

## A Mathematical Model for Predicting Draught Output of an Animal-Drawn Mouldboard Ridger

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**ABSTRACT:** -A Mathematical model for predicting the draught requirement of animal-drawn ridger implement was developed using implement and soil properties. The implement properties such as speed of operation, tillage depth, implement mass was considered in the model developed. The soil properties were soil moisture content and soil bulk density. The animal draught output model was developed by dimensional analysis, using the concept of Buckingham's Pi Theorem. The assumptions were derived from empirical results. The developed model was verified by comparing the predicted draught output by the model with the actual experimental results conducted in the field. The result reveals that the predicted model correlates well with the experimental results as could be seen from the  $r^2$  value of 0.9792. Also, a paired t-test revealed that the difference between the means of the predicted and measured experimental data was not statistically significant at 0.05 significant levels. The model can be used by animal-draught farmers in selecting the appropriate animal implement combination for any farm operation. The model can also be of value to scholars, consultants, government and international agencies interested in the development of animal drawn tillage implements especially in Nigeria.

**Keywords:** Model, Prediction, Animal-drawn, Mouldboard ridger, Draught requirement, dimensional analysis

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Date of Submission: 05-08-2019

Date of acceptance: 20-08-2019

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### I. INTRODUCTION

Despite the substantial progress made in mechanization of agriculture in the world, the challenges facing agricultural policy makers in the developing countries is how to increase agricultural production more than the subsistence production through increase in yield and labour saving. Nowadays, improvements in farm power availability through development and improvement of soil specific and farmer-friendly technologies for predominantly global agro-eco-regions, humans, draught animals and engines provide the motive power in various proportions for crop production, harvesting, transport and processing (Ogdigboh, 2000; Onwualuet al., 2006).

A number of researchers (Gbadamosi and Magaji, 2004; Sanni, 2008; Kawuyoet al., 2012) have all agreed that the introduction and adaption of draught animal technology in Africa and Nigeria in particular is an appropriate, affordable and sustainable technology. The most common work animals used are oxen generally worked in pairs with wither yokes, used for tilling and harrowing upland soils. Animal drawn tillage implements hold immense potentials for reducing the cost of production of crops especially if methods for reducing its perceived excessive power requirements in tillage operation can be found. Practically, this can be accomplished by carrying out studies on the soil-tool interaction system that can be used to study its performance (Ademiluyiet al., 2007; Kawuyo, 2011). Several mathematical models have been developed to predict the characteristics and performance of the ploughing tools and their action on the soil (Onwualuet al., 1991; Okokoet al., 2018). In spite of the capability of various methods to model soil-tool interaction, researchers (Amara and Guedioura, 2013; Okokoet al., 2018) are still working on new models to compensate for some shortcomings of the current models that are either complicated or ignoring some basic aspects that affect the results. Theoretical studies of animal-implement interaction have been made by Inns (1991), Chawatamaet al. (2003) and Bobobee (2007). These interactions influence the design and operational adjustment of both harnesses and implements, the development of which is still proceeding mainly on an empirical basis. Empirical models, by nature are limited in applicability, and can therefore not be used to design shares that may result in improved performance of the ploughs (Zelenin, 1950). Presently there are very few published data available on draught requirements of agricultural implements operating on soils of Nigeria. All the draught data presented in the ASAE Standards (1994) were based mostly on USA soils (Al-Janobi and Al-Suhaibaini, 1998; Kawuyoet al., 2012). This data constraint hinders farmers and extension workers from selecting the appropriate size of animals and implements for a particular farm operation (Fall and Faye, 2010). The purpose of this study was to develop and verify a mathematical model for predicting the draught output requirements of animal drawn

mouldboardridgertillage implement to provide a decision support system for agricultural planners and development agencies in achieving an efficient utilization of animal traction for crop production through choice of animals and implements combination to perform specific field operations.

## II. MATERIALS AND METHODS

This research work uses both theoretical as well as field experimental work. The theoretical part includes the development of the mathematical model for predicting the draught performance characteristics of the animal drawn ridger tillage tool. The field experiment involved the collection of data on-farm and on-station for development and validation of the mathematical model.

