International Journal of Engineering Inventions
ISSN: 2278-7461, www.ijeijournal.com
Volume 1, Issue 5 (September2012) PP: 56-59

Development of Design Charts for the Deflection of a Loaded Circular Plate Resting on the Pasternak Foundation

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Abstract—In this paper, nondimensional expression for the deflection of a thin circular elastic plate resting on the Pasternak foundation by the strain energy approach has been adopted. Various design charts has been developed for different nondimensional values of the various parameters like modulus of subgrade reaction (k_s*) and shear modulus (G*) for estimating the deflection (w*) of the circular plate.

Keywords—Pasternak Foundation; Modulus of Subgrade Reaction; Shear Modulus and Deflection.

Notations

a	Radius of the plate (m)
D	Flexural rigidity of the plate (Nm)
G	Shear modulus of the Pasternak foundation (N/m^2)
H	Thickness of the Pasternak shear layer (m)
k_s	Modulus of subgrade reaction of the Pasternak foundation (N/m^3)
q	Reaction for the foundation to applied concentrated load (N/m^2)
Q	Concentrated load applied at the centre of the plate (N)
r	radius vector of any point on the plate (m)
r_b	Contact radius (= b/a) (dimensionless)
w	Deflection of the plate (m)
υ	Poisson’s ratio of the plate material (dimensionless)

I. INTRODUCTION

The soil-foundation interaction problem is of importance to many engineering situations, which include the study of raft foundations, pavement of highways, water tanks, airport runways, buried pipelines and pile foundations. A major proportion of these studies are primarily concerned with elastic analysis of the interaction problem. A complete analysis of the interaction problem for elastic bodies generally requires the determination of stresses and strains within the individual bodies in contact. For generalized foundations, the model assumes that at the point of contact between plate and foundation there is not only pressure but also moments caused by interaction between the springs. Such a continuum model is often used to represent flexible structures that are essentially one-dimensional in geometry. The ground may be represented by foundation model like the one-parameter Winkler model or the two-parameter models proposed by Pasternak (1954) and Vlazov and Leont’ev (1966) or higher order models like those developed by Kerr (1964) and Reissner (1958). In recent years, efforts have been made to analyze the thin circular elastic plate resting on a two parameter Pasternak foundation under concentrated and distributed loads for determining approximate plate deflection. The analysis also considers the effect of tensionless characteristics of the Pasternak foundation, that is, lift off of the plate from the foundation surface. It was reported that lift off appears when the slopes of the foundation surface and that of the plate are not equal. A circumferential shear force develops along the contour of the plate due to the slope discontinuity of the plate and soil surfaces. It also reflects the lift off the plate and, consequently, the soil reactions develop within the range of contact radius.

II. REVIEW OF PAST WORKS

A system of equations was developed by Reissner (1945) for the theory of bending of thin elastic plates which takes into account the transverse shear deformability of the plate. Mindlin (1951) deduced the three dimensional equations of elasticity in to a two dimensional theory of flexural motions of isotropic elastic plates and computed the velocities of straight-crested waves and found to agree with those obtained from the three-dimensional theory. Galletly (1959) studied the circular plates on a generalized elastic foundation. Rodriguez (1961) presented the three-dimensional bending of a ring of uniform cross-sectional area and supported on a transverse elastic foundation. The modified Bessel function for the three-dimensional case of a finite beam or plate was given by Kerr (1964) by using the Kernal function. Weitsman (1970) analyzed the one parametric Winkler and Reissner foundation under a single concentrated load under the assumption that tensile stresses could not be transmitted across the interface between beams or plates and their supporting sub grade. This assumption forms no contact regions under the beam or plate. Due to no contact region the beam or plate lifts up away from its foundation and the sub grade does not support the structure. The lift off phenomenon under various boundary conditions based on the beam or plate was also reported.

Celep (1988) presented a phenomenon that a thin circular plate subjected to eccentric concentrated load and moment as well as uniformly distributed load has been supported on an elastic foundation of Winkler type foundation reacts in compression only and he reported that the plate will lift off when the foundation stiffness is low. It has been also reported that, if the edge of plate is not anchored against lifting off, the plate may have a partial contact with the foundation.

Celep and Turhan (1990) analyzed the axisymmetric vibrations of a circular plate subjected to a concentrated dynamic load at its center. The solution given by them was the extended solution given by Weitsman (1970). The solution was carried out by using the Galerkin's method and the modal functions of the completely free plate. This problem was a nonlinear one but linear theory of plate was used in the problem.

