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Modified Method for Fixed Charge Transportation Problem

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ABSTRACT: *In traditional method for solving fixed charge transportation problem, we introduce dummy column to make balance transportation problem from unbalance transportation problem. We set zero cost for the dummy column. It is used in both crisp environment as well as fuzzy environment. But if we use the maximum cost in each row for respective position in dummy column we get better result. A comparative study between the existing method and the modified method shows that the latter is much more effective.*

Key words: *Fuzzy Number, Fuzzy Transportation Problem, Fixed charge Transportation Problem.*
***Mathematics Subject Classification** : 90B06, 90C08.*

I. INTRODUCTION

In a transportation problem (TP) generally cost of transportation is directly proportional to amount of commodity which is to be transported. However, in many real world problems, in addition to transportation cost, a fixed cost, sometimes called a set up cost, is also incurred when a distribution variable assume a positive value. Such problem are called fixed charge transportation problem (FCTP). The FCTP differs from the linear TP only in the non-linearity of the objective function. While not being linear in each of the variables, the objective function has a fixed cost associated with each origin. The fixed charge TP was originally formulated by Hirsch and Dantzig[13]. In 1961, Balinski[4] presented a technique which provides an approximate solution for any given FCTP. Many problems in practice can be treated as FCTP. FCTP has been studies by many researchers [1,2,5,6,8,15,19,21,22,23,24]

The notion of fuzzy set has been introduced by L. A. Zadeh[25] in order to formalize the concept of regardless in class membership, in connection with the representation of human knowledge [18]. It was developed to define and solve the complex system with sources of uncertainty or imprecision which are non-statistical in nature. Fuzzy TP has been studied by many authors [3,7,9,10,11,12,14,16,17,20,26].

In this paper, we have done a modification. Unbalanced TP converted into balanced TP by introducing dummy destination with maximum cost in each row. Here we have done a comparative study between existing method and the new method. We see that our modification gives better result of the TP.

II. PRELIMINARIES

2.1.1 Basic Definition

Definition 2.1 Let A be a classical set and $\mu_A(x)$ be a function defined over A → [0,1]. A fuzzy set A^* with membership function $\mu_A(x)$ is defined by

$$A^* = \{(x, \mu_A(x)) : x \in A \text{ and } \mu_A(x) \in [0, 1]\}$$

Definition 2.2 A real fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$, where $a_1, a_2, a_3, a_4 \in$ R and two functions f(x); g(x) : R → [0; 1], where f(x) is non decreasing and g(x) is non increasing, such that we can define membership function $\mu_{\tilde{a}}(x)$ satisfying the following conditions

$$\mu_{\tilde{a}}(x) = \begin{cases} f(x) & if \;\; a_1 \le x \le a_2 \\ 1 & if \;\; a_2 \le x \le a_3 \\ g(x) & if \;\; a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

Definition 2.3 The membership function of trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$, is defined by

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & for \;\; a_1 \le x \le a_2 \\ 1 & for \;\; a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & for \;\; a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

Definition 2.4 The magnitude of the trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, a_3, a_4)$ is defined by

$$Mag(\tilde{a}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

Definition 2.5 The two fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ are said to be $\tilde{a} > \tilde{b}$ if

$$Mag(\tilde{a}) > Mag(\tilde{b}).$$

Definition 2.6 The two fuzzy numbers $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ are said to be equal if

$$Mag(\tilde{a}) = Mag(\tilde{b}).$$

2.1.2 Arithmetic operations: Let $\tilde{a} = (a_1, a_2, a_3, a_4)$ and $\tilde{b} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers, where $a_1, a_2, a_3, a_4; b_1, b_2, b_3, b_4 \in R$ then the arithmetic operation on $\tilde{a}$ and $\tilde{b}$ are:

1. **Addition**: The addition of $\tilde{a}$ and $\tilde{b}$ is $\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$.
2. **Subtraction**: $-\tilde{b} = (-b_4, -b_3, -b_2, -b_1)$, then the subtraction of $\tilde{a}$ and $\tilde{b}$ is
   $\tilde{a} \ominus \tilde{b} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$.
3. **Multiplication**: The multiplication of $\tilde{a}$ and $\tilde{b}$ is $\tilde{a} \otimes \tilde{b} = (t_1, t_2, t_3, t_4)$
   where $t_1 = \min\{ a_1b_1, a_1b_4, a_4b_1, a_4b_4 \}$; $t_2 = \min\{a_2b_2, a_2b_3, a_3b_2, a_3b_3 \}$.
   $t_3 = \max\{ a_2b_2, a_2b_3, a_3b_2, a_3b_3 \}$; $t_4 = \max\{ a_1b_1, a_1b_4, a_4b_1, a_4b_4 \}$.
4.

