International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 8, Issue 10 [October 2019] PP: 09-23

Heat Transfer on Steady Laminar Mixed Convective MHD Flow through an Inclined Channel

P. Sulochana
Professor, Department of MathematicsIntell Engineering College, Ananatapuramu
Andhra Pradesh, India, 515004

Abstract: We have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time. The expressions for the velocity field, the temperature field, the pressure drop, the friction factors and the Nusselt number at any plane parallel to the walls are obtained analytically. The effects of various emerging parameters on the velocity field, the temperature field, the pressure drop, the friction factors and the Nusselt number are discussed through graphically.

I. INTRODUCTION

The laminar mixed convection flow in an inclined channel has been the subject of much investigation due its possible application in many industrial and engineering processes. These include cooling of electronic equipment, heating of the Trombe wall system, gas cooling nuclear reactors geological system, agricultural engineering metallurgy, oil technology and others. The theory developed by viscous flow through porous media is useful in analyzing the influence of temperature and pressure on the flow of soil water. It is made use of in the energy extraction from geothermal regions. The flow of viscous fluid in an inclined channel with free surface has applications in coating to paper rolls, hydraulic engineering, and in the designs of drainage and irrigation canals. Until the eighteenth century the flow rate in a channel, the channels were related quantitatively on sound principles. A comparatively rapid development in the understanding of open channel phenomena has yielded a body of knowledge that is very successfully applied to the design of irrigation, flood control and sewerage systems as well as to the design of aqueducts, dam spillways and other structures in which water flows with a surface in contact with the atmosphere. Heat ransfer in porous medium has several applications in the situations viz: nuclear waste disposal, geothermal energy extraction, fossil fuels detection, regenerator bed etc. Understanding the development of hydrodynamic and thermal boundary layer along with the heat transfer characteristics is the basic requirement to further investigate the problem extensively and more exhaustively.In many chemical processing industries generally slurry adheres to the reactor vessels and gets consolidated. As a result of this, the chemical compounds within the reactor vessel percolates through the boundaries causing loss of production and then consuming more reaction time. The slurry thus formed inside the reactor vessel often acts as a porous boundary for the next cycle of chemical processing. Flow in a porous medium can be considered as an ordered flow in a disordered geometry. The transport process of fluid through a porous medium involves two substances, the fluid and the porous matrix, and therefore it will be characterized by specific properties of these two substances. A porous medium usually consists of a large number of interconnected pores each of which is saturated with the fluid. The exact form of the structure is highly complicated and differs from one medium to other medium. A porous medium may be either an aggregate of a large number of particles such as sand or gravel or solid containing many capillaries as seen in a porous rock (laterite stone). When the fluid percolates through a porous material, because of the complexity of microscopic flow in the pores, the actual path of an individual fluid particle cannot be followed analytically. In all such cases, one has to consider the gross effect of the phenomena represented by a macroscopic view applied to the masses of fluid, large compared to the dimensions of the pore structure of the medium. The process can be described in terms of equilibrium of forces. The driving force necessary to move a specific volume of fluid at a certain speed through a porous medium is in equilibrium with the resistance force generated by internal friction between the fluid and the pore structure. Such a resistive force is characterized by Darcy's [1] semi - empirical law. The simplest model for flow through a porous medium is the one dimensional model derived by Darcy. From such empirical evidence, Darcy's law indicates that, for an incompressible fluid flowing through a channel filled with a fixed uniform and isotropic porous matrix, the flow speed varies linearly with longitudinal pressure variation. Fluids possessing visco elasticity had become important industrially. Specifically in polymer processing applications as well as in chemical industry, one deals with flow of visco elastic fluids. A classical example of such a liquid is poly iso - butylene. With the development of general constitutive equations for visco elastic fluids, it has been a point of great concern for Non-Newtonian fluids. All such proposed constitutive equations should in principle lead to the definition of flow properties that need to be measured to define the rheology and also to the development of the equivalent Navier Stokes equations for the solution of all possible boundary values along with initial value problems that arises in several situations. Some of the analytical methods for complex flows of visco elastic fluids generally predict the nature of flow field and gives rise to more or less accurate solution though not a perfect solution. In all such situations, the methodology that is applied must be evolved and considered appropriately. It is pertinent to be quite specific about the experimental conditions applicable to the relevant phenomena. The inertia effects become all the more so important that in a sparsely packed porous medium and hence their effect on free convection problems needs to be investigated. The aim of the present investigation is, therefore, to study systematically the effect of inertial terms on combined free and forced convective heat transfer past a semi-infinite inclined plate embedded in a saturated porous medium with variable permeability, porosity and thermal conductivity under the influence of gravity. The results obtained under limiting conditions agree well with the existing ones and thus verify the accuracy of the method used. There is a fast growing belief that the many provocative experimental phenomena and dilemmas now have become a realistic possibility of being explained theoretically by Non-Newtonian Fluid Mechanics. An attempt is being made in this paper to illustrate such an optimistic thought in advocating visco elastic effect that occurs in several industrial and biological applications. The phenomenon of free convection arises in the fluid when temperature changes cause density variation leading to buoyancy forces acting on the fluid elements. This can be seen in our everyday life in the atmospheric flow, which is driven by temperature differences. Free convective flow past vertical plate has been studied extensively and more intensively by Ostrach [2], [3], and [4]. The transient free convection from a vertical flat plate has been examined by Siegel [5]. Although this problem is important in polymer processing applications Benenati and Brosilow [6] have shown that the permeability of a porous medium varies due to the variation of porosity from the wall to the interior of the porous medium. The problem of the exact solutions of two dimensional flows of a second order incompressible fluid has been examined by Pattabhi Ramacharyulu [7] by considering rigid boundaries while Kaloni [8] examined the fluctuating flow of a visco elastic fluid past an infinite porous plate subject to uniform suction. Thereafter, Merkin [9] investigated the mixed convection boundary layer flow on a semi-infinite vertical flat plate when the buoyancy forces aid and oppose the development of the boundary layer. In this study two series solutions were obtained, one of which is valid near the leading edge and the other is valid asymptotically. In the regions where the series solutions are not valid, numerical solutions were obtained by Lloyd and Sparrow [10]. The natural convection flows adjacent to both vertical and horizontal surfaces, which result from the combined buoyancy effects of thermal and mass diffusion, was first investigated by Gebhart and Pera [11] and Pera and Gebhart [12] while, Soundalgekar [13] investigated the situation of unsteady free convective flows wherein the effects of viscous dissipation on the flow past an infinite vertical porous plate was highlighted. In the course of analysis, it was assumed that the plate temperature oscillates in such a way that its amplitude is small. Oosthuizen and Hart [14] and Wilks [15] have carried out a numerical study of the combined forced and free convection flow over a vertical plate. Later, a linear analysis of the compressible boundary layer flow over a wall was presented by Lekoudis [16] et al while, Shankar and Sinha [17] studied the problem of Rayleigh for a wavy wall. Subsequently, Lessen and Gangwani [18] examined the effect of small amplitude wall waviness on the stability of the laminar boundary layer while, the free convective heat transfer on a vertical semi-infinite plate has been investigated by Berezovsky et al. [19]. The unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer was analysed by Soundalgekar et al [20] while for the same situation when the suction is variable was examined by Soundalgekar and Wavre [21]. The problem of free convection heat transfer from a vertical plate embedded in a fluid saturated porous medium is studied by Cheng and Minkowycz [22], who have obtained the similarity solutions for the problem considered. Murthy et al [23] had examined the dispersion effects due to a heated vertical flat plate. Subsequently, the problem of free convective heat transfer in a viscous incompressible fluid confined between vertical wavy wall and a flat wall was examined by Vajravelu and Shastri [24]. Cheng [25] has provided an extensive review of early works on free convection in porous media while, Mucoglu and Chen [26] had examined the mixed convection flow over an inclined surface for both the assisting and the opposing buoyancy forces. The linearity between speed and pressure variation breaks down for large enough flow speed was presented by Mac Donald et al. [27]. Later, Chen et al [28] studied the combined effect of buoyancy forces from thermal and mass diffusion on forced convection. Merkin and Mahmood [29] have obtained the similarity solution of the mixed convection flow over a vertical plate for the constant heat flux case, while Tsuruno and Iguchi [30] have investigated the effects of the surface mass transfer on the mixed convection flow on a permeable vertical surface. Plumb and Huenefeld [31] have investigated non-Darcy natural convection from vertical isothermal surfaces in saturated porous media. This was emphasized later by Joseph et al [32] who stressed force modeled by the Frochheimer acts in a direction opposite to the velocity vector. A numerical and experimental investigation of the effects of the presence of a solid boundary and initial forces on mass transfer in porous media was presented by Vafai and Tien [33]. The laminar free convection from a vertical plate has been studied by Martynenko et al. [34]. In all their papers, the plate was assumed to be maintained at a constant temperature, which is also the temperature of the surrounding stationary fluid. Bejan and Poulikakos [35] have used Forchheimer's model to study vertical boundary layer natural convection in a porous medium. The steady flow of an incompressible second grade fluid past an infinite porous plate subject to suction or blowing was investigated by Rajagopal and Gupta [36]. Chandrasekhar and Namboodiri [37] have shown the effectiveness of variable permeability of the porous medium on velocity distribution and heat transfer. Hong et al [38] have studied analytically the non-Darcian effects on a vertical plate natural convection in porous media. They used a combination of Rayleigh and Darcy numbers to describe the inertia and boundary terms and obtained similar solutions. They found that these effects decrease the velocity and reduce the heat transfer rate. Lai and Kulacki [39] have used both Darcy and non- Darcy models to study mixed convection from horizontal and vertical surfaces embedded in saturated porous media. Nakayama and Koyama [40] have obtained the similarity solution for the problem of free convection in the boundary layer adjacent to a vertical plate immersed in a thermally stratified porous medium. Ramachandran et al [41] have studied the mixed convection flow over vertical and inclined surfaces, theoretically as well as experimentally. Kumari et al [42] have investigated the non-Darcian effects on forced convection heat transfer over a flat plate in a highly porous medium. Subsequently, Ramanaiah and Malarvizhi [43] investigated the free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient. Wickern [44] has examined the influence of the inclination angle of the plate and the Prandtl number on the mixed convection flow over an inclined plate. Thereafter, Das and Ahmed [45] had studied the effects of thermal dispersion and dissipation effects on non – Darcy mixed convection problems and established the trend of heat transfer rate convection from a vertical plate in porous medium and investigated the flow and temperature fields. Hsieh et al [46] have obtained a non-similar solution for combined convection from vertical plates in porous media with variable surface temperatures or heat flux. Later, Knupp and Lage [47] analyzed the theoretical generalization to the tensor permeability case (anisotropic medium) of the empirically obtained Frochheimer extended Darcy unidirectional flow model. Hung and Chen [48] have studied non-Darcy free convection in a thermally stratified fluid saturated porous medium along a vertical plate with variable heat flux. It follows that, in multidimensional flow, the momentum equations for each velocity component derived by using the Frochheimer extended Darcy equation is atleast speculative while Patidar and Purohit [49] studied the free convective flow of a viscous incompressible fluid in porous medium between two long vertical wavy walls. Vighnesam and Soundalgekar [50] investigated the combined free and forced convection flow of water from a vertical plate with variable temperature. The transient free convection flow past an infinite vertical plate with periodic temperature variation was studied by Das et al [51]. Later, Kuznetsov [52] had investigated the effect of transverse thermal dispersion on forced convection in porous media and identified the situations favorable to heat transfer under dispersion effects. Thereafter, Mohammadien and El-Amin [53] studied the dispersion and radiation effects in fluid saturated porous medium on heat transfer rate for both Darcy and non-Darcy medium. In recent times, Hossain et al. [54] studied the influence of fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate. An explicit analytical technique namely homotopy analysis to solve the non - Darcy natural convection over a horizontal plate with surface mass flux and thermal dispersion was studied by Wang et al [55]. The wide range of its technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle on the surface acquires finite tangential velocity and hence "slips" along the surface. The flow regime is called the slip-flow regime and this effect cannot be neglected. Under the assumptions made by Sharma and Chaudhary [56] and Sharma and Sharma [57] have also discussed the free convection flow past a vertical plate in slip-flow regime. Several applications were cited in their papers that occur in several engineering applications wherein heat and mass transfer occurs at high degree of temperature differences. Subsequently, Taneja and Jain [58] had examined the problem of MHD flow with slip effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wall and a parallel flat plate. Recently, Ramana Murthy and Kulkarni [59] examined the problem of elastic - viscous fluid of second order type by causing disturbances in the liquid which was initially at rest and the bounding surface was subjected to sinusoidal oscillations. In view of the above studies, in this paper, we have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time.