### Theoretical Development: Modeling Factors Affecting Draught

Draught being a function of various factors is affected by soil type and condition, implement type and shape, and animals which pull the implements. In addition to these, depth of operation, operating speed, power requirement, mass of the implement, soil moisture content and density of the soil affect the draught of animal-drawn implements. In this study, modeling of the effects of several pertinent factors that affect implement draught was carried out. These factors include soil moisture content, soil bulk density, speed of operation, depth of soil tillage, and implement mass.

### Assumptions

The model development was based on certain assumptions in order to reduce the number of parameters involved in the draught determination to a manageable level: Soil in the test plots is assumed to be of homogenous type, the tillage treatments considered are economical of optimum draught characteristics with minimum labour requirements, the animals and operators used in this study are well experienced in carrying out field operations and the yoking system is efficient.

### Formulation of the Prediction Equations

The output draught requirement of animal drawn mouldboardridger model was developed by dimensional analysis using the Buckingham Pi theorem to derive a relationship between various physical quantities (Gajda and Biles, 1978). Considering some pertinent factors affecting draught, mathematical model of the implement draught can be represented as a function of dependent variable. The 'effect' (draught) could be hypothesized as a function of the 'cause' (draught characteristics) such as mass of the implement, depth of operation, speed of operation, soil bulk density and moisture content (Olatunji *et al.*, 2009; Okoko *et al.*, 2018). Abstracting from error terms, the 'cause-effect' function could be expressed as:

$$f(D, M, \rho, d, S, I_m) = 0 \tag{1}$$

Where:

D = Draught, N

$I_m$  = Implement mass, kg;

S = Speed of operation, m/s;

d = Depth of operation, m;

M = Soil moisture content, %;

$\rho$  = Soil bulk density,  $\text{kg/m}^{-3}$ ;

f = functional relationship between the variables.

The general relationship between the dependent and the independent variables may be expressed as:

$$D = f(M, \rho, d, S, I_m) \text{ or } f(D, M, \rho, d, S, I_m) = 0 \tag{2}$$

Using the three basic dimension systems of mass (M), length (L) and time (T), the variables and their corresponding dimensions used in the model development are given in Table 1. while the dimension matrix is presented in Table 2.

The procedure for applying the Buckingham's Pi Theorem to identify the dimensionless group to be formed is as follows:

Total number of quantities involved (n) = 6, Number of basic dimensions involved (b) = 3, the number of dimensionless (Pi terms) to be formed ( $N_p$ ) is given as:  $6-3=3$

**Table 1.** Variables and their Corresponding Dimensions

Variables	Symbol	Unit	Dimensional symbol (M,L,T)
Draught	D	N	$MLT^{-2}$
Soil moisture content	M	%	$M^0L^0T^0$
Soil bulk density	$\rho$	$\text{Kg/m}^3$	$ML^{-3}$
Depth of operation	d	m	L
Operating speed	S	m/s	$LT^{-1}$
Mass of the implement	$I_m$	Kg	M

**Table 2.** Dimensional Matrix of Variables

	D	M	P	D	S	I <sub>m</sub>
M	1	0	1	0	0	1
T	-2	0	0	0	-1	0
L	1	0	-3	1	1	0

Considering S, ρ, and d as repeating quantities

$$\pi_1 = f(S, \rho, d, D) = f[S]^a [d]^b [\rho]^c [D]$$

Equating the dimensions and solving for the exponents:

$$M^0 L^0 T^0 = [L T^{-1}]^a [L]^b [M L^{-3}]^c [M L T^{-2}] \quad \text{This is dimensionless}$$

$$\pi_1 = k_1 [S^{-2} \rho^{-1} d^{-2} D] = k_1 \frac{D}{S^2 \rho d^2} \quad 3$$

$$\pi_2 = f(S, \rho, d, I_m) = f[S]^a [\rho]^b [d]^c [I_m]$$

Equating the dimensions and solving for the exponents:

$$M^0 L^0 T^0 = [L T^{-1}]^a [M L^{-3}]^b [L]^c [M] \quad \text{This is also dimensionless}$$

$$\pi_2 = k_2 [S^0 \rho^{-1} d^{-3} I_m] = k_2 \frac{I_m}{\rho d^3} \quad 4$$

$$\pi_3 = k_3 M = M^0 L^0 T^0 \quad 5$$

$\pi_3 = k_3 M$ . Another dimensionless term

$k_1, k_2, k_3$ , represent an unknown function.

