

Sensitivity of Body Surface Potentials to Cardiac Size Using Boundary Element Methods

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Abstract:—Using numerical and an analytical mathematical model, we studied the effects of variations in heart size properties of the torso on epicardial and body surface potentials. The model consists of a spherical heart concentrically placed in a spherical torso (analytical model) region. The effects of the heart size variability are studied analytically simulating the concentric spheres model (with source placed at the centre) with different radii and then comparing the results with numerically computed ones. Simulation results show that there is a linear relationship between heart size and body surface potentials which confirms the laws of electromagnetics (Gauss' Law). Neglecting other inhomogeneous regions between heart and thorax, approximately 0.01% and 5.14% rise in body surface and epicardial potentials respectively are reported with a rise of 10% in heart size.

Key Words:—Body surface potentials, epicardial potentials, cardiac size, boundary element methods and concentric spheres model.

I. INTRODUCTION

Study of electrical activity of heart is carried out by two kinds of problems of Electrocardiographic (ECG) namely, Forward and Inverse Problems. Forward problem relates the epicardial potentials to body surface potentials with a known torso geometry and source distribution inside the heart. Whereas, inverse problem tries to adjust the source parameters responsible for a known body surface potential distribution and torso geometric information [1]. Today, almost 90% of the cardiology research is spanned by inverse problem. But, inverse problem is an ill posed problem, where a small error in computations may be due to measurement or geometric or body and heart tissue material property errors can cause irrelevant results. Therefore, one needs to be very much careful while working with inverse problem of ECG.

Very first step that comes into picture when dealing with inverse problem of ECG is to obtain the body surface potentials of the patient. This is done through clinical practices, which composes the forward problem of ECG, as stated above. Solution to the forward problem requires geometric and material property (heart and body tissues) information of the patient. It has been verified by many research groups in literature [2-7] that forward problem is sensitive to a number of factors (thoracic resistivity, conductivity, geometric parameters etc). Heart size variability is one of them. Heart size differs from person to person. But very few studies [5-7] have taken this factor into consideration. And still, a direct relationship has not been stated between heart size and body surface potentials neglecting other factors. Therefore, a thorough study is carried out to establish the relationship between body surface potentials and heart size variability. In this paper, we have solved the problem numerically using boundary element method and then accuracy of the results is verified with analytical model.

II. FORWARD PROBLEM OF ECG

1.1 Introduction

Forward problem of ECG is modeled by considering human body as a volume conductor. Volume conduction bioelectric problems are solved by Laplace's Equation. Assuming tissues are isotropic, for each compartment (torso and heart) we have a Laplace's equation to govern the potential behaviors according to the theory of the Quasi-static Maxwell's equations due to low-frequency response of human tissue.

$$\sigma_i \nabla^2 U_j = 0 \quad (1)$$

Where σ_i is the conductivity of each compartment when $i = T, H$ respectively, representing torso and heart surface. For $j=1, 2$; boundary conditions are defined as:

$$\begin{aligned} U_1 &:= U_T |_{\Gamma_1} \\ U_2 &:= U_H |_{\Gamma_2} \end{aligned} \quad (2)$$

Because current is continuous from heart surface to the body surface, Neumann boundary conditions are:

$$\begin{aligned} J_1 &:= \sigma_T \frac{\partial U_T}{\partial n} \Big|_{\Gamma_1} = 0 \\ J_2 &:= \sigma_H \frac{\partial U_H}{\partial n} \Big|_{\Gamma_2} = -\sigma_T \frac{\partial U_T}{\partial n} \Big|_{\Gamma_2} \end{aligned} \quad (3)$$

Where n is the outward normal vector of both boundaries. Using Green's theorem, boundary integral equations required to solve are:

$$c(k)u(k) - \int_{\Gamma} (u \frac{\partial G}{\partial n} - G \frac{\partial u}{\partial n}) d\Gamma = 0$$

$$\text{Where } c(k) = \begin{cases} 1 & k \in \Omega \\ \frac{1}{2} & k \in \Gamma \end{cases} \quad (4)$$

For each of the surfaces involved we can formulate corresponding boundary integral equation. The complete set of integral equations can be put into assembly form as:

$$WU = S \quad (5)$$

where W is a transformation matrix, potentials vector U includes the potentials on both surfaces with their respective conductivities and S denotes the equivalent source. To solve the potential from the surface integral equations, Boundary element discretization techniques have been used [8-10].

2.2 Solution Method

A forward model has been developed based on Helsinki BEM library [11]. It consists of 2 spheres respectively modeling the heart and torso regions, as shown in Figure 1. Different conductivity values assigned to 2 regions are as follows:

$$\begin{aligned} C_{iH} &= 10 \\ C_{oH} &= 5 \\ C_{iT} &= 5 \\ C_{oT} &= 0 \end{aligned} \quad (6)$$

where terms C_{iH} , C_{oH} , C_{iT} and C_{oT} refers to internal heart (endocardium), outside heart (epicardium), internal torso and outside torso surface conductivities. Outside torso being the air region, so its conductivity is set at zero value. This model assumes a dipole source placed at centre with dipole moment $[100 \ 0 \ 0]$.