Guler and Celep (1993) reported the extended study of axisymmetric vibrations of a circular plate on tensionless elastic foundation by Celep and Turhan (1990). The solution was given by using Galerkin's method using the modal functions of the completely free plate. Various contact curve and time variation with the central and edge deflection were shown. The most valuable result was given that increasing the load values, keeping loading eccentricities value constant, an increase in the displacements without changing the extent of the contact.

Utku et al. (2000) presented a new formulation for the analysis of axisymmetrically loaded circular plates supported on elastic foundation. In this formulation the circular plate was modeled as a series of concentric annular plates in such a way that each annular plate is considered as simply supported along the inner and outer edges, hence no contact develops between the plate and the foundation. The deflection and radial bending moment were chosen as the solution variables at each interior support. This analysis has been presented the circular plates on tensionless Winkler foundation; that reacts in the compression only.

Wang et al. (2001) presented the exact axisymmetric buckling solutions of Reddy circular plates on the Pasternak foundation. The solution was the extended solution of Kirchhof's buckling solution of circular plates without elastic foundation.

Guler (2004) analyzed a thin circular plate resting on a two parameter Pasternak type foundation under concentrated central and distributed loads. It was analyzed the effect of tensionless characteristics of the Pasternak foundation system. For the analytical expressions, lift off (plate takes lift from the foundation surface) had been considered. The system of nonlinear algebraic equations accomplished by using an iterative method. Galerkin technique has been adopted for the approximate analytical solutions.

Gupta, A. (2007) derived an expression for the deflection of a thin circular elastic plate resting on the Pasternak foundation under concentrated load by adopting the strain energy approach.

III. DESIGN CHARTS DEVELOPMENT

Design charts for deflection (w*) of the circular plate at different nondimensional modulus of subgrade reaction (k_s*)

$$w^* = \frac{Q^* {r^*}^2}{\pi\left(\frac{k_s^*}{2} - 2G^*\right)} + \frac{Q^*}{\pi}\left[\frac{1}{k_s^* - \frac{\left(2G^* - \frac{k_s^*}{2}\right)^2}{8(1+\nu) + \frac{k_s^*}{3} - 2G^*}}\right]\left[1 - \frac{{r^*}^2 k_s^*}{\left(\frac{k_s^*}{2} - 2G^*\right)}\right] \tag{1}$$

Considering nondimensional parameters as;

$$\mathrm{w}^* = \left(\frac{\mathrm{w}}{\mathrm{a}}\right) \tag{1a}$$

$$\mathrm{r}^* = \left(\frac{\mathrm{r}}{\mathrm{a}}\right) \tag{1b}$$

$$\mathrm{Q}^* = \left(\frac{\mathrm{Qa}}{\mathrm{D}}\right) \tag{1c}$$

$$\mathrm{G}^* = \left(\frac{\mathrm{Ga^2H}}{\mathrm{D}}\right) \tag{1d}$$

$$\mathrm{k_s}^* = \left(\frac{\mathrm{k_s a^4}}{\mathrm{D}}\right) \tag{1e}$$

Eq. (1) has been used to develop the design charts for different nondimensional values of the modulus of subgrade reaction (k_s*) for determining the deflection (w*) of the circular plate for different values of the shear modulus (G*) at any radial distance (r*), measured from the centre of the circular plate. By using these design charts for any values of modulus of subgrade reaction and shear modulus at any radial distance, it can be directly estimated the deflection of the circular plate. Four such design charts have been shown in Figs. 1, 2, 3 and 4.

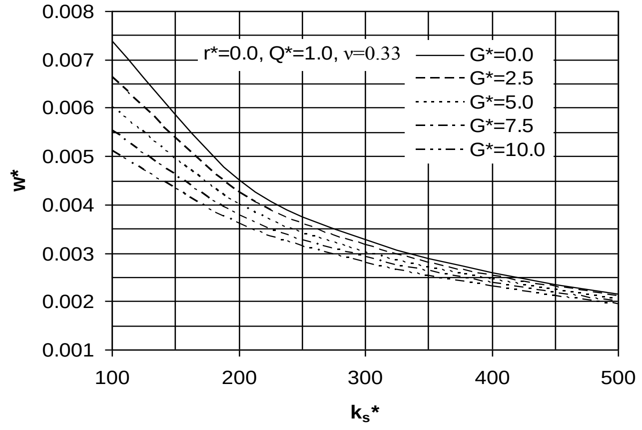

Figure 1. A design chart for estimating the deflection for different nondimensional k_s* at different G* and central distance r*=0