The three Pi terms required and the equation can be written as:

$$\pi_1 = f(\pi_2, \pi_3) \quad 6$$

Where:

$$\pi_1 = \frac{D}{S^2 d^2 \rho}, \quad \pi_2 = \frac{I_m}{\rho d^3}, \quad \pi_3 = M$$

These Pi terms specify the requirement for similarity and therefore included ratios defining geometric and dynamic similarities for the variables which are relevant to this problem. The general solution can therefore be written from the dimensional analysis including three dimensionless groups ( $\pi_1, \pi_2$  and  $\pi_3$ ) as:

$$\frac{D}{S^2 d^2 \rho} = f\left(\frac{I_m}{\rho d^3}, M\right) \quad 7$$

This involved an unknown function f. The formulation of the prediction equations involves the determination of the function for the general equation.

### Field Experimental Study

The main field studies was conducted at the Department of Agricultural and Environmental Engineering Research Farm of the ModibboAdama University of Technology Yola within the Savannah region, Yola area (9° 14' N and 12° 32' E) of Adamawa State – Nigeria. The area is at an elevation of 200m above sea level and falls within the Eastern Sudan Savanna ecological zone of Nigeria. The area is an agrarian tropical environment marked by dry season (November-April) and wet season (May-October) with mean annual rainfall usually ranges from 700mm to 1,050mm (Adebayo and Tukur, 1999; UBRBDA, 2013). The soils type in the area is predominantly of sandy loam textures (Kabri, et al., 2010). The field study focused on the use of a pair of Zebu bulls as draught animals. The implement considered in this study was the animal-drawn mouldboardridgerthe most commonly used in the study area (Haquet al., 2000). The variables observed include: Age of each animal (Yrs.), Weight of each animal (kgf), Pulling force (N), angle of pull in degree (°), distance travelled (m), working time (s), working speed (m/s), implement working depth (cm), implement working width (cm) and soil gravimetric water content (%). A 1 x 3 x 3 factorial experiment involving three speed levels (S<sub>1</sub>, S<sub>2</sub>, S<sub>3</sub>) and three depth levels (d<sub>1</sub>, d<sub>2</sub>, d<sub>3</sub>) in a Completely Randomized Design (CRD) was designed for the study. The experimental comprised of nine treatments replicated three times. These values were used as input data for the development of the model for prediction of draught requirement of animal-drawn ridger tillage implement. The Field data collected from the different treatment areas in real farmer environments within the study area were used as input data for the validation of the model developed.

### Determination of the Component Equations

The component equations are formed from the plots of  $\pi_1$  against  $\pi_2$  and  $\pi_1$  against  $\pi_3$  generated from experimental data (Table 3) using equation 6 above. The component equations may be combined by either summation or multiplication to form the general prediction equation, the selection of which depends on criteria a set of component equations satisfies (Kawuyo, 2011). The plot of the  $\pi$  –terms (Figures 1 – 2) showed that each

of the component equations formed a plane surface in linear space. These regression equations, being linear, favored combination by summation.

**Table 3.** Experimental Data for the Determination of  $\pi$ -terms

Treatment		Draught (D) N	Depth of Cut (d) cm	Bulk Density ( $\rho$ ) g/cm <sup>3</sup>	Moisture Content (MC) %	Implement Weight ( $I_m$ ) kg
1	s1d1	301.5	8.00	1.18	6.08	56.7
2	s1d2	308.0	13.52	1.43	5.78	
3	s1d3	320.0	18.06	1.59	5.47	
4	s2d1	314.1	8.00	1.26	4.81	
5	s2d2	337.1	13.52	1.58	4.20	
6	s2d3	346.3	18.06	1.78	4.70	
7	s3d1	323.2	8.00	1.45	4.31	
8	s3d2	342.0	13.52	1.65	4.43	
9	s3d3	356.4	18.06	1.86	5.11	

