence: By examining the topological characteristics of cyberthreats, TDA reveals hidden connections between them [21].

11. The study of astronomy
a. Galaxy Filament Analysis: By examining their topology, TDA investigates the universe's large-scale structure, including galaxy filaments and voids [22].
b. Dark Matter Distribution: Using gravitational lensing data, persistent homology aids in determining the distribution of dark matter.
c. Cosmic Microwave Background (CMB): TDA provides information about the early cosmos by extracting topological features from CMB maps.

12. Finding New Drugs
a. Molecular Docking: TDA analyzes the topological compatibility of target proteins to determine how well drug molecules fit with them.
b. Virtual Screening: Persistent homology identifies promising drug candidates by capturing their topological similarities to known active compounds.
c. Toxicity Prediction: TDA predicts the toxicity of chemicals by analyzing their topological fingerprints.

2. Algebraic Topology
(i) Homotopy Type Theory (HoTT): A fundamental framework that connects type theory and topology, with formal verification and computer science applications [23].

Types as Spaces: Terms of a type are thought of as points in topological spaces, which are understood as types. A homotopical approach to equality is made possible by modeling identity types (equality) as pathways between points: two words are "equal" if a path connects them [24].

(ii) Higher Category Theory: investigates structures such as $\infty-$categories, which have an impact on mathematical physics and algebraic geometry.

3. Geometric and Low-Dimensional Topology
a)3-Manifolds and Hyperbolic Geometry: developments in the study of hyperbolic structures, motivated by Thurston's geometrization initiative.
b) Knot Theory: Applications in quantum field theory (e.g., Khovanov homology), DNA topology, and polymer physics.
c) Exotic 4-Manifolds: investigation of smooth structures on 4D spaces, related to Seiberg-Witten invariants and gauge theory [25].

4. Symplectic and Contact Geometry*
a. Mirror Symmetry is a string theory duality that connects algebraic and symplectic geometry and has uses in enumerative geometry.
b. Floer Homology: An effective invariant for low-dimensional topology and Lagrangiansubmanifolds.

5. Topological Quantum Field Theory (TQFT)
connects topology with quantum physics, impacting both quantum computers (anyons in quantum error correction) and condensed matter physics (topological phases of matter, for example).

6. Applications in Physics and Materials Science
a. Topological Insulators: Materials related to quantum computing that have strong electrical characteristics shielded by topology.
b. MarynaViazovska's innovations in 8- and 24-dimensional sphere packing served as inspiration for sphere packing and lattice models.

7. Dynamical Systems and Topology
Study of fluid dynamics, climate modeling, and topological invariants in chaotic systems.

8. Interdisciplinary Innovations
a. Motion planning with configuration spaces in topological robotics.
b. Neurotopology: Using topological techniques to analyze neural networks and brain connectivity.

What Causes These Trends?
-Interdisciplinary Demand: TDA and other tools tackle the complexity of real-world data in AI and biology.
-Mathematical Physics: Interest in algebraic and geometric topology is fueled by string theory and quantum computing.
-Computational Advancements: Topological approaches are now available thanks to algorithms and software (such as GUDHI and Ripser).
These domains demonstrate how topology is becoming more and more important in both advanced applied applications and pure mathematics. Journals such as Journal of Topology or arXiv's math.AT (Algebraic Topology) and math.GT (Geometric Topology) are great places to look for more in-depth information.

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