II. MATHEMATICAL FORMULATION AND SOLUTION OF THE PROBLEM

We consider the laminar flow of a Newtonian fluid in the gap between two infinitely-wide plane parallel walls. The flow is assumed to be parallel such that U has the only non-vanishing component along the X -axis. The axis orthogonal to the walls, the gravitational acceleration 'g' and the X -axis lie on the same plane. The latter condition ensures that the flow can be considered as two-dimensional, i.e. both the velocity field and the temperature field depend only on two spatial coordinates. A uniform magnetic field B_0 is applied in the transverse direction to the flow. The electrical conductivity of the fluid is assumed to be small so that the magnetic Reynolds number is small and the induced magnetic field is neglected in comparison with the applied magnetic field. The external electric field is zero and the electric field due to polarization of charges is also negligible. Heat due to Joule dissipation is neglected. The system under consideration is sketched in Fig. 1, where the chosen coordinate axes (X, Y) are drawn. Let us assume that the wall at $Y=-L$ is kept isothermal with a constant temperature T_1, while the wall at $Y=L$ is subjected to an oscillating temperature

$$T\ (X,L,t)\ =\ T_2\ +\ \Delta T\ \cos(\omega t)\,. \tag{1}$$

Moreover, heat flow is assumed to occur only in the transverse direction, so that $\partial T/\partial X=0$. The latter assumption is conceivable since each wall is kept at a uniform temperature. The Boussinesq approximation is invoked, so that U is a solenoidal field and, as a consequence, $\partial U/\partial X=0$. A steady mass flow rate is prescribed; therefore the average velocity in a channel section, defined as

$$U_0\ =\ \frac{1}{2L}\int_{-L}^{L} U\ dy \tag{2}$$

is time independent.

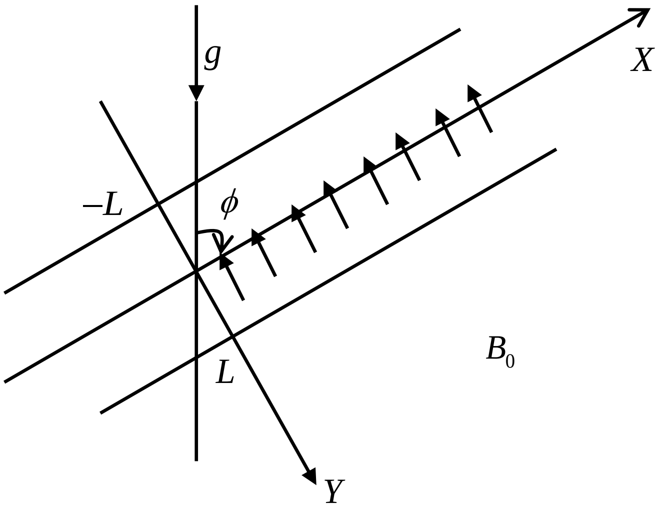


Fig.1 The Physical Model

The equation of state, $\rho=\rho(T)$ considered as linear,

$$\rho\ =\ \rho_0\ [\ 1\ -\ \beta\ (T-T_0)\ ] \tag{3}$$

Where T_0 is an average temperature both with respect to the interval $-L\le Y\le L$ and to the period $0\le t\le\frac{2\pi}{\omega}$, namely

$$T_0\ =\ \frac{\omega}{4\pi L}\int_0^{\frac{2\pi}{\omega}} dt\int_{-L}^{L} T\ dY \tag{4}$$

Since, $\partial T/\partial X=0$ the reference temperature T_0 is a constant. According to the Boussinesq approximation, the momentum balance equation yields, along the X and Y axes,

$$\rho_0\ \frac{\partial U}{\partial t}\ =\ \rho_0 g\ \beta\ (T-T_0)\ \cos\varphi-\ \frac{\partial P}{\partial X}+\ \mu\frac{\partial^2 U}{\partial Y^2}\ -\sigma B_0^2 U \tag{5}$$

$$\rho_0\ g\ \beta\ (T-T_0)\ \sin\varphi\ +\ \frac{\partial P}{\partial Y}\ =\ 0. \tag{6}$$

Where $P\ =\ p\ +\ \rho_0\ g\ (X\cos\varphi\ -\ Y\sin\varphi)$.

