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# Effects of Hall Current on an Unsteady MHD Flow of Heat and Mass Transfer along a Porous Flat Plate with Chemical Reaction and Viscous Dissipation

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**Abstract:-** The influence of Hall current, chemical reaction and viscous dissipation on an unsteady MHD flow and heat transfer along a porous flat plate with mass transfer is investigated. The transformed governing equations in the present study were solved numerically by using Galerkin Finite element method. The effects of Hall current, viscous dissipation, chemical reaction over primary velocity u, secondary velocity w, heat transfer and mass transfer are analyzed. Effects of magnetic parameter M, Prandtl number Pr, Schmidt number Sc, Modified Grashof number Gr for heat transfer and Modified Grashof number Gc for mass transfer on the velocity components and temperature is also examined.

Keywords:- MHD, Hall current, viscous dissipation, chemical reaction, finite element method.

## I. INTRODUCTON

The combined heat and mass transfer problems with chemical reaction are of importance in many processes, and therefore have received considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and flow in a desert cooler, the heat and mass transfer occurs simultaneously. Chemical reaction can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase; in contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be the first order reaction if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, a chemical reaction between foreign mass and fluid occur. Das et.al. [1] Studied the effect of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumarswamy and Ganesan,P [2,3] discussed, flow past an impulsively started infinite plate with constant heat and mass flux and effect of chemical reaction on moving isothermal surface with suction.

In the recent years theoretical study of MHD channel flows has been a subject of great interest due to its widespread applications in designed cooling systems with liquid metals, petroleum industry, purification of crude oil, polymer technology, centrifugal separation of matter from liquid, MHD generators, pumps, accelerators and flow meters. In an ionized gas where the density is low and/or the magnetic field is very strong, the conductivity normal to the magnetic field is reduced to the free spiraling of electrons and ions about the magnetic lines of force before suffering collision; also, a current is induced in a direction normal to both the electric and magnetic fields. The phenomenon, well known in the literature, is called the Hall Effect. The study of magnetohydrodynamic flows with Hall currents has important engineering applications in problems of magnetohydrodynamic generators and of Hall accelerators as well as in flight magnetohydrodynamics. Sato [4] has studied the effect of Hall currents on the steady hydromagnetic flow between two parallel plates. Various workers Sherman and Sutton [5], Hossain [6], Ram [7], Pop [8] studied the Hall effects. This effect can be taken into account within the range of magnetohydrodynamical approximation. Katagiri [9] studied the steady incompressible boundary layer flow past a semi-infinite flat plate in a transverse magnetic field at small magnetic Reynolds number considering with the effect of Hall current. Agarwal et.al [10] gave the solution of flow and heat transfer of a micro polar fluid over a stretching sheet using finite element technique. Sri Ramulu et.al [11] studied the Effect of Hall Current on MHD Flow and Heat Transfer along a Porous Flat Plate with Mass Transfer.

Hossain and Rashid [12] investigated the effect of Hall current on the unsteady free-convection flow of a viscous incompressible fluid with mass transfer along a vertical porous plate subjected to a time dependent transpiration velocity when the constant magnetic field is applied normal to the flow. And an analysis of heat transfer taking dissipation function into account, in boundary layer flow of a hydromagnetic fluid over a

stretching sheet in the presence of uniform transverse magnetic field has been given by Lodha and Tak [13], Gupta [14] studied the Hall current effects in the steady hydromagnetic flow of an electrically conducting fluid past an infinite porous flat plate. Sattar and Maleque [15] studied the effect of viscous dissipation as well as joule heating on two-dimensional steady MHD free-convection and mass transfer flow with Hall current of an electrically conducting viscous incompressible fluid past an accelerated infinite vertical porous plate with wall temperature and concentration. Recently, Bala Siddulu Malga et.al [16] studied the effects of viscous dissipation on unsteady free convection and mass transfer boundary layer flow past an accelerated infinite vertical porous flat plate with suction. In this paper, we investigated Hall current, viscous dissipation and chemical reaction of an unsteady free-convection MHD flow of an electrical conducting viscous incompressible fluid with mass transfer along an infinite vertical porous plate. We employed Galerkin finite element method to solve the governing partial differential equations. The effects of various important physical parameters on the flow and heat, mass transfer are analyzed.