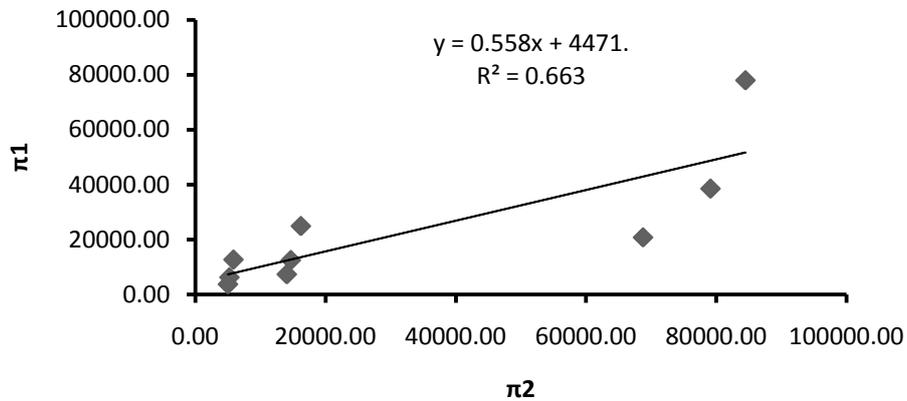


Figure 1. Plot of  $\pi_1$  against  $\pi_2$

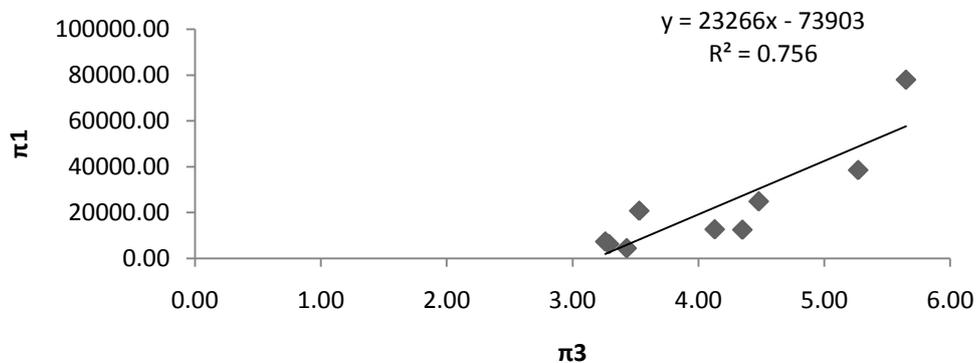


Figure 2. Plot of  $\pi_1$  against  $\pi_3$

The general condition for summation with respect to the three  $\pi$ -terms is given as:

$$\pi_1 = f(\pi_2, \bar{\pi}_3) + f(\bar{\pi}_2, \pi_3) - f(\bar{\pi}_2, \bar{\pi}_3) \tag{8}$$

Where,

$f(\bar{\pi}_2, \bar{\pi}_3)$  = constant of summation (C).

The regression equations representing the component equations are:

$$\pi_1 = 0.5585\pi_2 + 4471.9 \tag{9}$$

$$\pi_1 = 23266\pi_3 - 73903 \tag{10}$$

**Determination of the Constant of Summation (C)**

The constant of summation (C) for the three  $\pi$ -terms is:

$$C = f(\bar{\pi}_2, \bar{\pi}_3) \tag{11}$$

The constant C can be evaluated from any of the component equations

$$C = 0.5585\pi_2 + 4471.9 \quad 12$$

Or

$$C = 23266 \pi_3 - 73903 \quad 13$$

**2.5 Determination of Prediction Equation**

The general prediction equation for the system involving the three  $\pi$ -terms as indicated in equation (8) was formed by adding the component equations as:

$$\pi_1 = F(\pi_2, \pi_3) + F(\bar{\pi}_2, \pi_3) - C \quad 14$$

Substituting equations 10 and 11 into equation 12

$$\pi_1 = 0.5585 \pi_2 + 23266 \pi_3 - 118089.87 \quad 15$$

Therefore:

$$\frac{D}{s^2 d^2 \rho} = 0.5585 \frac{I_m}{\rho d^3} + 23266 M - 118089.87$$

$$D = s^2 d^2 (0.5585 I_m d^{-3} + 23266 \rho M - 118089.87 \rho) \quad 16$$

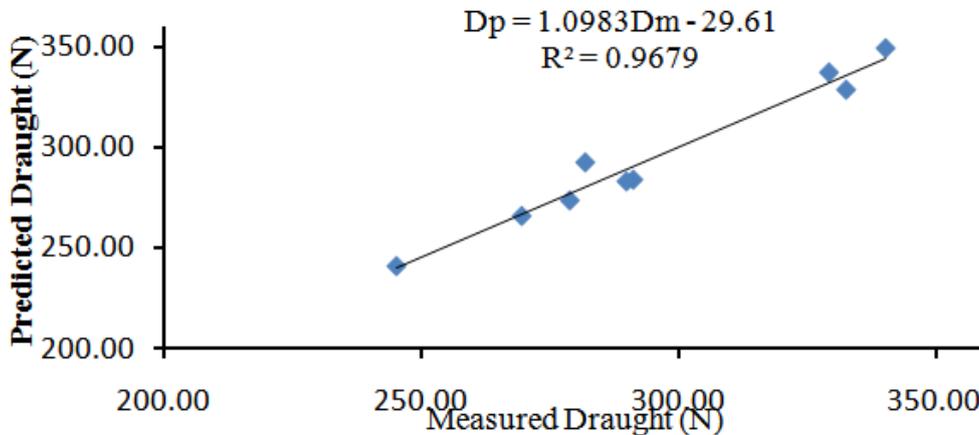
Equation 16 gives the required model for predicting the draught requirement of animal-drawn tillage tool.

**II. RESULTS AND DISCUSSIONS**

**Model Validation**

The proposed model Equation 16 was validated by comparing the results of the model generated and measured field experimental draught under farmers' field conditions. The result showed a very well agreement between the predicted and field experimental data with  $R^2 = 0.9679$  as shown in Figure 3 with the value of the slope and intercept (regression coefficient) as 1.0983 and 29.61 respectively. The regression equation obtained from the least square analysis for the predicted and measured results were:

$$D_p = 1.0983 D_m - 29.61 \quad 17.$$



**Figure 3.** Predicted Draught Output against the Measured Draught output for mouldboardRidger

A paired t-test conducted for the means value of the predicted and measured data at 0.05 significant levels showed that the calculated t-value (0.2357) is less than the t-critical value (2.306). The results show that the model has a high correlation with the measured data from the animal drawn ridger tool (Table 4).

**Table 4.**t-Test: Paired Two Sample for Means of Measured and Predicted Draught Data

	Measured Variable 1	Predicted Variable 2
Mean	295.454	294.892
Variance	1032.14	1286.31
Observations	9	9
Pearson Correlation	0.98384	
Hypothesized Mean Difference	0	
Df	8	
t Stat	0.23571	
P(T<=t) one-tail	0.40979	
t Critical one-tail	1.85955	
P(T<=t) two-tail	0.81958	
t Critical two-tail	2.306	

Considering the overall means, the measured output data was 1.1% greater than the predicted output. However, the likely discrepancies noted in the measured draught requirement values could be due to the assumptions regarding the homogeneity of the soil and strength parameters used in the model; and the unsteady interaction between the soil-tool interface arising from tool-frame instability and the soil movement over the mouldboard during the tillage operations.

#### IV. CONCLUSIONS

The equation  $D = s^2 d^2 (0.5585 I_m d^{-3} + 23266 \rho M - 118089.87 \rho)$  was formulated to predict the draught requirement of animal drawn mouldboardridger tillage implement using a mathematical model which includes parameters such as implement mass, speed of operation, soil moisture content and bulk density. The differences between the means of the predicted and measured draught output of the animal – drawn mouldboardridgerdraught requirement are not statistically significant at 5 % level of significance. The draught requirements prediction results obtained are promising and showed a very well agreement between the predicted and experimental data. The developed model provides a simplified representation of the processes in soil tillage and the interactions of the speed and depth of operation at the farm level. It can be used as a tool for the strategic planning in tillage operations to improve the efficiency of draught animal power in crop production by estimating effect of work stress, work requirement and energy requirements for maintenance in Bosindicus cattle.

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