Differentiating equations (5) and (6) on both sides with respect to X, we obtain,

$$\frac{\partial^2 P}{\partial X^2} = 0. \tag{7}$$

$$\frac{\partial^2 P}{\partial X \partial Y} = 0 \tag{8}$$

It is easily verified that Equations (7) and (8) imply the existence of two functions $A(Y,t)$ and $B(t)$ such that

$$P\ (X,Y,t) = A(Y,t) - B(t)\ X\ . \tag{9}$$

The energy balance equation is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial Y^2}. \tag{10}$$

Let us define the dimensionless quantities

$$\theta = \frac{T - T_0}{\Delta T},\ u = \frac{U}{U_0},\ y = \frac{Y}{D},\ \eta = \omega t,\ \lambda = \frac{D^2 B}{\mu U_0},\ \Omega = \frac{\omega D^2}{\nu},\ \Pr = \frac{\nu}{\alpha},\ \mathrm{Re} = \frac{U_0 D}{\nu},$$

$$Gr = \frac{g\beta \Delta T D^3 \cos\varphi}{\nu^2},\ \chi = \frac{T_2 - T_1}{\Delta T},\ \xi = \frac{T_1 - T_0}{\Delta T},\ M^2 = \frac{\sigma B_0^2 D^2}{\mu} \tag{11}$$

Where $D = 4L$ is the hydraulic diameter. By employing Equations (9) and (11), Equations (5) and (10) can be rewritten as

$$\Omega \frac{\partial u}{\partial \eta} = \frac{Gr}{\mathrm{Re}}\ \theta + \lambda + \frac{\partial^2 u}{\partial y^2} - M^2 u \tag{12}$$

$$\frac{\partial \theta}{\partial \eta} = \frac{1}{\Omega \Pr} \frac{\partial^2 \theta}{\partial y^2} \tag{13}$$

The no slip condition at the walls implies that

$$u(-1/4,\eta) = 0 = u(1/4,\eta) \tag{14}$$

While the dimensionless thermal boundary conditions are

$$\theta(\frac{-1}{4},\eta) = \xi \tag{15}$$

$$\theta(\frac{1}{4},\eta) = \xi + \chi + \cos\eta \tag{16}$$

Equations (2) and (4) imply the following constraints on the functions $u(y,\eta)$ and $\theta(y,\eta)$:

$$\int_{-1/4}^{1/4} u(y,\eta)\ dy = \frac{1}{2} \tag{17}$$

$$\int_0^{2\pi} d\eta \int_{-1/4}^{1/4} \theta(\mathrm{y},\eta)\ dy = 0 \tag{18}$$

Obviously, Equation (17) yields the further constraint

$$\int_{-1/4}^{1/4} \frac{\partial u(y,\eta)}{\partial \eta}\ dy = 0 \tag{19}$$

On account of Equation (19), an integration of Equation (12) with respect to y in the range $[-1/4,1/4]$ yields

$$\left.\frac{\partial u}{\partial y}\right|_{y=-1/4} - \left.\frac{\partial u}{\partial y}\right|_{y=1/4} = \frac{\lambda}{2} + \frac{Gr}{\mathrm{Re}} \int_{-1/4}^{1/4} \theta(y,\eta)\ dy \tag{20}$$

The friction factors f_1 and f_2 at the walls $Y = -L$ and $Y = L$ respectively are defined as

$$f_1 = \frac{2v}{U_0^2}\frac{\partial U}{\partial Y}\bigg|_{Y=-L} = \frac{2}{\text{Re}}\frac{\partial u}{\partial y}\bigg|_{Y=-1/4} \qquad f_2 = \frac{-2v}{U_0^2}\frac{\partial U}{\partial Y}\bigg|_{Y=L} = \frac{-2}{\text{Re}}\frac{\partial u}{\partial y}\bigg|_{y=1/4} \tag{21}$$

On account of Equation (3.20), the friction factors and the parameter λ are related as follows:

$$(f_1 + f_2)\text{Re} = \lambda + 2\frac{Gr}{\text{Re}}\int_{-1/4}^{1/4}\theta(y,\eta)\,dy. \tag{22}$$

Analytical solution for steady periodic regime:

Since Equations (12) – (18) are linear, one can define the complex valued functions namely the velocity $u^*(y,\eta)$, temperature $\theta^*(y,\eta)$ and friction factor $\lambda^*(\eta)$ which fulfill the equations

$$\Omega\frac{\partial u^*}{\partial\eta} = \frac{Gr}{\text{Re}}\theta^* + \lambda^* + \frac{\partial^2 u^*}{\partial y^2} - M^2 u \tag{23}$$

$$\frac{\partial\theta^*}{\partial\eta} = \frac{1}{\Omega\,\text{Pr}}\frac{\partial^2\theta^*}{\partial y^2} \tag{24}$$

$$u^*(-1/4,\eta) = \xi \tag{25}$$

$$\theta^*(1/4,\eta) = \xi + \chi + e^{i\eta} \tag{26}$$

$$\int_{-1/4}^{1/4} u^*(y,\eta)\,dy = \frac{1}{2},\ \int_0^{2\pi} d\eta \int_{-1/4}^{1/4}\theta^*(y,\eta)\,dy = 0, \tag{27}$$

and are such that $u = \text{Re}(u^*), \theta = \text{Re}(\theta^*), \lambda = \text{Re}(\lambda^*)$.

In steady-periodic regime, a solution of Equations (27) can be written in the form

$$\begin{aligned} u^*(y,\eta) &= u_a(y) + \frac{Gr}{\text{Re}}u_b(y)\,e^{i\eta} \\ \theta^*(y,\eta) &= \theta_a(y) + \theta_b(y)\,e^{i\eta} \\ \lambda^*(\eta) &= \lambda_a + \frac{Gr}{\text{Re}}\lambda_b\,e^{i\eta}. \end{aligned} \tag{28}$$

Due to the linearity of Equations (27), if we substitute Equations (28) into Equations (27) we obtain two distinct boundary value problems. The first is given by

$$\frac{d^2 u_a}{dy^2} + \frac{Gr}{\text{Re}}\theta_a + \lambda_a - M^2 u_a = 0 \tag{29}$$

$$\frac{d^2\theta_a}{dy^2} = 0 \tag{30}$$

$$\begin{aligned} u_a(-1/4) &= 0 = u_a(1/4) \\ \theta_a(-1/4) &= \xi,\quad \theta_a(1/4) = \xi + \chi \end{aligned} \tag{31}$$

$$\int_{-1/4}^{1/4} u_a(y)\,dy = \frac{1}{2},\ \int_{-1/4}^{1/4}\theta_a(y)\,dy = 0 \tag{32}$$

While the second one is given by

$$\frac{d^2 u_b}{dy^2} - i\,\Omega\,u_b + \lambda_b - M^2 u_b = 0 \tag{33}$$

$$\frac{d^2\theta_b}{dy^2} - i\,\Omega\,pr\,\theta_b = 0 \tag{34}$$

$$u_b(-1/4) = 0 = u_b(1/4),$$
$$\theta_b(-1/4) = 0\,,\ \theta_b(1/4) = 1 \qquad (35)$$

$$\int_{-1/4}^{1/4} u_b^*(y)\, dy = 0, \quad \theta_a(y) = 2\,\chi\, y \qquad (36)$$