#### II. MATHEMATICAL FORMULATION

An unsteady free-convection flow of an electrically conducting viscous incompressible fluid with mass transfer along an infinite vertical porous plate has been considered. The flow is assumed to be in x' direction which is taken along the plate in upward direction. The y' axis is taken to be normal to the direction of the plate. At time  $t \ge 0$ , the temperature and the species concentration at the plate are raised to  $T_w$  ( $\ne T_\infty$ ) and  $C_w$  ( $\ne C_\infty$ ) and thereafter maintained uniform. The level of species concentration is assumed to be very low and hence species thermal diffusion and diffusion thermal energy effects can be neglected. A magnetic field of uniform strength is assumed to be applied transversely to the porous plate. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field can be neglected. The equation of conservation of electric change  $\nabla J = 0$  gives  $j_y$  =constant, where  $J = (j_x, j_y, j_z)$ . It is also assumed that the plate is nonconducting. This implies  $j_v = 0$  at the plate and hence zero everywhere. When the strength of magnetic field is very large the generalized Ohm's law, in the absence of electric field takes the form  $J + \frac{w_{eT_e}}{B_0}JB = \sigma(\mu_e VB + \frac{1}{en_e}\nabla P_e)$ 

$$J + \frac{w_{eT_e}}{B_0} JB = \sigma(\mu_e VB + \frac{1}{en_e} \nabla P_e) \tag{1}$$

where V is the velocity vector;  $\sigma$ , the electric conductivity;  $\mu_e$ , the magnetic permeability;  $\omega_e$ , the electron frequency;  $\tau_e$ , the electron collision time; e, the electron charge;  $n_{e_i}$  the number density of the electron; and  $P_e$ , the electron pressure. Under the assumption that the electron pressure (for weakly ionized gas), the thermoelectric pressure and ion-ship are negligible, equation (1) becomes

$$\begin{cases} j_{x} = \frac{\sigma \mu_{e} B_{0}}{1+m^{2}} (mu - w) \\ j_{z} = \frac{\sigma \mu B_{0}}{1+m^{2}} (u + mw) \end{cases}$$
(2)

Where u is the x component of V; w, the z component of V and m ( $=\omega_e \tau_e$ ), the Hall parameter. Within this frame work, the equations which govern the flow under the usual Boussinesq approximation are:

$$\frac{\partial V}{y'} = 0 \tag{3}$$

$$\frac{\partial u'}{\partial t'} + V' \frac{\partial u'}{\partial y'} = v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \mu_e^2 B_0^2}{\rho (1 + m^2)} (u' + mw') + g\beta (T - T_\infty) + g\beta^* (C' - C'_\infty)$$

$$\tag{4}$$

$$\frac{\partial w'}{\partial t'} + V' \frac{\partial w'}{\partial y'} = v \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma \mu_e^2 B_0^2}{\rho (1+m^2)} (w' - mu') \tag{5}$$

this frame work, the equations which govern the flow under the usual Boussinesq approximation are:

$$\frac{\partial V'}{y'} = 0$$

$$\frac{\partial u'}{\partial t'} + V' \frac{\partial u'}{\partial y'} = v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \mu_e^2 B_0^2}{\rho (1+m^2)} (u' + mw') + g\beta (T - T_\infty) + g\beta^* (C' - C'_\infty)$$

$$\frac{\partial w'}{\partial t'} + V' \frac{\partial w'}{\partial y'} = v \frac{\partial^2 w'}{\partial y'^2} - \frac{\sigma \mu_e^2 B_0^2}{\rho (1+m^2)} (w' - mu')$$
(5)
$$\frac{\partial T}{\partial t'} + V' \frac{\partial T}{\partial y'} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y'^2} + \frac{\mu}{\rho C_p} \left(\frac{\partial u'}{\partial y'}\right)^2$$
(6)
$$\frac{\partial C'}{\partial t'} + V \frac{\partial C'}{\partial y'} = D \frac{\partial^2 C'}{\partial y'^2} - K' C'$$

$$\frac{\partial C'}{\partial t'} + V^{\frac{\partial C'}{\partial y'}} = D \frac{\partial^2 C'}{\partial y'^2} - K_1' c'$$
(7)