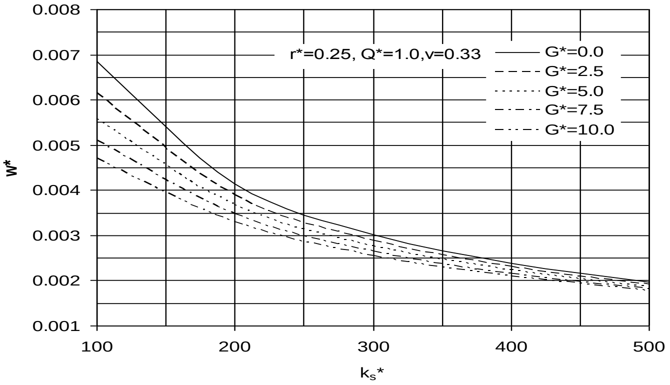

Figure 2. A design chart for estimating the deflection for different nondimensional k_s* at different G* and central distance r*=0.25

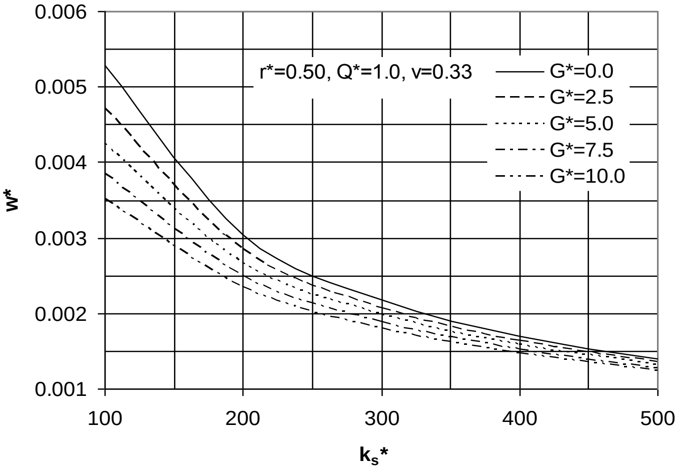

Figure 3. A design chart for estimating the deflection for different nondimensional k_s* at different G* and central distance r*=0.50

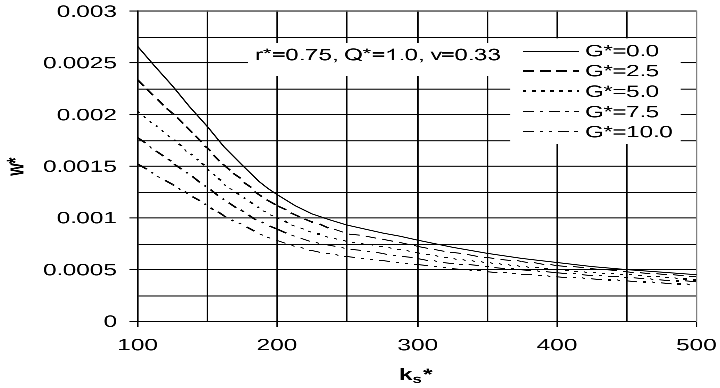

Figure 4. A design chart for estimating the deflection for different nondimensional k_s* at different G* and central distance r*

IV. CONCLUSIONS

Estimation of deflection of circular plates is of great importance as it is associated with many geotechnical engineering structures. As described earlier, it has been found that a large number of studies and various analytical methods and the numerical techniques have been adopted for the analysis of circular plates resting on the different foundation models under concentrated loading condition. Various efforts have been made to analyze a thin circular elastic plate resting on the Winkler foundation and Pasternak foundation by adopting different approaches in order to present the analytical expressions for the plate deflection as well as for the study of the plate lift off. Strain energy approach has been also reported for two-parameter Pasternak foundation soil case by Gupta, A. (2007) and With the help of the complete derivations by Gupta, A. (2007), an effort has been done for analyzing the circular plates resting on two-parameter foundation soil in the present study with some design charts. The general conclusions of the present study are summarized as follows:

- Design charts has been developed for different nondimensional values of the modulus of subgrade reaction (k_s*) for estimating the deflection (w*) of the circular plate considering several values of the shear modulus (G*).
- For any values of the modulus of subgrade reaction (k_s*) and the shear modulus (G*) the deflection (w*) at any radial distance (r*), can be directly interpolated with the help of design charts as illustrated by a numerical example.

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