Solving the Equations making use of corresponding boundary conditions,
we get

$$u_a = -\frac{\lambda a}{M^2 \cosh(\frac{m}{4})} \cosh My - \frac{Gr\chi \sinh my}{2\,\mathrm{Re}\, M^2 \sinh(\frac{m}{4})} + \frac{2Gr\chi}{\mathrm{Re}\, M^2} y + \frac{\lambda a}{M^2} \qquad (37)$$

Where, $\lambda_a = -\dfrac{M^2}{(4\tanh(\frac{M}{4}) - \frac{1}{M^2})}$.

and $$\theta_b(y) = \frac{\cosh(\sigma y)}{2\cosh(\sigma/4)} + \frac{\sinh(\sigma y)}{2\sinh(\sigma/4)} \qquad (38)$$

$$u_b(y) = \frac{\cosh NY}{2\cosh(N/4)} [\frac{1}{\sigma^2 - N^2} - \frac{2\lambda_b}{N^2}] + \frac{\sinh Ny}{2\sinh(N/4)(\sigma^2 - N^2)}$$
$$- \frac{1}{2(\sigma^2 - N^2)} \{\frac{\cosh(\sigma y)}{\cosh(\sigma/4)} + \frac{\sinh(\sigma y)}{\sinh(\sigma/4)}\} + \frac{\lambda b}{N^2} \qquad (39)$$

where $\lambda_b = \dfrac{N^2}{(\sigma^2 - N^2)}$.

By employing Equation (11), the heat flux per unit area can be written as

$$q = k\,\frac{\partial T}{\partial Y} = \frac{k\Delta T}{D}\,\frac{\partial \theta}{\partial y} \qquad (40)$$

In analogy with the literature (10) – (13), we will define a dimensionless heat flux per unit area, called Nusselt number, as follows:

$$Nu = \frac{qD}{k\Delta T} = \frac{\partial \theta}{\partial y} = \mathrm{Re}(\frac{\partial \theta^*}{\partial y}) = \mathrm{Re}(Nu^*), \qquad (41)$$

Where Nu^* is defined as

$$Nu^* = \frac{\partial \theta_a}{\partial y} + \frac{\partial \theta_a}{\partial y}\, e^{i\eta} = Nu_a + Nu_b e^{i\eta} \qquad (42)$$

Finally, Equations (37) and (39) allow one to express the quantities $f_1\,\mathrm{Re}$ and $f_2\,\mathrm{Re}$ as

$$f_1\,\mathrm{Re} = 2\,\mathrm{Re}(\frac{\partial u_a}{\partial y}\Big|_{y=-1/4} + \frac{Gr}{\mathrm{Re}}\frac{\partial u_b}{\partial y}\Big|_{y=-1/4} e^{i\eta}) = \mathrm{Re}(f_{1a}\,\mathrm{Re} + \frac{Gr}{\mathrm{Re}} f_{1b}\,\mathrm{Re}\, e^{i\eta}) \qquad (43)$$

$$f_2\,\mathrm{Re} = -2\,\mathrm{Re}(\frac{\partial u_a}{\partial y}\Big|_{y=1/4} + \frac{Gr}{\mathrm{Re}}\frac{\partial u_b}{\partial y}\Big|_{y=1/4} e^{i\eta}) = \mathrm{Re}(f_{2a}\,\mathrm{Re} + \frac{Gr}{\mathrm{Re}} f_{2b}\,\mathrm{Re}\, e^{i\eta}) \qquad (44)$$

where $f_{1a}\,\mathrm{Re}$, $f_{1b}\,\mathrm{Re}$, $f_{2a}\,\mathrm{Re}$ and $f_{2b}\,\mathrm{Re}$ are given by

$$f_{1a}\,\mathrm{Re} = 2\frac{\partial u_a}{\partial y}\Big|_{y=-1/4} \qquad f_{1b}\,\mathrm{Re} = 2\frac{\partial u_b}{\partial y}\Big|_{y=-1/4}, \qquad (45)$$

$$f_{2a}\,\mathrm{Re} = -2\frac{\partial u_a}{\partial y}\Big|_{y=1/4} \qquad f_{2b}\,\mathrm{Re} = -2\frac{\partial u_b}{\partial y}\Big|_{y=1/4}, \qquad (46)$$

$$f_{1a}\,\mathrm{Re} = -\frac{2\lambda_a}{M}\tanh(M/4) - \frac{Gr\,\chi}{\mathrm{Re}\,M}\, COth(M/4) - 4\frac{Gr\,\chi}{\mathrm{Re}\,M^2}$$

$$f_{2a}\,\mathrm{Re} \;=\; -\;\frac{2\lambda_a}{M}\;\tanh(M/4)\;+\;\frac{Gr\;\chi}{\mathrm{Re}\;M}COth(M/4)\;-\;4\frac{Gr\;\chi}{\mathrm{Re}\;M^2}$$

$$f_{1b}\,\mathrm{Re} = -N\tanh(N/4)\left(\frac{1}{(\sigma^2-N^2)}-\frac{2\lambda_b}{N^2}\right)+\frac{N\coth(N/4)}{\sigma^2-N^2}-\frac{\sigma}{\sigma^2-N^2}\{-\tanh(\sigma/4)+\coth(\sigma/4)\}$$

$$f_{2b}\,\mathrm{Re} = -N\tanh(N/4)\left(\frac{1}{(\sigma^2-N^2)}-\frac{2\lambda_b}{N^2}\right)-\frac{N\coth(N/4)}{\sigma^2-N^2}+\frac{\sigma}{\sigma^2-N^2}\{\tanh(\sigma/4)+\coth(\sigma/4)\}$$

III. RESULTS AND DISCUSSIONS

We have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time. The velocity field, temperature field, the pressure drop the friction factor and the Nusselt number obtained so for are discussed through graphically for various values of the physical parameters namely the Prandtl number Pr, angular frequency Ω, Hartmann number M.

Figs. (2 – 3) represents the effects of Prandtl number Pr, angular frequency Ω and Hartmann number M on the velocity $|u_b|$ (amplitude of the velocity oscillations). Fig.(2) shows the effect of Pr on the $|u_b|$ with $r=10$ and $M=1$. It is found that, for $\mathrm{Pr}=0.7,\ |u_b|$ is exactly symmetric with respect to the midplane of the channel on the other hand, for $\mathrm{Pr}=7$ and $\mathrm{Pr}=100$, the $|u_b|$ is not symmetric with respect to the midplane of the channel. Fig.(3) depicts the effect of Ω on $|u_b|$ with $\mathrm{Pr}=7$ and $M=1$ .It is observed that, for $\Omega=1,\ |u_b|$ is symmetric with respect to the midplane of the channel, whereas for $\Omega=10$ and $\Omega=100$, the $|u_b|$ is not symmetric with respect to the midplane of the channel. Fig.(4) presents the effect of Hartmann number on $|u_b|$ with $\mathrm{Pr}=7,\Omega=10.$ It is observed that, $|u_b|$ decreases with increasing M at any position except at the else, where velocity oscillations cannot occur.

Fig.(5) shows the effect of Pr on the $|\lambda_b|$ as a function of Ω with $M=1$. It is observed that, the friction factor $|\lambda_b|$ decreases with increase of Ω. Further it is observed, the $|\lambda_b|$ decreases with an increase in Pr. The effect of Hartmenn number M on $|\lambda_b|$ as a function of Ω with $\mathrm{Pr}=0.7$ is shown in Fig.(6). It is found that, the $|\lambda_b|$ decreases with an increase in M . Fig.6(a) represents enlargement of Fig.(6).

