*International Journal of Engineering Inventions*
*e-ISSN: 2278-7461, p-ISSN: 2319-6491*
*Volume 3, Issue 3 (October 2013) PP: 12-14*

# Some Studies on Semirings and Ordered Semirings

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***ABSTRACT:*** *In this paper, we study the properties of Semirings and ordered semirings satisfying the identity* ***a + ab = a****. It is proved that Let (S, +, .) be a t.o.semiring satisfying the condition a + ab = a, for all a, b in S. If (S, +) is p.t.o (n.t.o), then (S, .) is non-positively ordered (non-negatively ordered).*

***Keywords:*** *left singular semigroup, mono semiring, non-positively ordered, rectangular band.*
***2000 Mathematics Subject Classification****: 20M10, 16Y60.*

## I. INTRODUCTION

The concept of semiring was first introduced by Vandiver in 1934. However the developments of the theory in semirings have been taking place since 1950. Semirings have been studied by various researchers in an attempt to broaden techniques coming from the semigroup theory or ring theory or in connection with applications. Using the techniques of (ordered) semigroups, M. satyanarayana examines in his paper whether the multiplicative structure of semirings determines the order structure as well as the additive structure of the semirings. Here, in this paper, there are two sections. we are going to study the nature of semirings and ordered semirings with the identity a + ab = a subject to different constrains.

## II. PRELIMINARIES

A triple (S, +, .) is called a semiring if (S, + ) is a semigroup; (S, .) is semigroup; a (b + c) = ab + ac and (b + c ) a = ba + ca for every a, b, c in S. (S, +) is said to be band if a + a = a for all a in S. A (S, +) semigroup is said to be rectangular band if a + b + a = a for all a, b in S. A semigroup (S, .) is said to be a band if $a = a^2$ for all a in S. A semigroup (S, .) is said to be rectangular band if aba = a. A semiring (S, +, .) is said to be Mono semiring if a + b = ab for all a, b in S.

**Definition 2.1 :** A semigroup (S, .) is said to be left ( right ) singular if ab = a (ab = b) for all a, b in S.

**Definition 2.2 :** A semigroup (S, +) is said to be left ( right ) singular if a + b = a (a + b = b) for all a, b in S.

**Theorem 2.3:** If (S, +, .) is a semiring and S contains multiplicative identity which is also additive identity 1, then (S, . ) is left singular if and only if a + ab = a for all a, b in S.

**Proof :** Consider a + ab = a for all a, b in S

$\Rightarrow$ a (e + b ) = a

$\Rightarrow$ ab = a

i.e ( S, . ) is left singular

Converse is also evident

**Definition 2.4 :** A semiring (S, +, .) with additive identity zero is said to be zerosumfree semiring if x + x = 0 for all x in S.

**Theorem 2.5:** If (S, +, .) is a Semiring satisfying the identity a + ab = a for all a, b in S and S contains a multiplicative identity which is also an additive identity. Then the following are true.

**( i )** ( S, + ) is left singular.

**( ii )** S is a mono semiring

**( iii )** (S, .) is band

**(iv)** $(ab)^n + (ba)^n = a + b$

**Proof: (i)** Consider a + ab = a for all a , b in S …… (1)

$\Rightarrow$ a + ab + b = a + b $\Rightarrow$ a + (a + e ) b = a + b

$\Rightarrow$ a + ab = a + b $\Rightarrow$ a = a + b ……. (2)

Similarly b + ba = b for all b, a in S

$\Rightarrow$ b + ba + a = b + a $\Rightarrow$ b + ( b + e ) a = b + a

$\Rightarrow$ b + ba = b + a $\Rightarrow$ b = b + a ……. (3)

From (2) and (3)

(S, +) is left singular

**(ii)** Consider a + ab = a for all a , b in S

$\Rightarrow$ a + ab + b = a + b $\Rightarrow$ a [ e + b ] + b = a + b

$\Rightarrow$ ab + b = a + b $\Rightarrow$ [ a + e ] b = a + b
$\Rightarrow$ ab = a + b
which is a mono semiring
**(iii)** From (1) we have $a + a^2 = a$ for all a in S
a (e + a ) = a $\Rightarrow$ a.a = a $\Rightarrow$ $a^2 = a$ for all a in S
**(iv)** (ab) + (ba) = a + b ( $\because$ from Theorem 2.3 )
$\Rightarrow$ $(ab)^2 + (ba)^2 = a^2 + b^2 = a + b$ ( $\because$ from ( iii ) )
Continuing like this we get $(ab)^n + (ba)^n = a + b$
The following is the example for both theorem 2.3 and 2.5
**Example 2.6: S ={ e, a, b }**

| + | e | a | b |
|---|---|---|---|
| e | e | a | b |
| a | a | a | a |
| b | b | a | b |

| . | e | a | b |
|---|---|---|---|
| e | e | a | b |
| a | a | a | a |
| b | b | a | b |