$$\rho \otimes \tilde{a} = \begin{cases} (\rho a_1, \rho a_2, \rho a_3, \rho a_4) & for \ \ \rho \geq 0. \\ (\rho a_4, \rho a_3, \rho a_2, \rho a_1) & for \ \ \rho \leq 0. \end{cases}$$

2.2 Problem Formulation

In 1994, Basu et. al.[6] consider a fixed charge transportation problem in crisp environment as follows:

$$P_1: \quad Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} F_i$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i\ ; \qquad \text{for } i = 1,2,3,\ldots\ldots\ldots,m.$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ ; \qquad \text{for } j = 1,2,3,\ldots\ldots\ldots,n.$$

and $x_{ij} \geq 0$

where

x_{ij} = the amount of product transported from the i^{th} origin to the j^{th} destination,

c_{ij} = the cost involved in transporting per unit product from the i^{th} origin to the j^{th} destination,

F_i = the fixed cost (or fixed charge) associated with origin i,

a_i = the number of units available at the i^{th} origin,

b_j = the number of units required at the j^{th} destination.

m is the number of origin and n is number of destination.

In 2010, A. Kumar et. al.[16] consider fixed charge transportation problem in fuzzy environment as follows:

$$P_2: \quad Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \oplus \sum_{i=1}^{m} \tilde{f}_i$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq a_i\ ; \qquad \text{for } i = 1,2,3,\ldots\ldots\ldots,m.$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ ; \qquad \text{for } j = 1,2,3,\ldots\ldots\ldots,n.$$

and $x_{ij} \geq 0$

where $\tilde{f}_i$ = the fixed cost associated with origin i, and all other notation are defined in above.

In both cases the solution procedure as follows:

First we have to balanced the problem P_1 and P_2 using dummy destinations. Then we have

$$P_3: \quad Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n+1} c_{ij} x_{ij} + \sum_{i=1}^{m} F_i$$

subject to

$$\sum_{j=1}^{n+1} x_{ij} = a_i\ ; \qquad \text{for } i = 1,2,3,\ldots\ldots\ldots,m.$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ ; \qquad \text{for } j = 1,2,3,\ldots\ldots\ldots,n,n+1.$$

and $x_{ij} \geq 0$

where $\quad c_{i,n+1} = \max\limits_{1 \leq j \leq n} \{ c_{ij} \}, \quad 1 \leq i \leq m$

and

$$P_4: \quad Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n+1} \tilde{c}_{ij} x_{ij} \oplus \sum_{i=1}^{m} \tilde{f}_i$$

subject to

$$\sum_{j=1}^{n+1} x_{ij} = a_i\ ; \qquad \text{for } i = 1,2,3,\ldots\ldots\ldots,m.$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ ; \qquad \text{for } j = 1,2,3,\ldots\ldots\ldots,n,n+1.$$

and $x_{ij} \geq 0$

where $\quad \tilde{c}_{i,n+1} = \max\limits_{1 \leq j \leq n} \{ \tilde{c}_{ij} \}, \quad 1 \leq i \leq m$

In problem P_3 and P_4, we consider the costs associated with the dummy cells are all maximum in each corresponding row. Find a basic feasible solution of the problem P_3 and P_4 with respect to the variable costs. Let B be the current basis.

III. ALGORITHM

In both the cases algorithm are same while first one is in crisp environment and second one is in fuzzy environment.

Step 1: Convert into balanced transported problem.

Step 2: Set k=1, where is the number of iterations in the algorithm.

Step 3: Find a basic feasible solution of the problem P_3 with respect to the variable costs. Let B be the current basis.

Step 4: calculate the fixed cost of the current basic feasible solution (without considering dummy cells) and denote this by F^1(current), where $F^1(\text{current}) = \sum_{i=1}^{m} F_i$

Step 5: Find $(C_{ij} - u_i - v_j)$; for all (i; j) $\notin$ B and denote it by $(C_{ij})_1$; where u_i, v_j are the dual variables for i = 1, 2, 3,…….,m; j = 1, 2, 3,…………..,n, n + 1.