Fig.(7). shows the effect of Pr on the amplitude of oscillations of $|f_{1b}\,\mathrm{Re}|$ with $M=1.$ It is observed that, the $|f_{1b}\,\mathrm{Re}|$ decreases with increasing Pr. Fig.(8) depicts the effect of M on the $|f_{1b}\,\mathrm{Re}|$ with $\mathrm{Pr}=7$. It is found that, the $|f_{1b}\,\mathrm{Re}|$ decreases with an increase in M . Fig.(9) presents the effect of Pr on the amplitude of the $|f_{2b}\,\mathrm{Re}|$ (resonance frequency for the oscillations of the friction factor at the wall $Y=L$) with $M=1$. It is observed that, with increase in the Pr, decreases the $|f_{2b}\,\mathrm{Re}|$. The effect of M on the $|f_{2b}\,\mathrm{Re}|$ with $\mathrm{Pr}=7$ is shown in Fig.(10). It was found that, the $|f_{2b}\,\mathrm{Re}|$ decreases with increasing M .

The modules of Nu_b, the steady-temperature wall ($Y=-0.25$), at $Y=-0.05$, and at the midplane of the channel $(Y=0)$ is plotted versus $\Omega\mathrm{Pr}$ in Fig.(11), in the range $0\le\Omega\mathrm{Pr}\le400$. As is illustrated by this figure, the amplitude of the oscillations of the Nusselt number is a decreasing function of $\Omega\mathrm{Pr}$ in the whole interval $-0.25\le y\le0$. On the contrary, is the open interval $0\le y\le0.25$, for every value of y there exists a value of $\Omega\mathrm{Pr}$ which maximizes the modules of Nu_b i.e. there exists a reasonable frequency for the fluctuation of the Nusselt number which is proportional to the inverse of Pr. In this interval, the value of $\Omega\mathrm{Pr}$ which maximizes the modules of Nu_b is an increasing function of y. Finally, at the right wall, the modules of Nu_b are an increasing function of $\Omega\mathrm{Pr}$, and no resonance occurs. These phenomena are illustrated in fig.(12) & (13). In Fig.(12), plots of the modulus of Nu_b versus $\Omega\mathrm{Pr}$ are reported in the range $0\le\Omega\mathrm{Pr}\le1200,$

for $y = 0.1, 0.17 \& 0.2$. For the plots reported in Fig.(12) the resonance frequencies correspond to $\Omega\,\mathrm{Pr} = 93.61, 312.7 \text{and} 800.0.$ In Fig.(13), plots of the modules of Nu_b versus $\Omega\,\mathrm{Pr}$ are reported in the range $0 \le \Omega Pr \le 40,000,$ for $y = 0.22, 0.24 \& 0.25$ (right wall). The first plot presents a resonance frequency for $\Omega Pr = 2222,$ the second presents a resonance frequency for $\Omega Pr = 20,000,$ while the third presents no resonance.

Fig.(14) shows the effect of $\sigma\,\mathrm{Pr}$ as the amplitude of temperature oscillations $|\theta_b|$. The figure shows that, for $\Omega\,\mathrm{Pr} \le 10,$ the amplitude of the temperature oscillations looks like a line as function of y on the other hand, when the value of $\Omega\,\mathrm{Pr}$ becomes higher and higher, the temperature oscillations land to be sensible only in a narrow region of the channel adjacent to the wall $Y = L$.

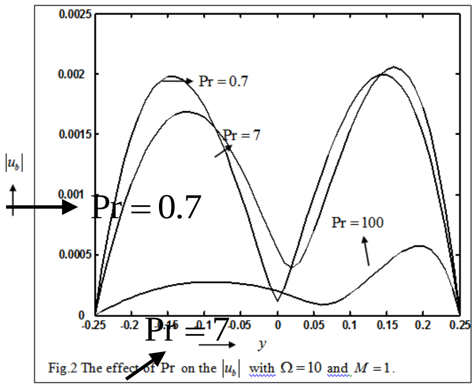


Fig.2 The effect of Pr on the $|u_b|$ with $\Omega = 10$ and $M = 1$.

$|u_b|$

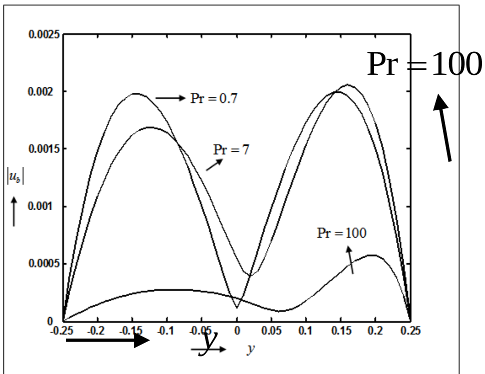


Fig.2 The effect of Pr on the $|u_b|$ with $\Omega = 10$ and $M = 1$.

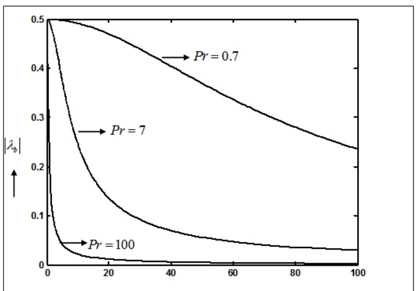


Fig.5 The effect of $Pr\Omega$ on the $|\lambda_b|$ with $M = 1$.

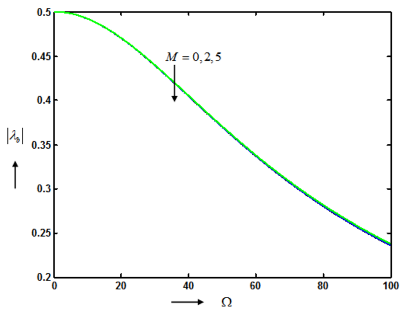


Fig.6(i) The effect of M on the $|\lambda_b|$ with $Pr = 0.7$.

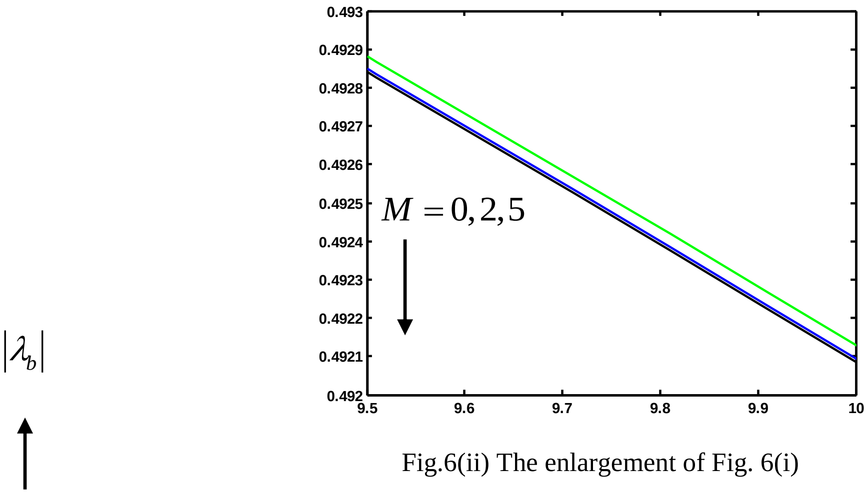


Fig.6(ii) The enlargement of Fig. 6(i)