Where g is the acceleration due to gravity;  $\beta$ , the volumetric coefficient of thermal expansion;  $\beta^*$ , the coefficient of volume expansion with species concentration; T, the temperature of the fluid within the boundary layer; C', the species concentration;  $\rho$ ,  $\mu$ ,  $\nu$ , k,  $c_p$  are respectively density, viscosity, kinematic viscosity, thermal conductivity, specific heat at constant pressure; and D, the chemical molecular diffusivity, Chemical reaction parameter  $K_1$ . The initial and boundary conditions of the problem are

parameter 
$$K_1$$
. The initial and boundary conditions of the problem are  $t' \le 0$ :  $u' = 0$ ,  $w' = 0$ ,  $T = T_{\infty}$ ,  $C' = C_{\infty}$  for all  $y'$   $t' > 0$ :  $\begin{cases} u' = 0, \ w = 0, T = T_{\omega}, \ C' = C_{\omega} \ at \ y' = 0 \\ u' = 0, w' = 0, T = T_{\infty}, \ C' = C_{\infty}, \ at \ y' \to \infty \end{cases}$  (8)

The non-dimensional quantities introduced in the equations (3) to (7) are

$$t = \frac{t'U_0^2}{v}, \ y = \frac{y'U_0}{v}, (u, V, w) = \frac{(u', V', w')}{U_0}$$

$$\theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \ C = \frac{(C' - C_\infty)}{(C_w - C_\infty)}, K_1 = \frac{K_1'v}{V_0^2}$$
(9)

Where  $U_0$  is the reference velocity. Therefore, the governing equation in the dimensionless form are obtained as

$$\frac{\partial V}{\partial v} = 0 \tag{10}$$

$$\frac{\partial u}{\partial t} + V \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{M}{1+m^2} (\mathbf{u} + \mathbf{m}\mathbf{w}) + G_r \theta + G_c C$$

$$\frac{\partial w}{\partial t} + V \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{1+m^2} (\mathbf{w} - \mathbf{m}\mathbf{u})$$
(12)

$$\frac{\partial w}{\partial t} + V \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{1 + m^2} (w - mu) \tag{12}$$

$$\frac{\partial \theta}{\partial t} + V \frac{\partial \theta}{\partial y} = \frac{1}{P_{r}} \frac{\partial^{2} \theta}{\partial y^{2}} + Ec \left(\frac{\partial u}{\partial y}\right)^{2}$$

$$\frac{\partial \mathcal{C}}{\partial t} + V \frac{\partial \mathcal{C}}{\partial y} = \frac{1}{S_{c}} \frac{\partial^{2} \mathcal{C}}{\partial y^{2}} - K_{1}c$$
(13)

$$\frac{\partial C}{\partial t} + V \frac{\partial C}{\partial v} = \frac{1}{S_c} \frac{\partial^2 C}{\partial v^2} - K_1 c \tag{14}$$

 $M = \frac{\sigma \mu_e^2 B_0^2 v}{\rho U^2}$  is Magnetic field parameter,  $P_r = \frac{\mu C_p}{k}$  is Prandtl number

 $Sc = \frac{v}{D}$  is Schmidt number,  $Ec = \frac{U_0^2}{Cp(T_w - T_{cr})}$  is Eckert number

 $G_r = \frac{vg\beta(T_w - T_\infty)}{U_0^3}$  is Modified Grashof number for heat transfer  $G_c = \frac{vg\beta^*(C_w - C_\infty)}{U_0^3}$  is Modified Grashof number for mass transfer

And the boundary conditions (equation (8)) in the non-dimensional form are

$$t \le 0: u = 0, w = 0, T = 0, C = 0 \text{ for all } y$$

$$t > 0: \begin{cases} u = 0, w = 0, \ \theta = 1, \ C = 1 \text{ at } y = 0 \end{cases}$$

$$t > 0: \begin{cases} u = 0, w = 0, \ \theta = 0, \ C = 0 \text{ as } y \to \infty \end{cases}$$
From equation (10), it is seen that  $v$  is either constant or a function of time t. Similarly, solutions of equations

(11) to (14) with the boundary conditions equation (15) exist only if

$$V = -\lambda t^{-\frac{1}{2}} \tag{16}$$

Where  $\lambda$  is a non-dimensional transpiration parameter. For suction  $\lambda > 0$  and for blowing  $\lambda < 0$ . From equation (16), it can be observed that the assumption is valid only for small values of time variable.