Step 6: Find $A_{ij}^1 = (C_{ij})_1 \times (E_{ij})_1$, where A_{ij}^1 is the change in cost occurs for introducing a non-basic (i; j) cell with value $(E_{ij})_1$ (for all i, j $\notin$ B) into the basis by making reallocation.

Step 7: Find F_{ij}^1 (Difference) = F_{ij}^1(NB) – F^1(current), where F_{ij}^1(NB) is the total fixed cost involved for introducing the variable x_{ij} with values $(E_{ij})_1$ (for all i, j $\notin$ B) into the current basis to form a new basis.

Step 8: Add F_{ij}^1 (Difference) and A_{ij}^1; and denote it by Δ_{ij}^1, i.e. $\Delta_{ij}^1 = F_{ij}^1$ (Difference) + A_{ij}^1, for all i, j $\notin$ B.

Step 9: If all $\Delta_{ij}^1 \geq 0$, then goto Step 10; otherwise find min { Δ_{ij}^1, $\Delta_{ij}^1 \leq 0$, $\forall$ i,j $\notin$B }. Then the variable x_{ij} associated with min (Δ_{ij}^1) will enter into the basis, where I, j $\notin$ B. Continue this procedure until all $\Delta_{ij}^1 \geq 0$. Goto Step 3.

Step 10: Let Z_1^* be the optimum cost of P_1 and X_1^* be the optimum solution corresponding to Z_1^*.

Similar Algorithm for the problem P_4.

IV. NUMERICAL EXAMPLE

Basu et. al. [6] consider the fixed charge transportation problem which is tabulated in Table 1.

Table 1

	D_1	D_2	D_3	a_i
O_1	5	9	9	19
O_2	4	6	2	10
O_3	2	1	1	11
b_j	5	8	15	

The fixed cost are

$$F_{11} = 100;\quad F_{12} = 50;\quad F_{13} = 50$$
$$F_{21} = 150;\quad F_{22} = 50;\quad F_{23} = 50$$
$$F_{31} = 200;\quad F_{32} = 30;\quad F_{33} = 50$$

Where $F_i = \sum_{l=1}^{3} \delta_{il} F_{il}$ for i = 1; 2; 3

where $\delta_{i1} = 1$; if $\sum_{l=1}^{3} x_{ij} > 0$ for i = 1, 2, 3:
= 0; otherwise;

where $\delta_{i2} = 1$; if $\sum_{l=1}^{3} x_{ij} > 7$ for i = 1, 2, 3:
= 0; otherwise;

where $\delta_{i3} = 1$; if $\sum_{l=1}^{3} x_{ij} > 10$ for i = 1, 2, 3:
= 0; otherwise;

In [6] the optimum solution is $X^* = \{x_{11} = 5,\ x_{13} = 14,\ x_{32} = 8,\ x_{33} = 1\}$, with optimum cost $Z^* = 660$.

Introducing dummy destination D_4 with maximum cost of the corresponding row in Table 1, we get Table 2.

Table 2

	D_1	D_2	D_3	D_4	a_i
O_1	5	9	9	9	19
O_2	4	6	2	6	10
O_3	2	1	1	2	11
b_j	5	8	15	12	

The optimum solution of this problem are tabulated in Table 3.

Table 3.

	D_1	D_2	D_3	D_4	a_i
O_1	5 $\boxed{5}$	9 $\boxed{8}$	9 $\boxed{5}$	9 $\boxed{1}$	19
O_2	4	6	2 $\boxed{10}$	6	10
O_3	2	1	1	2 $\boxed{11}$	11
b_j	5	8	15	12	

The optimum solution is $X_1^* = \{ x_{11} = 5, x_{12} = 8, x_{13} = 5, x_{23} = 10 \}$, with optimum cost $Z_1^* = 562$.

Kumar et. al.[16] consider the fixed charge transportation problem in fuzzy environment which is tabulated in Table 4.