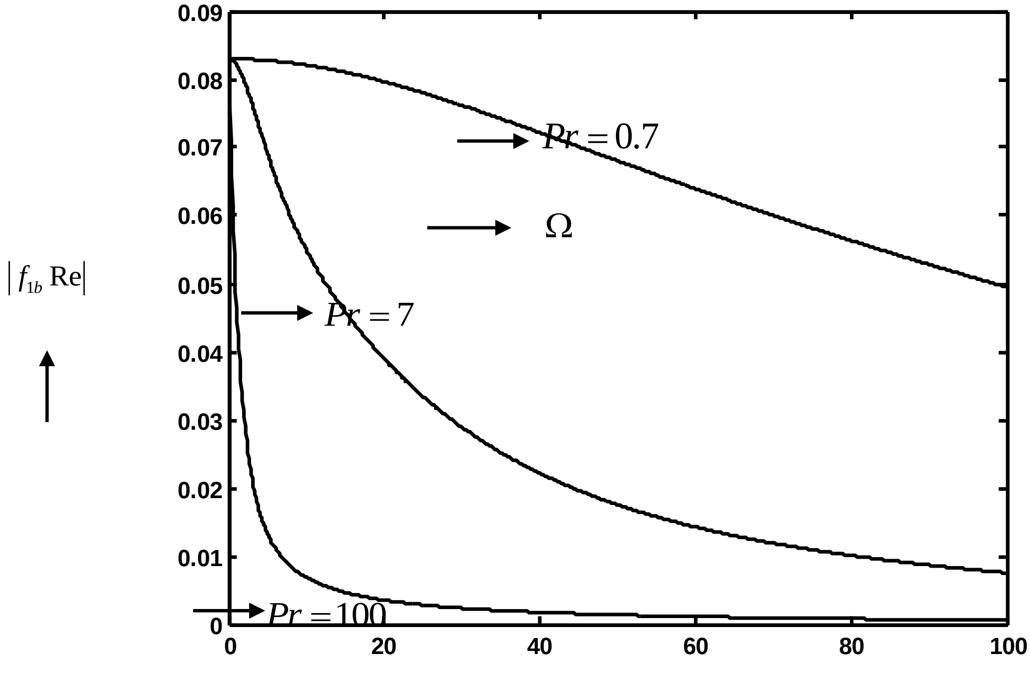


Fig.7 The effect of Pr on the $|f_{1b} \mathrm{Re}|$ with $M = 1$.

$\longrightarrow \Omega$

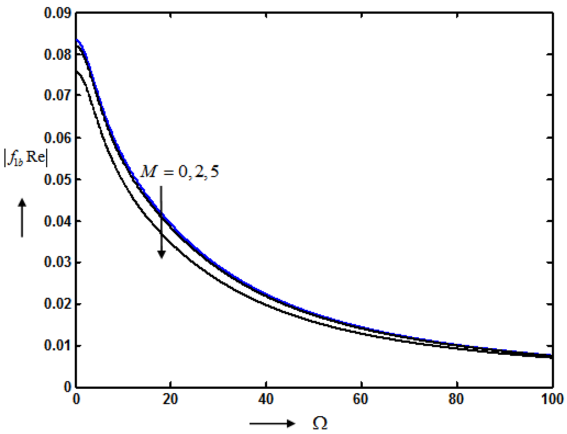


Fig.8 The effect of M on the $|f_{1b}\,\mathrm{Re}|$ with $Pr = 7$.

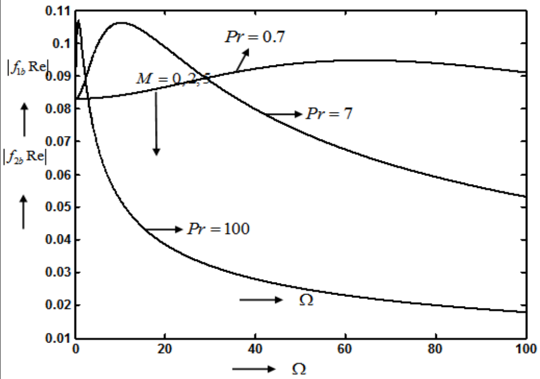


Fig.9 The effect of Pr on the $|f_{2b}\,\mathrm{Re}|$ with $M = 1$.

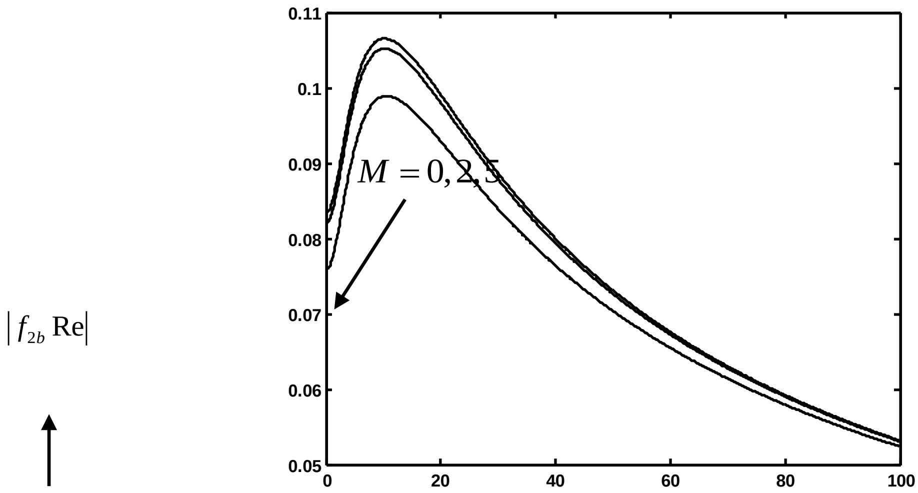
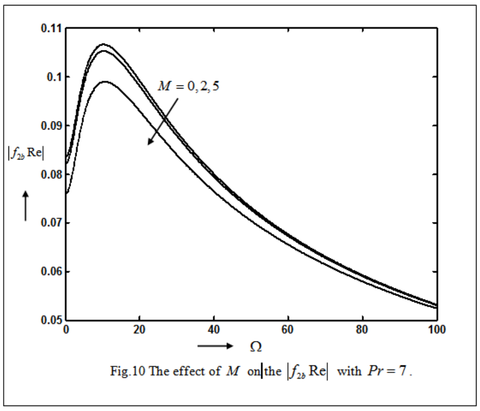


Fig.10 The effect of M



the $|f_{2b}\,\mathrm{Re}|$ with $Pr = 7$.

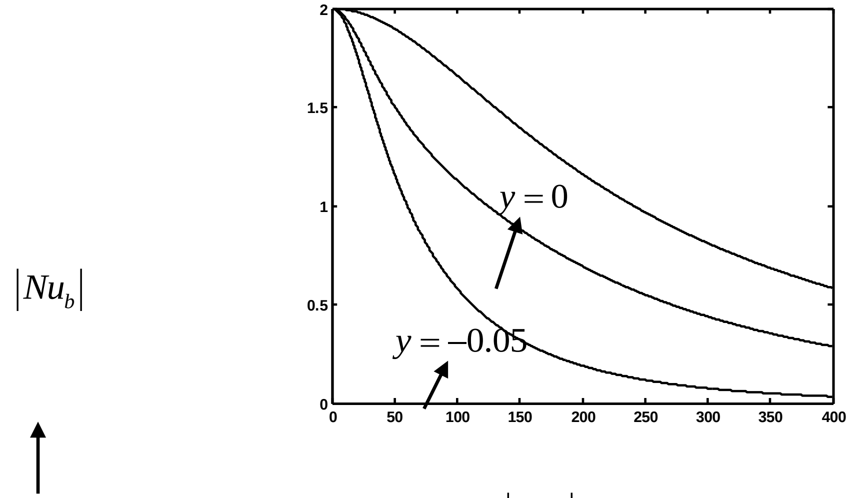


Fig.11 Effect of ΩPr on $|Nu_b|$ in the range $0 \le \Omega Pr \le 400$

$y = -0.25$

for $y = -0.25, -0.05$ and 0.