#### METHOD OF SOLUTION III.

By applying the Galerkin finite element method for equation (11) over a typical two-noded linear element (e)  $(y_i \le y \le y_k)$  is

$$\int_{y_j}^{y_k} N^T \left[ \frac{\partial^2 u^{(e)}}{\partial y^2} - \frac{\partial u^{(e)}}{\partial t} - V \frac{\partial u^{(e)}}{\partial y} - \frac{M}{1+m^2} \left( m\omega + u^{(e)} \right) + \left( G_r \theta + G_c \mathcal{C} \right) \right] dy \tag{17}$$

$$\int_{y_{j}}^{y_{k}} \left[ \frac{\partial N}{\partial y} \cdot \frac{\partial u^{(e)}}{\partial y} - N^{T} \left( -V \frac{\partial u^{(e)}}{\partial y} - \frac{\partial u^{(e)}}{\partial t} - \left( \frac{M}{1+m^{2}} \right) u^{(e)} + R^{*} \right) \right] dy = 0 
where  $R^{*} = \left( -\left( \frac{Mm}{1+m^{2}} \right) \omega + G_{r}\theta + G_{c}C \right), \quad N = [N_{j}, N_{k}], \quad \emptyset^{(e)} = \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix}, 
u^{(e)} = N. \, \emptyset^{(e)}, \quad N_{j} = \frac{y_{k} - y}{l^{(e)}}, \quad N_{k} = \frac{y - y_{j}}{l^{(e)}}, \quad l^{(e)} = y_{k} - y_{j} = h$ 
(18)$$

The element equation given by

$$\int_{y_{j}}^{y_{k}} \begin{bmatrix} N_{j}^{'} N_{j}^{'} & N_{j}^{'} N_{k}^{'} \\ N_{k}^{'} N_{j}^{'} & N_{k}^{'} N_{k}^{'} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} dy - V \begin{bmatrix} N_{j} N_{j}^{'} & N_{j} N_{k}^{'} \\ N_{k} N_{j}^{'} & N_{k} N_{k}^{'} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} dy + \begin{bmatrix} N_{j} N_{j} & N_{j} N_{k} \\ N_{k} N_{j} & N_{k} N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} dy + \begin{bmatrix} M_{j} N_{j} & N_{j} N_{k} \\ N_{k} N_{j} & N_{k} N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} dy - R^{*} \begin{bmatrix} N_{j} \\ N_{k} \end{bmatrix} dy = 0$$
(19)

$$\int_{\gamma_i}^{\gamma_k} \left( S^{(e)} + A^{(e)} + R \right) dy = 0 \tag{20}$$

where 
$$S^{(e)} = \begin{bmatrix} N_{j}' N_{j}' & N_{j}' N_{k}' \\ N_{k}' N_{j}' & N_{k}' N_{k}' \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} - V \begin{bmatrix} N_{j} N_{j}' & N_{j} N_{k} \\ N_{k} N_{j}' & N_{k} N_{k}' \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} + \left(\frac{M}{1+m^{2}}\right) \begin{bmatrix} N_{j} N_{j} & N_{j} N_{k} \\ N_{k} N_{j} & N_{k} N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix},$$

$$A^{(e)} = \begin{bmatrix} N_{j} N_{j} & N_{j} N_{k} \\ N_{k} N_{j} & N_{k} N_{k} \end{bmatrix} \begin{bmatrix} u_{j} \\ u_{k} \end{bmatrix} \text{ and } R = R^{*} \begin{bmatrix} N_{j} \\ N_{k} \end{bmatrix}$$

Here the prime and dot denote differentiation with respect to y and t. we obtain

$$\begin{split} S^{(e)} &= \frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} - \frac{V}{2} \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} + \left( \frac{M}{1+m^2} \right) \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u_j \\ u_k \end{bmatrix} \\ A^{(e)} &= \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \dot{u}_j \\ \dot{u}_k \end{bmatrix} \ \ and \ R = R^* \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{split}$$

