International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 2, Issue 7 (May 2013) PP: 01-06

Analysis and Performance Prediction of Students Using Fuzzy Relations and Interval Data

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ABSTRACT: *An attempt has been made using the approach formulated by Adlassnig [1] and Adlassnig and Kolarz [2] in the design of CADIAG-2, to find the possible marks obtained by a student in the final exam using his capacity or intelligence and the knowledge obtained using fuzzy relations, interval data and the comparison of intervals.*
Key Words: *Fuzzy relation, Interval data, CADIAG-2 and comparison of intervals.*

I. INTRODUCTION

Education today is based on the information – collection and information- giving. In this state, it is difficult to analyze the end purpose of education itself. Levels of grasping, absorbing and then expressing vary according to individuals. The output by the students depends on the student's capacity and the knowledge obtained by them. Hence by using fuzzy relations it is possible to confirm the possible marks obtained by the students in the final examinations.

Moreover, the linguistic terms high, very high, low, very low are ambiguous. Hence we use the interval data for the corresponding terms.

The model proposes two types of relations to exist between capacity and the knowledge obtained.

(1) Existence relation
(2) Assurance relation

The first relation gives information about how much a student has basic intelligence or capacity. The second relation assures the presence of intelligence and the knowledge obtained by the student in certain topics.

II. PRELIMINARIES

The distinction between assurance and existence is important and is useful because a student may be quite intelligent but may not have obtained the knowledge about one subject. On the other hand, a student with less I.Q but with the knowledge obtained might get good marks.

Let C denote the crisp universal set of all capacities, K be the crisp universal setoff all knowledge obtained by the students and S be the crisp universal set of all students.

Let us define a fuzzy relation R_c on the set $S \times C$ in which membership grades $R_c(s,c)(where\ s \in S, c \in C)$ indicates degree to which the capacity c is present in student in S. For instance, if c represents the capacity level in Calculus and the test marks is roughly 3.6 to 5.1, then a test result of 5.1 for a student S could lead to a membership grade $R_c(s,c) = 0.5$

Let us further define a fuzzy relation R_e on the universal set where $R_e(c,k)$ (where $c \in C, k \in K$) indicates the existence of capacity with the knowledge K. Let R_a also be a fuzzy relation in the same universal set (C, K) where $R_a(c,k)$ corresponds to the degree to which the capacity together with the knowledge assures the maximum marks.

We assign membership grades of 1, 0.9, 0.6, 0.3, and 0 in fuzzy sets R_e and R_a for the linguistic terms very high, high, medium, low, and very low respectively. We use a concentration operation to model the linguistic modifier very such that $A_{very}(x) = A^2(X).$

Assume that the following documentation exists concerning the relations of capacities C_1, C_2, C_3 to the Knowledge obtained K_1, K_2, K_3.

- Capacity C_1 is very high in Calculus and the knowledge obtained in Calculus K_1 is low.

- Capacity C_1 is high and K_2 the knowledge obtained in Algebra is very high.
- Capacity C_2 is very low in Algebra K_2 is high.
- Capacity C_3 in Differential equations is medium and K_3 the knowledge obtained in the same is very high.
- Capacity C_3 is very low and K_1 is low.

All missing relational pairs of capacities and the knowledge obtained are assumed to be unspecified and are given a membership grade of 0.5.

2.1 Interval Arithmetic:

Interval arithmetic is the arithmetic of quantities that lie within specified ranges (ie intervals) instead of having definite known values. Interval arithmetic can be especially useful when working with data that is subject to measurement errors or uncertainties. It can be considered a rigorous version of significance arithmetic. Refer [11].

Definition: 2.1

Let R be the set of all real numbers. The set of all real compact intervals I = {A: A = [a, b], a, b $\in$ R} and we identify the set of point intervals [a, a] $\in$ I with R. Let A, B $\in$ I. Then the interval arithmetic operations are defined by

$A*B = \{\alpha*\beta: \alpha \in A, \beta \in B\}$

where $* \in \{+, -, ., /\}$. Here / is undefined when $0 \in B$.