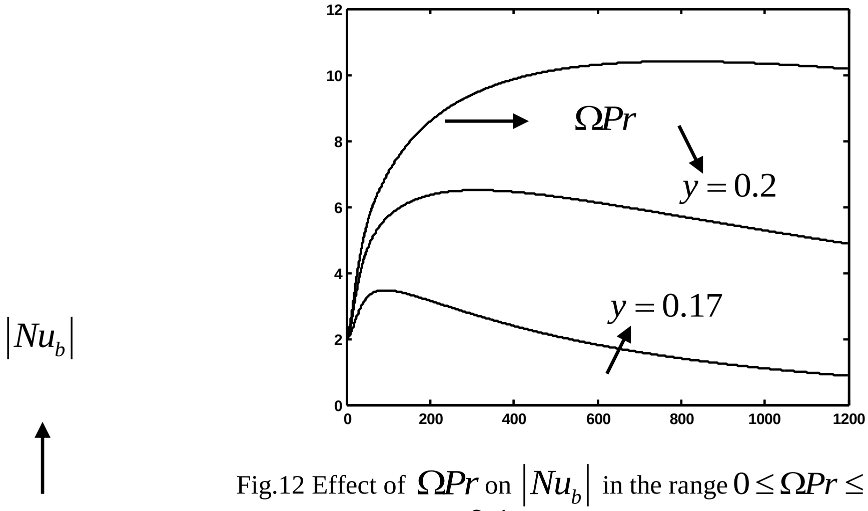


Fig.12 Effect of ΩPr on $|Nu_b|$ in the range $0 \le \Omega Pr \le 1200$

$y = 0.1$

For $y = 0.1, 0.17$ and 0.2.

ΩPr

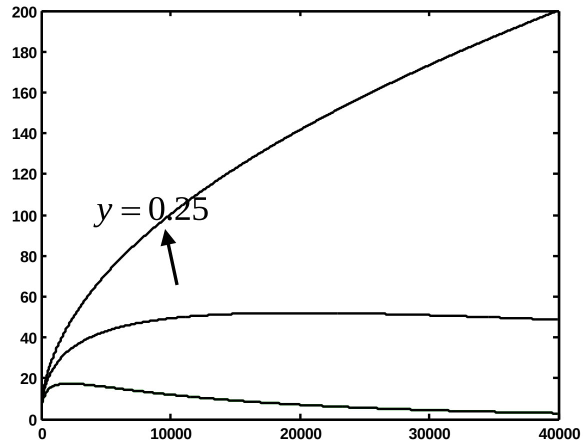


Fig.13 Effect of ΩPr on $|Nu_b|$ in the range $0 \le \Omega Pr \le 40000$ for $y = 0.22, 0.24$ and 0.25.

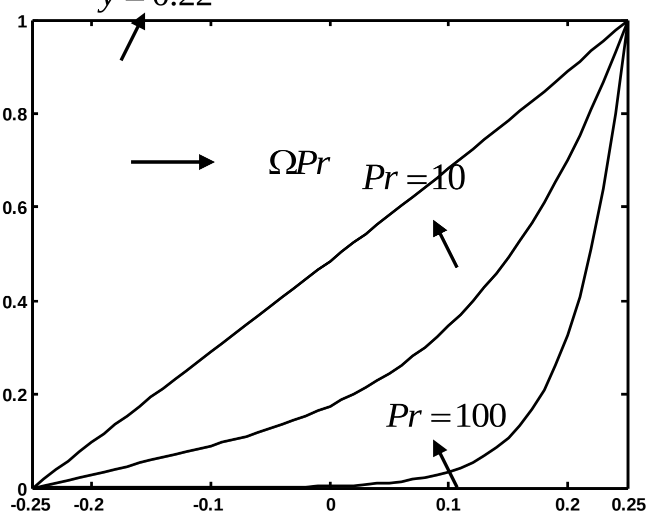


Fig.14 Effect of ΩPr on $|\theta_b|$ for $\Omega \mathrm{Pr} = 10, 100$ and 1000.

IV. CONCLUSIONS

We have studied the MHD steady periodic regime of laminar mixed convective flow through an inclined channel under uniform transverse magnetic field and the temperature of one wall is constant, while the temperature of the other wall is sinusoidal function of a time. The conclusions are made as the following.

[1]. The amplitude of the velocity oscillations is exactly symmetric with respect to the midplane of the channel for Pr=0.71 on the other hand, the same is not symmetric with respect to the midplane of the channel for Pr=7.

[2]. The friction factor $|\lambda_b|$ decreases with increase of angular frequency Ω and Hartmann number M.

[3]. The resonance frequency for the oscillations of the friction factor decreases with increase in Prandtl number Pr and Hartmann number M.

[4]. The amplitude of the temperature oscillations $|\theta_b|$ looks like a line as fraction y on the other hand when the value of ΩPr becomes higher and higher the temperature oscillations land to be sensible only in a narrow of the channel adjacent to the wall.