We write the element equation for the elements  $y_{i-1} \le y \le y_i$  and  $y_i \le y \le y_{i+1}$ . Assembling these element

equations, we get
$$\frac{1}{l^{(e)}} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_{i+1} \end{bmatrix} - \frac{V}{2} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_{i+1} \end{bmatrix} +$$

$$\left(\frac{M}{1+m^2}\right) \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_{i+1} \end{bmatrix} + \frac{l^{(e)}}{6} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} u_{i+1} \\ u_{i+1} \end{bmatrix} = R^* \frac{l^{(e)}}{2} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{21}$$
Now put row corresponding to the node ito zero from equation (21) the difference schemes with  $l^{(e)} = h$ , is

Now put row corresponding to the node i to zero, from equation (21) the difference schemes with  $l^{(e)} = h$  is

$$\frac{h}{6}(\dot{u}_{i-1}+4\dot{u}_i+\dot{u}_{i+1})+\left(\left(\frac{M}{1+m^2}\right)\frac{h}{6}-\frac{V}{2}-\frac{1}{h}\right)u_{i-1}+\left(\frac{2}{h}+\left(\frac{2M}{1+m^2}\right)\frac{h}{3}\right)u_i+\frac{h}{6}u_{i-1}+\frac{h}{6}u_{i$$

$$\left( \left( \frac{M}{1+m^2} \right) \frac{h}{6} + \frac{V}{2} - \frac{1}{h} \right) u_{i+1} = R \text{ and here } R = R^* \frac{l^{(e)}}{2} \begin{bmatrix} 1\\2\\1 \end{bmatrix}$$
 (22)

Using the Cranck-Nicolson method to the equation  $(\overline{22})$ , we obtain:

$$A_1 u_{i-1}^{j+1} + A_2 u_i^{j+1} + A_3 u_{i+1}^{j+1} + A_4 u_{i-1}^{j} + A_5 u_i^{j} + A_6 u_{i+1}^{j} = R$$

$$(23)$$

$$B_1 w_{i-1}^{j+1} + B_2 w_i^{j+1} + B_3 w_{i+1}^{j+1} + B_4 w_{i-1}^{j} + B_5 w_i^{j} + B_6 w_{i+1}^{j} = R_A$$

$$(24)$$

Using the Cranck-Nicolson method to the equation (22), we obtain:
$$A_{1}u_{i-1}^{j+1} + A_{2}u_{i}^{j+1} + A_{3}u_{i+1}^{j+1} + A_{4}u_{i-1}^{j} + A_{5}u_{i}^{j} + A_{6}u_{i+1}^{j} = R$$
Similarly, the equations (12), (13), (14) are becoming as follows:
$$B_{1}w_{i-1}^{j+1} + B_{2}w_{i}^{j+1} + B_{3}w_{i+1}^{j+1} + B_{4}w_{i-1}^{j} + B_{5}w_{i}^{j} + B_{6}w_{i+1}^{j} = R_{A}$$

$$C_{1}\theta_{i-1}^{j+1} + C_{2}\theta_{i}^{j+1} + C_{3}\theta_{i+1}^{j+1} + C_{4}\theta_{i-1}^{j} + C_{5}\theta_{i}^{j} + C_{6}\theta_{i+1}^{j} = R_{B}$$

$$D_{1}c_{i-1}^{j+1} + D_{2}c_{i}^{j+1} + D_{3}c_{i+1}^{j+1} + D_{4}c_{i-1}^{j} + D_{5}c_{i}^{j} + D_{6}c_{i+1}^{j} = 0$$
The initial and boundary conditions (15) reduce to
$$C_{1}(i,0) = 0 \quad w(i,0) = 0 \quad x(i,0) = 0 \quad x(i$$

$$D_1 c_{i-1}^{j+1} + D_2 c_i^{j+1} + D_3 c_{i+1}^{j+1} + D_4 c_{i-1}^j + D_5 c_i^j + D_6 c_{i+1}^j = 0 (26)$$

$$u(i,0) = 0, w(i,0) = 0, \theta(i,0) = 0, C(i,0) = 0 \quad for \ all \ i$$

$$u(0,j) = 0, w(0,j) = 0, \theta(0,j) = 1, C(0,j) = 1$$

$$u(100,j) = 0, w(100,j) = 0, \theta(100,j) = 1, C(100,j) = 0$$

$$A_1 = \left(\frac{1}{6} - \frac{k}{6} \left(\frac{M}{1+m^2}\right) - r + \frac{pV}{2}\right), A_2 = \left(\frac{4}{6} + 2r + \frac{2k}{3} \left(\frac{M}{1+m^2}\right)\right),$$