**Theorem 2.7:** If (S,+, .) is a semiring satisfying the identity a + ab = a for all a, b in S and (S, .) is right singular semigroup, then (S , +) is left singular semigroup.
**Proof:** a + ab = a for all a, b in S ……..(1)
Let (S , . ) is right singular
ab = b $\Rightarrow$ a + ab = a + b $\Rightarrow$ a = a + b ( $\because$ from (1) )
And ba = a $\Rightarrow$ b + ba = b + a
$\Rightarrow$ b = b + a ( $\because$ b + ba = b for all b, a in S)
Therefore, (S, +) is left singular semigroup
**Theorem 2.8:** If ( S, +, . ) is a semiring satisfying the identity a + ab = a for all a, b in S and ( S, . ) is rectangular band, then (S, + ) is a band.
**Proof:** Consider b + ba = b for all b, a in S ….. (1)
(S, .) is rectangular band implies bab = b ……. (2)
From (1) and (2) we have
b + ba = bab $\Rightarrow$ ab + aba = abab
$\Rightarrow$ ab + a = ab ( $\because$ (S, . ) is rectangular band aba = a )
$\Rightarrow$ a + ab + a = a + ab $\Rightarrow$ a + a = a ( $\because$ a + ab = a )
Therefore, (S, +) is band
**Theorem 2.9:** Let (S, +, .) be a semiring satisfying the identity a + ab = a for all a, b in S. If (S, +) is commutative and (S, +) is rectangular band, then $a + b^n = a$.
**Proof: S**ince (S , +) is rectangular band implies a + b + a = a …… (1)
Consider a + ab = a for all a, b in S ….. (2)
$\Rightarrow$ a + (a + b + a ) b = a $\Rightarrow$ $a + ab + b^2 + ab = a$
$\Rightarrow$ $a + b^2 + ab = a$ $\Rightarrow$ $a + ab + b^2 = a$ ( $\because$ (S, +) is commutative )
$\Rightarrow$ $a + b^2 = a$ $\Rightarrow$ $ab + b^2 b = ab$ ( $\because$ from (2) )
$\Rightarrow$ $ab + b^3 = ab$ $\Rightarrow$ $a + ab + b^3 = a + ab$
$\Rightarrow$ $a + b^3 = a$ ( $\because$ from (2) )
Continuing like this we get $a + b^n = a$
**Theorem 2.10** Let (S, +, .) be a semiring satisfying the identity $a + a^2 = a$ for all a, b in S and PRD, then b + ab = b, for all b in S.
**Proof:** By hypothesis, $a + a^2 = a$, for all 'a' in S
Consider $a + a^2 = a$ $\Rightarrow$ a.1 + a.a = a.1 $\Rightarrow$ a (1 + a) = a.1
$\Rightarrow$ (1 + a) = 1$\Rightarrow$ (1 + a) b = 1.b $\Rightarrow$ b + ab = b
$\therefore$ b + ab = b, for all a, b in S

## III. ORDERED SEMIRINGS

**Definition 3.1:** A totally ordered semigroup (S, ·) is said to be non-negatively (non-positively) ordered if every one of its elements is non-negative (non-positive). (S, ·) is positively ( negatively ) ordered in strict sense if $xy \geq x$ and $xy \geq y$ ( $xy \leq x$ and $xy \leq y$ ) for every x and y in S.
**Theorem 3.2:** If (S, +, .) is a totally ordered semiring satisfying the identity a + ab = a for all a, b in S and (S, +) is p.t.o (n.t.o), then (S, .) is n.t.o (p.t.o ).
**Proof:** Let a + ab = a for all a, b in S

$\Rightarrow a = a + ab \geq ab \Rightarrow a \geq ab$ ( $\because$ ( S, + ) is p.t.o )
Suppose $b \leq ab$
$\Rightarrow a + b \leq a + ab \Rightarrow a + b \leq a$
Which is a contradiction to (S, +) is p.t.o
$\therefore b \geq ab$
Therefore, $ab \leq a$ and $ab \leq b$
Hence, (S, .) is n.t.o
Similarly, we can prove that, (S, .) is p.t.o
**Example 3.3 : 1 < b < a**

| + | 1 | a | b |
|---|---|---|---|
| 1 | 1 | a | b |
| a | a | a | a |
| b | b | a | b |

| . | 1 | a | b |
|---|---|---|---|
| 1 | 1 | a | b |
| a | a | a | a |
| b | b | a | b |

**Theorem 3.4:** Let (S, +, .) be a t.o.semiring satisfying the condition $a + ab = a$, for all a, b in S. If (S, +) is p.t.o (n.t.o), then (S, .) is non-positively ordered (non-negatively ordered).
**Proof:** Given that $a + a^2 = a$, for all `a' in S
Since (S, +) is p.t.o
Then $a + a^2 \geq a$ and $a^2$
$\Rightarrow a = a + a^2 \geq a^2$
$\Rightarrow a \geq a^2$
$\Rightarrow a^2 \leq a$
Therefore , (S, .) is non–positively ordered
Similarly, If (S, +) is n.t.o, (S, .) is non–negatively ordered

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