Let A = [a, b], B = [c, d]. It can be shown that

(i) A + B = [a+c, b + d]

(ii) A - B = [a - d, b - c]

(iii) A.B = [min{ac, ad, bc, bd}, max{ ac, ad, bc, bd}]

(iv) A/B = [a, b].[1/d, 1/c] if $0 \notin$ [c, d]

Moreover A < B iff b < c

A = B iff a = c and b = d.

A > B iff b > c.

Definition 2.2

Comparison of intervals:

Comparison of intervals is very important problem in interval analysis. In this section we consider the order relation ($\leq and \geq$) between intervals.

Let a = [a_L, a_R] and b = [b_L, b_R] and if $a_R < b_L$ then a < b crisply which is similar to the definition of comparison used in [11].

Definition 2.3

Equal intervals:

a = b iff a $\leq$ b and b $\leq$ a

Definition 2.4

Complement of an interval:

1 - [a, b] = [1 - b,1 - a]

Definition 2.5

Overlapping Intervals:

If $a_L < b_L < a_R < b_R$ then the intervals are overlapping. For any x in [a_L, b_L] a < b.

If x $\in$ [b_L, a_R], then every x is less than or equal to b.

Therefore a $\leq$ b. (crisply)

If y $\in$ [a_R, b_R] then y in b $\geq$ a.

That is b $\geq$ a (crisply).

Definition 2.6

Nested sub intervals:

If $b_L < a_L < a_R < b_R$ then the intervals are said to be nested.

In terms of fuzzy membership, we define,

$$a \prec b = \frac{b_R - a_R}{b_R - b_L}.$$

If $b \subseteq a$ then,

$$a \prec b = \frac{b_L - a_L}{a_R - a_L}.$$

Consolidating the above discussions we define the fuzzy operators $\prec$ and $\succ$ as follows.

Definition 2.7
The binary fuzzy operators $\prec$ and $\succ$ of two intervals a and b returns a real number between 0 and 1 as follows:

$$a \prec b = \begin{cases} 1 \text{ if } a = b \text{ or } a_R < b_L, a \neq b; \text{or } a_L < b_L < a_R < b_R \\ 0 \text{ if } b_R < a_L, b_L < a_L < b_R < a_R \\ \dfrac{b_L - a_L}{a_R - a_L} \text{ if } a_L < b_L < b_R < a_R \\ \dfrac{b_R - a_R}{b_R - a_L} \text{ if } b_L < a_L < a_R < b_R \end{cases}$$

These values $\frac{b_L - a_L}{a_R - a_L}$ and $\frac{b_R - a_R}{b_R - b_L}$ moves to 0 when x moves from a_L *to* b_L *and* a_R *to* b_R left to right.

Only when a = b, $a \prec b$ takes the value 1.Using simple algebraic operations, it can be seen that the membership value for $b \succ a = 1 - (a \prec b)$.

Definition 2.8
The binary fuzzy operator $\succ$ of two intervals is defined as follows:

$$a \succ b = \begin{cases} 1 \\ 0 \\ \dfrac{a_R - b_R}{a_R - a_L} \\ \dfrac{a_L - b_L}{b_R - b_L} \end{cases}$$

The relations of two intervals can now be either crisp or fuzzy as described below.

Definition 2.9
If the values of $a \prec b$ is exactly 1 or 0 then we say that a and b are crisply related. Otherwise we say that they are fuzzily related.

III. EVALUATION

We construct the following matrices of relations R_e, $R_a \in (C, K)$ with interval data as the values are uncertain [3, 4, 5].

$$R_e = \begin{bmatrix} [0.8,0.9] & [0.3,0.6] & [0.1,0.4] \\ [0.5,0.8] & [0.7,0.9] & [0.34,0.65] \\ [0.6,0.8] & [0.5,0.7] & [0.8,1] \end{bmatrix}$$

$$R_a = \begin{bmatrix} [0.7,0.9] & [0.4,0.6] & [0,1] \\ [0.6,0.8] & [0.8,1] & [0.5,0.7] \\ [0,0.25] & [0.3,0.5] & [0.9,1] \end{bmatrix}.$$