$$A_3 = \left(\frac{1}{6} + \frac{k}{6} \left(\frac{M}{1+m^2}\right) - \frac{pV}{2} - r\right), A_4 = \left(-\frac{k}{6} \left(\frac{M}{1+m^2}\right) + r - \frac{pV}{2} - \frac{1}{6}\right),$$

$$A_5 = \left(-2r - \left(\frac{M}{1+m^2}\right) \frac{2k}{6}, A_6\right), A_6 = \left(-\frac{k}{6} \left(\frac{M}{1+m^2}\right) + r - \frac{pV}{2} - \frac{1}{6}\right),$$

$$B_1 = \left(\frac{1}{6} + \frac{k}{6} \left(\frac{M}{1+m^2}\right) - r + \frac{pV}{2}\right), B_2 = \left(\frac{4}{6} + 2r + \frac{2k}{3} \left(\frac{M}{1+m^2}\right)\right),$$

$$B_3 = \left(\frac{1}{6} + \frac{k}{6} \left(\frac{M}{1+m^2}\right) - r + \frac{pV}{2}\right), B_4 = \left(-\frac{k}{6} \left(\frac{M}{1+m^2}\right) + r - \frac{pV}{2} - \frac{1}{6}\right),$$

$$B_5 = \left(-2r - \left(\frac{1}{1+m^2}\right) \frac{2k}{3} - \frac{4}{6}\right), B_6 = \left(-\frac{k}{6} \left(\frac{M}{1+m^2}\right) + r - \frac{pV}{2} - \frac{1}{6}\right),$$

$$C_1 = \left(\frac{1}{6} - \frac{r}{sc} + \frac{pV}{2}\right), C_2 = \left(\frac{4}{6} + \frac{2r}{sc}\right), C_3 = \left(\frac{1}{6} - \frac{r}{sc} - \frac{pV}{2}\right), C_4 = \left(\frac{1}{6} + \frac{r}{sc} - \frac{pV}{2}\right),$$

$$C_5 = \left(\frac{4}{6} - \frac{2r}{sc}\right), C_6 = \left(\frac{1}{6} + \frac{r}{sc} + \frac{pV}{2}\right),$$

$$D_1 = \left(1 - 6r \frac{1}{s_c} + 3pV + K_1k\right), D_2 = \left(4 + 12r \frac{1}{s_c} + 4K_1k\right),$$

$$D_3 = \left(1 - 6r \frac{1}{s_c} - 3pV + K_1k\right), D_4 = \left(1 + 6r \frac{1}{s_c} - 3pV - K_1k\right),$$

$$D_5 = \left(4 - 12r \frac{1}{s_c} - 4K_1k\right), D_6 = \left(1 + 6r \frac{1}{s_c} + 3pV - K_1k\right),$$

$$R_4 = \frac{Mm}{1+m^2}u, R_8 = E_c\left(\frac{U_{i+1} - U_{i-1}}{2h}\right)^2$$

Here  $r = \frac{k}{h^2}$  where k, h is mesh sizes along y direction and time direction respectively. Index i refers to space and j refers to time. The mesh system consists of h=0.1 for velocity profiles and concentration profiles and k=0.05 has been considered for computations. In equation (23)-(26), taking i=1(1) n and using initial and boundary conditions (15), the following system of equation are obtained.

$$A_i X_i = B_i$$
,  $i = 1,2,3...$  (28)

Where  $A_i$ 's are matrices of order n and  $X_i$  and  $B_i$ 's are column matrices having n-components. The solution of above system of equations are obtained using Thomas algorithm for velocity, temperature and concentration. Also, numerical solutions for these equations are obtained by MATLAB-Program, Galerkin finite

element method. The same MATLAB-Program was run with slightly changed values of h and k and no significant change was observed in the values of  $u, w, \theta, c$ .

