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Ramanujan Integral involving the Product of Generalized Zeta-function and $\bar{H}$ -function

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Abstract: *In the present paper, some Ramanujan Integrals associated with product of generalized Riemann Zeta-function and $\bar{H}$ -function of one variable evaluated.*

Importance of the results established in this paper lies in the fact they involve $\bar{H}$-function which is sufficiently general in nature and capable to yielding a large of results merely by specializing the parameters their in.

Key words: *Zeta-function, $\bar{H}$ -function etc.*

Mathematics subject classification: 2000, 24A33

I. Introduction

The $\bar{H}$ -function is a generalization of Fox's H-function [2], introduced by Inayat Hussain [5] and studied by Buchman and Shrivastava [1] and other, is defined and represented in the following manner ;

$$\bar{H}_{p,q}^{m,n}[z] = \bar{H}_{p,q}^{m,n}\left[z \left| \begin{matrix} (e_j, E_j; \in_j)_{1,n}, (e_j; E_j)_{n+1,p} \\ (f_j, F_j)_{1,m}, (F_j, f_j; \tau_j)_{m+1,q} \end{matrix} \right. \right]$$

$$= \frac{1}{2\pi\omega} \int_L \tilde{\phi}(\xi) z^{\xi} d\xi, \tag{1.1}$$

Where,

$$\tilde{\phi}(\xi) = \frac{\prod_{j=1}^{m} \Gamma(f_j - F_j\xi_i) \prod_{j=1}^{n} \left[\Gamma(1 - e_j + E_j\xi)\right]^{\in_j}}{\prod_{j=m+1}^{q} \left[\Gamma(1 - f_j + F_j\xi)\right]^{\tau_i}, \prod_{j=n+1}^{p} \Gamma(e_j - E_j\xi)}$$

(1.2)

And the contour L is a line from $c-\omega\infty$ to $c+\omega\infty$ suitable indented to keep the poles of $\Gamma(f_j - F_j\xi,), \ \forall\ j = 1,....,m$ to the right of the path and the singularities of $\left[\Gamma(1 - e_j + E_j\xi)\right]^{\tau_i}$, $j = 1,....,n$ to the left path.

The following sufficient condition for absolute convergence of the integral defined in equation (1.1) have been recently given by Gupta, Jain and Agrawal [4, p. 169-172] as follow,

(i) $\left|\arg(z)\right| < \frac{1}{2}\Omega\pi$ and $\Omega > 0,$

(ii) $\left|\arg(z)\right| < \frac{1}{2}\Omega\pi$ and $\Omega \geq 0$

Where,

$$\Omega=\prod_{j=1}^{M}F_j+\prod_{j=1}^{N}\in_j E_j-\prod_{j=M+1}^{Q}F_j\tau_j-\prod_{j=N+1}^{P}E_j \tag{1.3}$$

II. Formula required

In the present paper we will use following formula.

$$\int_0^\infty x^{\rho-1}\left\{x+\sqrt{\left(1+x^2\right)}\right\}^{-n}dx=\frac{\Gamma(n+1)\,\Gamma(\rho)\,\Gamma\left(\frac{n-\rho}{2}\right)}{2^{P+1}\Gamma n\left\{(n+\rho+2)/2\right\}}$$

(2.1)

Where, $n>\rho>0,$

The Riemann Zeta-function given to by Goyal and Laddha [3] are used for Drive these Integrals:

$$\phi_{(\mu)}(z,s,a,g)=\sum_{g=0}^{\infty}(\mu)_g(a+g)^{-s}\frac{z^g}{g!},$$

$\mu\geq 1, |z|<1, \mathrm{Re}(a)>0,$

(2.2)

If $\mu=1$ in (2.2) then it reduces to generalized Riemann zeta function.

$$\phi(z,s,a,g)=\sum_{g=0}^{\infty}(a+g)^{-s}\frac{z^g}{g!},$$

$|z|<1, \mathrm{Re}(a)>0,$

(2.3)

If $\mu=1$, and s = 1, in (2.2) reduce to Hypergeometric function i.e.,

$$\phi(z,1,a,g)=\frac{1}{a}\sum_{r=0}^{\infty}\frac{(1)_r(a)_r}{g!}.\frac{z^r}{r!},\ =\frac{1}{a}.\frac{1}{q!}\ {}_2F_0[1,a:z] \tag{2.4}$$

III. Integrals

In this section we will established integrals involving product of $\bar{H}$ -function and Riemann zeta function.