Table 4

	D_1	D_2	D_3	a_i
O_1	(1,4,5,10)	(3,6,9,18)	(3,6,9,18)	19
O_2	(1,3,4,8)	(2,4,6,12)	(0,1,2,5)	10
O_3	(0,1,2,5)	(0,0.5,1.5,2)	(0,0.5,1.5,2)	11
b_j	5	8	15	

The fixed cost are

$\tilde{f}_{11} = (70, 80, 100, 150)$; $\tilde{f}_{12} = (30, 40, 50, 80)$; $\tilde{f}_{13} = (30, 40, 50, 80)$;
$\tilde{f}_{21} = (90, 100, 200, 210)$; $\tilde{f}_{22} = (30, 40, 50, 80)$; $\tilde{f}_{23} = (30, 40, 50, 80)$;
$\tilde{f}_{31} = (100; 150; 200; 350)$; $\tilde{f}_{32} = (70, 80, 100, 150)$; $\tilde{f}_{33} = (30, 40, 50, 80)$;

Where $\tilde{f}_i = \sum_{l=1}^{3} \delta_{il} \tilde{f}_{il}$ for $i = 1; 2; 3$

where $\delta_{i1} = 1$; if $\sum_{l=1}^{3} x_{ij} > 0$ for $i = 1, 2, 3$:
$= 0$; otherwise;

where $\delta_{i2} = 1$; if $\sum_{l=1}^{3} x_{ij} > 7$ for $i = 1, 2, 3$:
$= 0$; otherwise;

where $\delta_{i3} = 1$; if $\sum_{l=1}^{3} x_{ij} > 10$ for $i = 1, 2, 3$:
$= 0$; otherwise;

In [16] the optimum solution is $\tilde{X}^* = \{ \tilde{x}_{11} = 5, \tilde{x}_{13} = 14, \tilde{x}_{32} = 8, \tilde{x}_{33} = 1\}$, with optimum cost $\tilde{Z}^* = (347, 498.5, 664.5, 1130)$.

Here also $\tilde{f}_i$ has consider three steps as above. Introducing dummy destination D_4 with maximum cost of the corresponding row in Table 4, we get Table 5

Table 5

	D_1	D_2	D_3	D_4	a_i
O_1	(1,4,5,10)	(3,6,9,18)	(3,6,9,18)	(3,6,9,18)	19
O_2	(1,3,4,8)	(2,4,6,12)	(0,1,2,5)	(2,4,6,12)	10
O_3	(0,1,2,5)	(0,0.5,1.5,2)	(0,0.5,1.5,2)	(0,1,2,5)	11
b_j	5	8	15	12	

The optimum solution of this problem are tabulated in Table 6

Table 6

	D_1	D_2	D_3	D_4	a_i
O_1	(1,4,5,10) $\boxed{5}$	(3,6,9,18) $\boxed{8}$	(3,6,9,18) $\boxed{5}$	(3,6,9,18) $\boxed{1}$	19
O_2	(1,3,4,8)	(2,4,6,12)	(0,1,2,5) $\boxed{10}$	(2,4,6,12)	10
O_3	(0,1,2,5)	(0,0.5,1.5,2)	(0,0.5,1.5,2)	(0,1,2,5) $\boxed{11}$	11
b_j	5	8	15	12	

The optimum solution is $\tilde{X}_1^* = \{\tilde{x}_{11} = 5, \tilde{x}_{12} = 8, \tilde{x}_{13} = 5, \tilde{x}_{23} = 10\}$, with optimum cost $\tilde{Z}_1^* =$ (299, 408, 612, 934).

Comparative study between Basu et. al. , Kumar et. al. and modified method, is given in Table 7.

Table 7

	Optimum solution	Optimum cost
Basu et. al	$X^*=\{x_{11} = 5, x_{13} = 14, x_{32} = 8, x_{33} = 1\}$	$Z^*=660$
Modified Method	$X_1^* =\{ x_{11} = 5, x_{12}= 8, x_{13} = 5, x_{23} = 10\}$	$Z_1^* =562$
Kumar et. al.	$\tilde{X}^* =\{ \tilde{x}_{11} = 5, \tilde{x}_{13} = 8, \tilde{x}_{32} = 8, \tilde{x}_{33} = 10\}$	$\tilde{Z}^*$=(347, 498.5, 664.5, 1130). Mag.($\tilde{Z}^*$) = 660
Modified Method	$\tilde{X}_1^* =\{ \tilde{x}_{11} = 5, \tilde{x}_{12} = 8, \tilde{x}_{13} = 5, \tilde{x}_{23} = 10\}$	$\tilde{Z}_1^*$ =(299, 408, 612, 934). Mag.($\tilde{Z}_1^*$) = 562

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