### IV. RESULTS AND DISCUSSION

To get the physical insight of the problem, the results are discussed through graphs. The numerical results are obtained for the velocity profiles of main (primary) flow and cross (secondary) flow and the temperature profiles, concentration profiles have been shown graphically for different flow parameters, such as magnetic field parameter M, Hall current m, Eckert number Ec, Chemical reaction parameter  $K_1$  Modified Grashof number Gr for heat transfer, Modified Grashof number Gc for mass transfer, Schmidt number Sc, transpiration parameter  $\lambda$ , and time t. The primary velocity profiles are shown in fig.1 (a), fig.2 (a) and fig.3 (a). The Hall current effect is to increases the primary velocity component u, and this is because of the fact that Hall Effect, in general reduces the level of Lorentz forces effect on the fluid, whose tendency is to suppress the flow. The primary velocity flow is decreases with the increase of magnetic field parameter M and the transpiration parameter  $\lambda$ . It is also observed that the primary velocity u increases with the time t. It can be seen that the effect of Gr, and Gc is to increase the primary velocity profiles u, which is due to with an increase in buoyancy force due to temperature differences, the flow is accelerated in the boundary layer. Here the positive of Gr (>0) correspond to cooling of the plate by natural convection. The effect of Prandtl number Pr is reduces velocity profiles, the reason is that the smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of Pr. The influence of Schmidt number Sc is to reduce the velocity profiles. It is also noticed that the influence of Eckert number Ec, on the velocity profiles is increases the primary velocity profiles u, and designates the ratio of kinetic energy of the flow into the boundary layer enthalpy differences. It embodies the conversion of kinetic energy to internal energy by work done against the viscous fluid stresses. Hence, greater viscous dissipative heat causes a rise in the velocity profiles.

The secondary velocity profiles w are shown graphically in fig.1 (b), fig.2 (b) and fig.3 (b). The secondary velocity profiles w increase with time t, Hall parameter m and the magnetic field parameter M. But decreases owing to increase of transpiration parameter  $\lambda$ , it is observed from figures. It also can be observed that the secondary velocity profiles w increases as the Grashof number Gr (for heat transfer) and Gc (for mass transfer) increases, but it decreases as Prandtl number Pr and Schmidt number Sc increases. It also seen that effects of viscous dissipation increases the secondary velocity profiles w.

From the fig.5 it is evident that the temperature of the fluid increases with the increase of time t. But it decreases with the increase of transpiration parameter  $\lambda$  and Prandtl number Pr. The viscous dissipation effect on temperature profiles shown in fig.3(c). The viscous dissipation heat causes a rise in the temperature which is shown from the figure.

Fig.6 is drawn for the concentration profiles. From the figure it can be seen that the concentrations profiles are increases with the increase of time t, while it is decreases with the increase of transpiration parameter  $\lambda$  and Schmidt number Sc.

The effect of chemical reaction parameter  $K_1$  on primary velocity and secondary velocity profiles, temperature and concentration profiles are shown in fig.4 (a)-(d) respectively. From the fig.4 (a) & (b) it can be observed that the effect of  $K_1$ , leads to decreases the primary velocity profiles as well as secondary velocity profiles. Fig4(c) reviles that an increase of chemical reaction parameter  $K_1$  leads to decrease the temperature profiles. Fig.4 (d) shows that there is fall in concentration profiles due to the increasing the value of chemical reaction parameter  $K_1$  is noticed.