First Integral

$$\int_0^\infty x^{\rho-1}\left\{x+\sqrt{1+x^2}\right\}^{-n}\phi_\mu(z,s,a,g).\bar{H}\left[zx^{\sigma_1}.\left\{x+\sqrt{1+x^2}\right\}^{\rho_1}\right]dx.$$

$$=\sum_{g=0}^{\infty}(\mu)_g(a+g)^{-s}\left(\frac{c^g}{g!}\right)2^{-(1+\rho+vg)}\bar{H}^{m+2,\,n}_{p+1,\,q+3}\left[z\middle|\begin{matrix}R_1\\R_2\end{matrix}\right]$$

Where R_1 and R_2 are sets of parameter giving as follows

$$R_1:(1+n;\rho_1),\ \left(\frac{n-\rho}{2};\frac{\rho_1-\sigma_1}{2}\right),\ \left(e_j,E_j;\in_j\right)_{1,n},\left(e_j;E_j\right)_{n+1,p}$$

$$R_2:\left(\frac{-n}{2};\frac{\sigma_1-\rho}{2},1\right),\ \left(f_j,F_j\right)_{1,m},\left(F_j,f_j;\tau_j\right)_{m+1,q},(n;\rho_1,1)$$

Provided,

$$n+\xi g+\rho_1\left(\frac{F_j}{f_j}\right)>0, \qquad \rho+\sigma_1\left(F_j/f_j\right)>0,$$

$$|\arg(z)|\leq\frac{1}{2}\Omega\pi,$$

(3.1)

Where Ω is mention in (1.3)

Second Integral:

$$\int_0^\infty x^{\rho-1}\left\{x+\sqrt{1+x^2}\right\}^{-n}\phi_\mu(z,s,a,g).\bar{H}\left[zx^{\sigma_1}.\left\{x+\sqrt{1+x^2}\right\}^{-\rho_1}\right]dx.$$

$$=\sum_{g=0}^{\infty}(\mu)_g(a+g)^{-s}\frac{c^g}{\lfloor g}.2^{(\rho+1+vg)}.\bar{H}_{p+3,\,q+2}^{m+1,\,n+2}\left[z\middle|\begin{matrix}R_3\\R_4\end{matrix}\right]$$

Where R_3 and R_4 are sets of parameter giving as follows

$$R_3:(-n;\rho_1,1),\ (-1+n-\rho;\rho_1+\sigma_1,1),\ (e_j,E_j;\in_j)_{1,n},(e_j;E_j)_{n+1,p},(P,\sigma_1)$$

$$R_4:\left(\frac{n+\rho+1}{2};\frac{\rho_1-\sigma_1}{2}\right),\ (f_j,F_j)_{1,m},(F_j,f_j;\tau_j)_{m+1,q},(1-n;\rho,1)$$

Provided,

$$n+\in g+\rho_1\left(\frac{F_j}{f_j}\right)>0, \qquad \rho-\sigma_1\left(F_j/f_j\right)>0,$$

$$|\arg(z)|\leq\frac{1}{2}\Omega\pi,$$

Where Ω is mention in (1.3)

Proof:

To prove the first integral we first express $\bar{H}_{p.q}^{m,n}[z]$ occurring in the left hand side of (3.1) in term of Mellin-Barnes contour integral with the help of (1.1) and changing the order of integration, which is permissible under the conditions stated, using the (2.1) and (2.2) and interpreting the expression thus obtained in term of $\bar{H}$ -function with the help of (1.1), we arrive at the right hand side of (3.1) after a little simplification.

The proof of second integral (3.2) can be developed proceeding on similar line to that of (3.1).

IV. Special Cases

The importance of our Integrals involving $\bar{H}$ -function and Remain zeta function lie in their manifold generality.

In view of the generality of the $\bar{H}$ -function, on specializing the various parameters, we can obtain from our integrals, several results involving a remarkable wide Varity of useful function,

Which are expressible in terms of [2] Fox H-function, G-function and Remain zeta-function they are as follows :

(1) If we take $\in_j=\tau_j=1,(i=1,....,n;\ j=m+1,....,q)$ in (3.1) and (3.2), the relation is reduce with $\bar{H}$ -function and Fox's H-function [2].

(2) If we reduce $\bar{H}_{p.q}^{m,n}[z]$ to generalized Wright hypergeometric function ${}_{p+1}\bar{\psi}_{q-1}$ [6, pp.271, Eq. (7)]

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