We assume that we are given a fuzzy relation R_c specifying the degree of capacities C_1 , C_2 , C_3 for three students S_1 , S_2 , S_3 as follows:

$$R_c = \begin{bmatrix} [0.3,0.5] & [0.2,0.5] & [0,0.25] \\ [0.8,1] & [0,0.15] & [0.4,0.6] \\ [0.7,0.9] & [0.5,0.75] & [0.6,0.75] \end{bmatrix}$$

Using the relations R_a, R_e, R_c we can calculate four different indication relations defined on the set SXC of students and capacities. The first indication is defined as,

$$R_1 = R_c \, o \, R_e = \begin{bmatrix} [0.3, 0.5] & [0.3, 0.5] & [0.2, 0.5] \\ [0.8, 0.9] & [0.4, 0.6] & [0.4, 0.6] \\ [0.7, 0.9] & [0.5, 0.75] & [0.6, 0.75] \end{bmatrix}.$$

The assurance indication relation is given by

$$R_2 = R_c o R_a$$

$$R_2 = \begin{bmatrix} [0.3,0.5] & [0.3,0.5] & [0.3,0.5] \\ [0.7,0.9] & [0.4,0.6] & [0.8,1] \\ [0.7,0.9] & [0.5,0.75] & [0.7,0.9] \end{bmatrix}$$

The non-existence relation R_3 is given by $R_3 = R_c o(1 - R_e)$.

$$R_3 = \begin{bmatrix} [0.2,0.5] & [0.3,0.5] & [0.3,0.5] \\ [0.2,0.4] & [0.4,0.7] & [0.6,0.9] \\ [0.2,0.5] & [0.4,0.7] & [0.6,0.9] \end{bmatrix}.$$

Finally the non-capacity indication R_4 is given by

$$R_4 = (1 - R_c) o R_e$$

$$R_4 = \begin{bmatrix} [0.6,0.8] & [0.5,0.8] & [0.8,1] \\ [0.5,0.8] & [0.7,0.9] & [0.4,0.6] \\ [0.25,0.5] & [0.25,0.5] & [0.25,0.5] \end{bmatrix}.$$

From these four indication relations, we may draw different types of conclusions. If R_2(S, K) = 1, we may make confirmed analysis of a student's knowledge. If R_3(S, K) = 1 or if R_4(S, K) = 1 may make an excluded capacity K in student S. In our example, we may exclude the capacity or knowledge for the student S_3.In our example K_1, K_2, K_3 are suitable knowledge hypotheses for students S_1, S_2, S_3.This system incorporates relations not only between knowledge and capacity but also between the knowledge themselves and capacities themselves and between combinations of knowledge and capacities.

IV. CLUSTER ANALYSIS

Another alternative approach to model the student's performance analysis utilizes fuzzy cluster analysis. This type of technique is used by Fordon and Bezdek [9] and Esogbue and Elder [6, 7, 8]. Models that use cluster analysis usually perform a clustering algorithm on the set of students by examining the similarity of the existence and assurance of capacity patterns exhibited by each. The level of capacity present can be designated with degrees of membership in fuzzy sets representing each capacity category. Often the similarity measure is computed between the capacities of the student in question and the capacities of a student possessing the prototypical capacity pattern for each possible student. The student to be analyzed is then clustered to varying degrees with the prototypical students whose capacities are most similar. The most likely diagnostic candidates are those knowledge clusters in which the student's degree of membership is the greatest.

We describe a simplified adaptation of the method employed by Esogbue and Elder [6, 7, 8] to illustrate this technique.

Let us assume that we are given a student x who displays the capacities $c_1, c_2, c_3 \,\&\, c_4$ at the levels given by the fuzzy set

$A_x = 0.1/s_1 + 0.7/s_2 + 0.4/s_3 + 0.6/s_4$ where $A_x(s_i) \in [0,\ 1]$ denotes the grade of membership in the fuzzy set characterizing student x and defined on the set

c = { c_1, c_2, c_3, c_4 }

This indicates the level of the capacity c_i for the student.