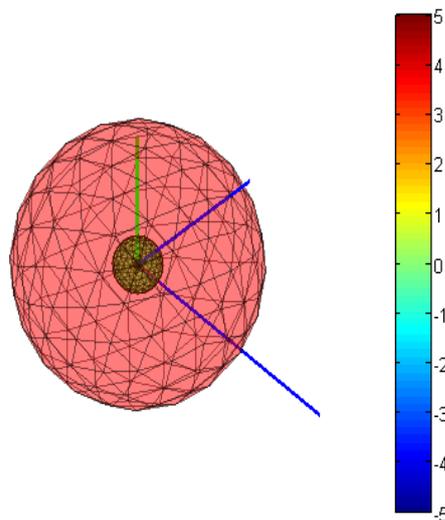


Figure 1: 2 Concentric spheres model with dipole position $[0 \ 0 \ 0]$ and dipole moment $[100 \ 0 \ 0]$

III. RESULTS AND DISCUSSION

Based on 2 concentric spheres model shown in Figure 1; effect of cardiac size on epicardial and body surface potentials is visualized as per the Table 1. Radius of the outer sphere r_2 is kept constant at 5 units; whereas radius of the inner sphere r_1 is varied in 11 steps from 1.0 to 2.0 units. Terms listed in the table refers to EP=epicardial potential computed numerically, BSP= body surface potential computed numerically, EPANA= epicardial potential computed analytically, BSPANA=body surface potential computed analytically, RE1=relative error between analytical and numerically computed potentials at heart surface, RE2= relative error between analytical and numerically computed potentials at body surface, CC1=correlation coefficient at heart surface and CC2= correlation coefficient at body surface.

As shown in Figure 2, there is a linear relationship between heart size and body surface potentials. This linear relationship is in confirmation with laws of electromagnetics which states that there is an inverse relationship between potential at observation point and distance between source and the point of observation (Gauss' Law).

Table1: Simulation results of 2concentric spheres model with r1=1.0 to 2.0 units

r1 (variable), r2 (fixed)=5	EP	BSP	EPANA	BSPANNA	RE1	RE2	CC1	CC2
1.0	-0.3019	-0.0352	-0.2985	-0.0347	0.010964	0.011591	1.000000	0.999999
1.1	-0.2505	-0.0351	-0.2476	-0.0347	0.010966	0.011618	1.000000	0.999999
1.2	-0.2114	-0.0351	-0.2090	-0.0346	0.010968	0.011650	1.000000	0.999999
1.3	-0.1811	-0.0350	-0.1790	-0.0346	0.010972	0.011688	1.000000	0.999999
1.4	-0.1571	-0.0350	-0.1553	-0.0345	0.010977	0.011732	1.000000	0.999999
1.5	-0.1378	-0.0349	-0.1362	-0.0344	0.010983	0.011783	1.000000	0.999999
1.6	-0.1221	-0.0348	-0.1207	-0.0343	0.010990	0.011841	1.000000	0.999999
1.7	-0.1091	-0.0347	-0.1078	-0.0342	0.010999	0.011906	1.000000	0.999999
1.8	-0.0982	-0.0345	-0.0971	-0.0341	0.011011	0.011978	1.000000	0.999999
1.9	-0.0891	-0.0344	-0.0881	-0.0340	0.011025	0.012059	1.000000	0.999999
2.0	-0.0814	-0.0343	-0.0804	-0.0338	0.011043	0.012148	1.000000	0.999999

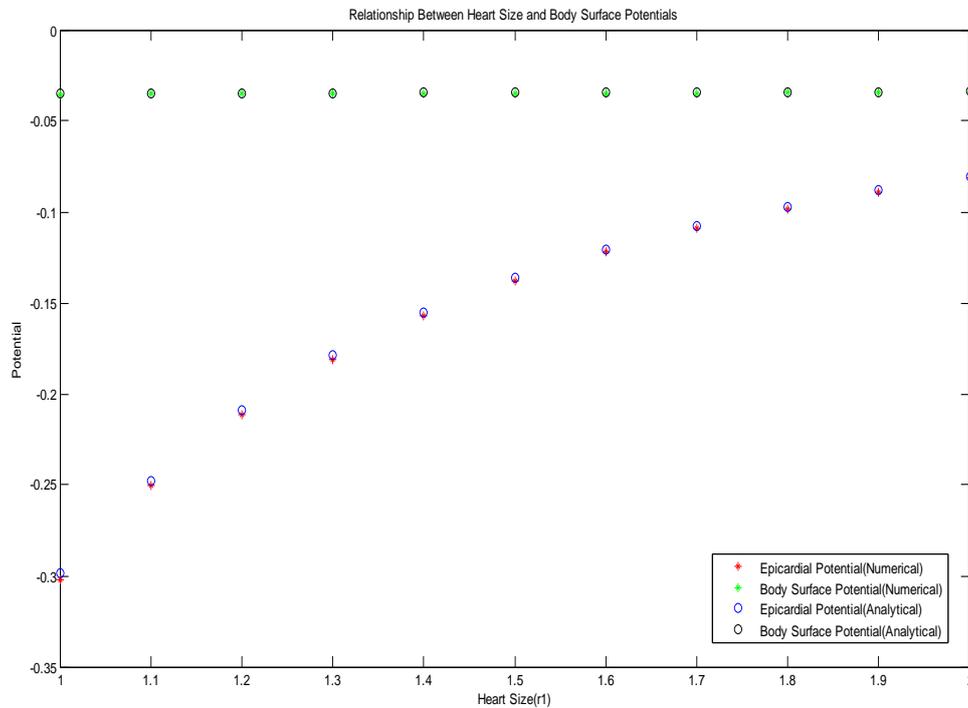


Figure 2: Relationship between heart size and body surface potentials

IV. CONCLUSION

With increase in heart size, distance between body surface (ECG measurement surface) and source location (heart) is decreased, hence rise in potential results. It is found that about 10% rise in heart size results into approximately 0.01% and 5.14% rise in body surface and epicardial potentials respectively.

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