REFERENCES

[1]. Darcy H., Les Fountaines Publiques de la ville de Dijon, Victor Dulmont, Paris (1856).
[2]. Ostrach, S., Laminar natural convection flow and heat transfer of fluids with and without heat sources in channels with constant wall temperature, NACATN, Vol. 2863 (1952).
[3]. Ostrach, S., New aspects of natural convection heat transfer, Trans. Am. Soc. Mech. Engrs., Vol. 75, pp. 1287-1290, (1953).
[4]. Ostrach, S., Combined natural and forced convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperature, NACA TN, Vol. 3141, (1954).
[5]. Siegel, R., Transient free convection from a vertical flat plate, Transactions of the ASME, Vol. 30, pp. 347-359 (1958).
[6]. Benenati R.F. and BrosilowC.B., Void fraction distribution in beds of spheres, A.I. Che. J., Vol.8, pp. 359-361, (1962).
[7]. Pattabhi Ramacharyulu Ch. N., Exact solutions of two dimensional flows of second order fluid, App. Sci. Res., Sec-A, Vol.15, pp. 41-50, (1964).
[8]. Kaloni P. N., Fluctuating flow of an elastico-viscous fluid past a porous flat plate, Phy. Of Fluids, Vol.10, pp. 1344-1346, (1966).
[9]. Merkin J. H., The effects of buoyancy forces on the boundary layer flow over a semiinfinite vertical flat plate in a uniform stream, J. Fluid Mech., Vol. 35, pp. 439-450, (1969).
[10]. Lyod J.R. and Sparrow E.M., Combined forced and free convection flow on vertical surfaces, Int. J. Heat and Mass Transfer, Vol. 13, pp. 434-438, (1970).
[11]. Gebhart, B. and L. Pera, The nature of vertical natural convection flow from the combined buoyancy effects on thermal and mass diffusion, Int. J. Heat Mass Transfer, Vol. 14, pp. 2024-2050, (1971).
[12]. Pera, L. and B. Gebhart, Natural convection flows adjacent to horizontal surface resulting from the combined buoyancy effects of thermal and mass diffusion, Int. J. Heat Mass Transfer, Vol. 15, pp. 269-278 (1972).
[13]. Soundalgekar, V.M., Viscous dissipation effects on unsteady free convective flow past an infinite vertical porous plate with constant suction, Int. J. Heat Mass Transfer, Vol. 15, pp. 1253-1261 (1972).
[14]. Oosthuizen P. H. and Hart H. A., Numerical study of laminar combined convection flow over a flat plate, Int. J. Heat and Mass Transfer, Vol. 95, pp. 60-63, (1973).
[15]. Wilks G., Combined forced and free convection flow on vertical surfaces, Int. J. Heat and Mass Transfer, Vol. 16, pp. 1958-1966, (1973).
[16]. Lekoudis S.G., Nayef A.H. &Saric., Compressible boundary layers over wavy walls, Phy. Of Fluids, Vol. 19, pp. 514-519, (1976).
[17]. Shankar P.N. and Sinha U.N., The Rayeigh problem for wavy wall, J. Fluid Mech., Vol. 77, pp. 24-56, (1976).
[18]. Lessen M. and Gangwani S.T., Effects of small amplitude wall waviness upon the stability of the laminar boundary layer, Phy. Of Fluids, Vol. 19, pp. 510-513, (1976).
[19]. Berezovsky, A.A., O.G. Martynenko and Yu. A. Sokovishin, Free convective heat transfer on a vertical semi infinite plate, J. Engng. Phy., Vol. 33, pp. 32-39, (1977).
[20]. Soundalgekar, V.M. and P.D. Wavre, Unsteady free convection flow past an infinite vertical plate with constant suction and mass transfer, Int. J. Heat Mass Transfer, Vol. 20, pp. 1363-1373, (1977).
[21]. Soundalgekar, V.M. and P.D. Wavre, Unsteady free convection flow past an infinite vertical plate with variable suction and mass transfer, Int. J. Heat Mass Transfer, Vol. 20, pp. 1375-1380 (1977).
[22]. Cheng P. and Minkowycz W.J., Free convection about a vertical plate embedded in a porous medium with application to heat transfer from a dike, J. Geo. Phy. Res.,Vol. 82, pp. 2040-2044, (1977).
[23]. Murthy P.V.S.N. and Singh P., Thermal dispersion effects on non-Darcy natural convection with lateral mass flux, Int. J. Heat and Mass Transfer, Vol. 33, pp. 1-5, (1977).
[24]. Vajravelu K. and Sastri K.S., Free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat plate, J. Fluid Mech., Vol. 86, pp. 365-383, (1978).
[25]. Cheng P., Heat transfer in geothermal systems, Adv. Heat Transfer, Vol. 4, pp.1-105, (1978).
[26]. Mucoglu A. and Chen T.S., Mixed convection on inclined surfaces, J. Heat Transfer, Vol. 10, pp. 422-426, (1979).
[27]. Mac Donald I. F., El-Sayed, Mow M.S. and Dullien, Flow through porous medium the Ergun equation revisited, Indust. Chem. Fundam., Vol. 18, pp.199-208, (1979).
[28]. Chen, T.S., C.F. Yuh and A. Moutsoglou, Combined heat and mass transfer in mixed convection along a vertical and inclined plates, Int. J. Heat Mass Transfer, Vol. 23, pp. 527- 537 (1980).
[29]. Merkin J.H. and Mahmood, Mixed convection boundary layer flow on a vertical surface in a saturated porous medium, J. Eng. Mathematics, Vol. 14, No.4, pp. 301-313, (1980).
[30]. Tsuruno S and Iguchi. I., Prediction of combined free and forced convective heat transfer along a vertical plate with blowing. ASME, J Heat Transfer, Vol. 102, pp. 168-170, (1980).
[31]. Plumb O.A. and Huenefeld J.C., Non-Darcy natural convection from heated surfaces in saturated porous media, Int. J. Heat and Mass Transfer, Vol. 24, pp.765-768. (1981).
[32]. Joseph D.D., Nield D.A. and Papanicolaou G., Non – linear equation governing flow in a saturated porous medium, Water Resources Res. ,Vol. 18, pp.1049-1052, (1982).
[33]. Vafai K. and Tien C.L., Boundary and inertia effects on convective mass transfer in porous media, Int. J. Heat and Mass Transfer, Vol. 25, pp.1183-1190, (1982).
[34]. Martynenko, O.G., A.A. Berezovsky and Yu. A. Sokovishin, Laminar free convection from a vertical plate, Int. J. Heat Mass Transfer, Vol. 27, pp. 869-881, (1984).
[35]. Bejan A. and Poulikakos D., The non-Darcy regime for vertical boundary layer natural convection in a porous medium, Int. J. Heat Mass Transfer, Vol. 27, pp.717-722. (1984).
[36]. Rajagopal K. and Gupta A. S., An exact solution for the flow of a non-Newtonian fluid past an infinite porous plate , Meccanica, Vol. 19, pp.158-160, (1984).
[37]. Chandrasekhara B.C. and Namboodiri P.M.S., Influence of variable permeability on combined vertical surfaces in porous medium, Int. J. Heat Mass Transfer, Vol. 28, pp.199- 206, (1985).
[38]. Hong J.T., Yamada Y. and Tien C.L., Effect of non-Darcian and non-uniform porosity on vertical plate natural convection in porous media, Int. J. Heat and Mass Transfer, Vol.109, pp.356-382, (1987).
[39]. Lai F.C. and Kulacki F.A., Non-Darcy convection from horizontal impermeable surface in saturated porous medium, Int. J. Heat and Mass Transfer, Vol. 30, pp.2189-2192, (1987).
[40]. Nakayama A. and Koyama H., Effect of thermal stratification on free convection within a porous medium, J. Therm. Phy. and Heat Transfer, Vol. 1, pp.282-285, (1987).
[41]. Ramachandran N. and Armaly B.F. and Chen T.S., Measurements of laminar mixed convection flow adjacent to an inclined surface, J. Heat Transfer, Vol. 109, pp. 146-151, (1987).

[42]. Kumari M., Pop I. and Nath G., Non-Darcian effects on forced convection heat transfer over a flat plate in a highly porous medium, Acta Mechanica, Vol. 84, pp.201-207, (1990).

[43]. Ramanaiah, G. and G. Malarvizhi, Free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient, ActaMech., 87, pp.73-80 (1991).

[44]. Wickern G., Mixed convection from an arbitrarily inclined semi-infinite flat plate-I The influence of the inclination angle, Int. J. Heat and Mass Transfer, Vol. 34, pp.1935-1945, (1991).

[45]. Das U.N. and Ahmed N., Free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, Ind. J. Pure Appl. Math., Vol. 23, pp. 295-304, (1992).

[46]. Hsieh J.C., Chen T.S. and Armaly B.F., Non-similarity solutions for mixed convection from vertical surfaces in porous medium with variable surface temperature or heat flux, Int. J. Heat Mass Transfer, Vol. 38, No.4, pp.1485-1493, (1993).

[47]. Knupp P.M. and Lage J.L., Generalization of the Forchheimer –extended Darcy flow model to the tensor permeability case via a variational principle, J. Fluid Mech., Vol. 299, pp. 97- 104, (1995).

[48]. Hung C.I. and Chen C.B., Non-Darcy free convection in a thermally stratified porous medium along a vertical plate with variable heat flux, Int. J. Heat and Mass Transfer, Vol. 33, pp.101-107, (1997).

[49]. Patidar R.P. and Purohit G.N., Free convection flow of a viscous incompressible fluid in a porous medium between two long vertical wavy walls, Ind. J. Math., Vol. 40, pp. 76-86, (1998).

[50]. Vighnesam, N.V. and V.M. Soundalgekar, Combined free and forced convection flow of water at 4oC from a vertical plate with variable temperature, Ind. J. of Engineering and Material Sciences, Vol. 5, pp. 124-126 (1998).

[51]. Das, U.N., R.K. Deka and V.M. Soundalgekar, Transient free convection flow past an infinite vertical plate with periodic temperature variation, J. HeatTransfer (ASME), Vol. 121, pp. 1091-1094 (1999).

[52]. Kuznetsov A.V., Investigation of the effect of transverse thermal dispersion on forced convection in porous media, Acta Mechanica, Vol. 145, pp. 35-43, (2000).

[53]. Mohammadien A.A. and El-Amin M.F., Thermal dispersion – radiation effects on nonDarcy natural convection in a fluid saturated porous medium, Transport in porous media, Vol. 40, pp. 153-163, (2000).

[54]. Hossain, M.A., S. Hussain and D.A.S. Rees, Influence of fluctuating surface temperature and concentration on natural convection flow from a vertical flat plate, ZAMM, Vol. 81, pp. 699-709 (2001).

[55]. Wang C., Liao S.J. and Zhu J.M., An explicit analytic solution for non-Darcy natural convection over horizontal plate with surface mass flux and thermal dispersion effects, Acta Mechanica, Vol. 165, pp. 139-150, (2003).

[56]. Sharma, P.K. and R.C. Chaudhary, Effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime, Emirates Journal for Engineering Research, Vol. 8, pp. 33-38 (2003).

[57]. Sharma, P.K. and B.K. Sharma, "Influence of variable suction on unsteady free convective flow from a vertical flat plate and heat transfer in slip-flow regime", Ganita Sandesh, Vol. 18, pp. 55-62 (2004).

[58]. Taneja R and Jain N.C., MHD flow with slip effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall, Def. Sci. J., Vol. 54 (1), pp. 21-29, (2004).

[59]. Ramana Murthy Ch.V. and Kulkarni S.B., On the class of exact solutions of an incompressible fluid flow of second order type by creating sinusoidal disturbances, Def. Sci. J., Vol. 56, pp. 733-741, (2005).