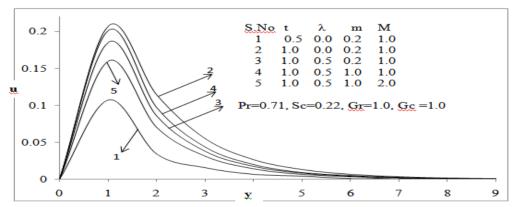


Fig.1(a): The Primary velocity profiles

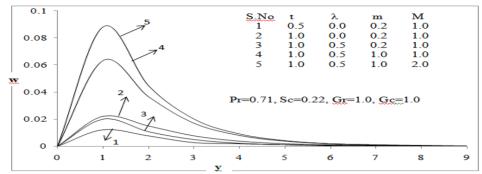


Fig.1 (b): The Secondary velocity profiles

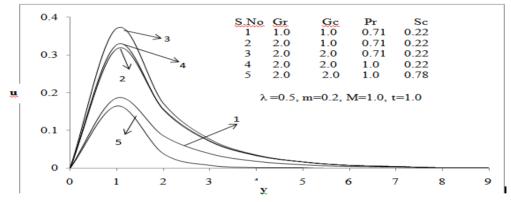


Fig.2 (a): The Primary velocity profiles

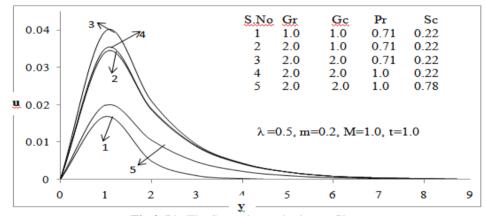


Fig.2 (b): The Secondary velocity profiles

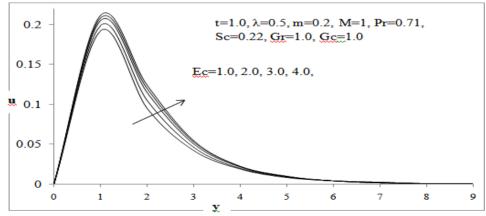


Fig.3 (a): The Primary velocity profiles for different values of Ec

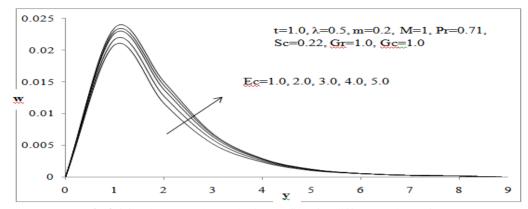


Fig.3 (b): The Secondary velocity profiles for different values of Ec

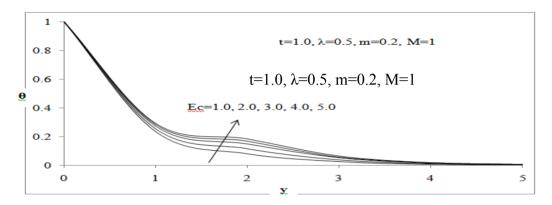
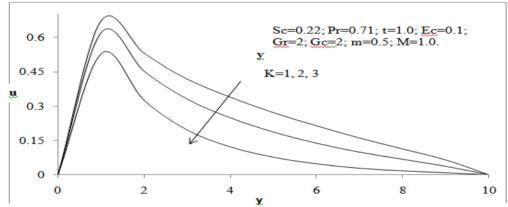
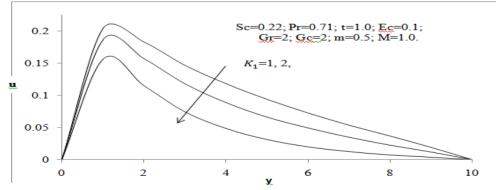


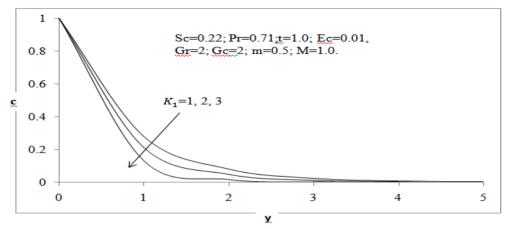
Fig.3 (c): Temperature Profiles for different values of Ec



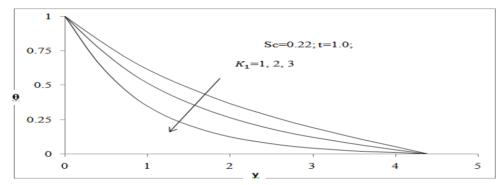
**Fig.4** (a): The Primary Velocity Profile for different values of  $K_1$ 



**Fig.4** (b): The Secondary Velocity Profile for different values of  $K_1$ 



**Fig.4(c):** The Temperature Profile for different values of  $K_1$ 



**Fig.4** (d): Concentration Profile for different values of  $K_1$ 

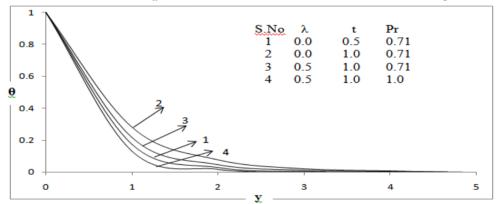


Fig.5: The Temperature Profiles

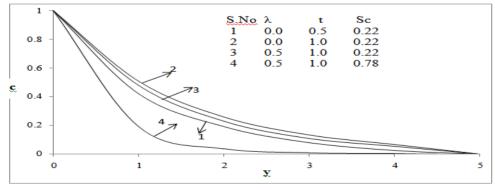


Fig.6: The Concentration Profiles

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