We must determine an analysis for this student among three possible knowledge obtained in 3 areas as $k_1,\ k_2\ and\ k_3$. Each of these knowledge is described by a matrix giving the upper and lower bounds of the normal range of level of each of the four capacities that can be expected in a student with the knowledge. The knowledge $k_1, k_2 \,\&\, k_3$ are described in this way by the matrices

$$B_1 = \begin{array}{c} Lower \\ Upper \end{array}\begin{bmatrix} 0 & 0.5 & 0.6 & 0.2 \\ 0.3 & 0.4 & 0.8 & 0.7 \end{bmatrix}$$

$$B_2 = \begin{array}{c} Lower \\ Upper \end{array}\begin{bmatrix} 0 & 0.8 & 0.2 & 0.3 \\ 0 & 0.9 & 0.1 & 0.5 \end{bmatrix}$$

$$B_3 = \begin{array}{c} Lower \\ Upper \end{array}\begin{bmatrix} 0.2 & 0.5 & 0.3 & 0.1 \\ 0.4 & 0.8 & 0.2 & 0.7 \end{bmatrix}$$

For each j = 1, 2, 3 matrix B_j defines fuzzy sets $B_{jl}(c_i)\ \ and\ \ B_{ju}(c_i)$ denote respectively the lower and upper bounds of capacity c_i for knowledge k_j. The relation W of these weight of relevance is given by

$$W(C,\ K) = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{array}\begin{array}{c} \begin{array}{ccc} k_1 & k_2 & k_3 \end{array} \\ \begin{bmatrix} 0.5 & 0.9 & 0.5 \\ 0.4 & 0.7 & 0.1 \\ 0.7 & 0.1 & 0.3 \\ 0.8 & 0.2 & 0.5 \end{bmatrix} \end{array}$$

where $w(c_i,\ k_i)$ denote the weight of capacity for the knowledge obtained in subject k_j. In order to discuss the student's condition performance, we use a clustering technique to determine to which performance cluster (as specified by matrices $B_1, B_2\ and\ B_3$), the student is most similar. The clustering is performed by computing a similarity measure between the student's capacities and those typical of each knowledge k_j.

To compute this similarity, we use a distance measure based on the Minkowski distance that is appropriately modified. It is given by the formula

$$D_p(k_j,\ x) = \left[\sum_{i \in I_l} \left| W(c_i,\ k_j)(B_{jl}(c_i) - (A_x(c_i)) \right|^p + \sum_{i \in I_u} \left| W(c_i,\ k_j)(B_{ju}(c_i) - A_x(c_c)) \right|^p \right]^{1/p}$$

where I_l = $\{\ i \in N_m | A_x(c_i) < \mathrm{B}_{ju}(c_i)\}$

$I_U = \{\ i \in N_m | A_x(c_i) > \mathrm{B}_{ju}(c_i)\}$, And m denotes the total number of student's capacities. Choosing, for example the Euclidean distance we use (1) with p = 2 to calculate the similarity between the student x and knowledge's k_1, k_2, k_3 in our example as follows:-

$$D_2(k_1,\ x) = \left[|(0.7)(0.6 - 0.4)|^2 + |(0.4)(0.4 - 0.7)|^2 \right]^{\frac{1}{2}} = 0.18$$

$$D_2(k_{2,}\ x) = \left[|(0.7)(0.8-0.7)|^2 + |(0.9)(0-0.1)|^2 + |(0.2)(0.5-0.6)|^2 \right]^{\frac{1}{2}} = 0.12$$

$$D_3(k_3,\ x) = \left[|(0.5)(0.2-0.1)|^2 + |(0.3)(0.2-0.4)|^2 \right]^{\frac{1}{2}} = 0.032$$

The most likely knowledgeable candidate is the one for which the similarity measure attains the minimum values. In this case, the students' capacities are most similar to those typical of knowledge k_3.

V. CONCLUSION

Thus it can be inferred that the students can be identified to score a higher percentage by (1) the cardiac method and through (2) the cluster analysis method .Thus the groups of achievers can be clustered in order to train them for excellence.

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