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Improved Particle Swarm Algorithm to Solve the Vehicle Routing Problem

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Abstract: *Vehicle routing problem is a NP hard problem. To solve the premature convergence problem of the particle swarm optimization, an improved particle swarm optimization method was proposed. In the first place, introducing the neighborhood topology, defining two new concepts lepton and hadron. Lepton are particles within the scope of neighborhood, which have weak interaction between each other, so they update speed and position according to the individual extreme value. Hardon are the local optimal particles, which through collide to produce strong reaction, so set the local optimal particles collision with the global optimal particle to update speed and position. Finally, when the algorithm step to stagnation, we take particle decay to increase the population diversity. Simulated experiments show that stagnation behavior of basic particle swarm optimization are avoided in improved particle swarm algorithm and a good ability of searching better solution in process of solving vehicle routing problem.*

Keywords: Vehicle routing problem, Hadron, Lepton, Decay

I. INTRODUCTION

With the development of market economy and the raising level of specialization logistics technology, logistics distribution got rapid development, so vehicle routing problem(VRP) became an important problem needed to resolve. VRP was proposed by Dantzig and Ramser in 1959. It refers to some customer points have different demands of goods, arranged by distribution center according to the appropriate path, and require to meet the goal of shortest total path length, minimum cost and the least time consuming under the given conditions[1]. At present, the use of heuristic algorithm such as genetic algorithm, ant colony algorithm and simulated annealing algorithm have obtain good effect[2]. Particle swarm optimization(PSO) algorithm as a new kind of swarm intelligence algorithm was put forward by professor Kennedy and Eberhart in 1995[3]. It has the characteristics of parallel processing, good robustness, easy realization, and large probability to find the global optimal solution, so it cause wide concern among scholars. PSO have be applied to function problem, neural network training, pattern classification, fuzzy control system and so on[4]. In solving VRP, particle swarm optimization based on collision was put forward to solve the vehicle problem with time windows by Qin jia-jiao, etc[5]. In literature [6], using scanning method to generate initial feasible solution and taking in PSO to solve again. By introducing a neighbor factor, strengthen the function of particle study, Zhang nian-zhi, etc, have improve the convergence precision of the algorithm[7]. Under the enlightenment of all above literature, an improved particle swarm optimization(IPSO) algorithm is put forward.

II. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

The vehicle scheduling problem can be described as: There is one distribution center and l customers. The distribution center have n vehicles with load capacity q. The demand of each customer is $g_i (i = 1,2,\cdots,l)$ $q > g_i$. Vehicles set out from distribution center and finally go back to the center. Each customer point can only be supplied by one car. Customer demands can't exceed the sum weight of per car. In order to show the model conveniently, now define as follow: The number of distribution center is 0 and the number of each customer is $1,2,\cdots,l$. c_{ij} said the transportation cost from customer i to customer j . x_{ijk} indicates whether the vehicle transport from point i to point j . y_{ik} indicates whether the point j distributed by vehicle k and y_{0k} equal to 1. With this symbols to establish the mathematical model of vehicle scheduling optimization problems is as follow:
The objective function:

$$\min Z = \sum_{k=1}^{m} \sum_{i=0}^{l} \sum_{j=0}^{l} c_{ij} x_{ijk} \tag{1}$$

Constraints:

$$x_{ijk} = 0 or 1,\ i, j = 0,1,\cdots,l;\ k = 1,2,\cdots,n \tag{2}$$

$$y_{ik} = 0 or 1,\ i = 1,2,\cdots,l;\ k = 1,2,\cdots,n \tag{3}$$

$$y_{0k} = 1,\ k = 1,2,\cdots,n \tag{4}$$

$$\sum_{k=1}^{n} y_{ik} = 1,\ i = 1,2,\cdots,l \tag{5}$$

$$\sum_{i=0}^{l} x_{ijk} = y_{jk},\ j = 0,1,\cdots l;\ k = 1,2,\cdots,n \tag{6}$$

$$\sum_{j=0}^{l} x_{ijk} = y_{ik},\ i = 0,1,\cdots l;\ k = 1,2,\cdots,n \tag{7}$$

$$\sum_{i=1}^{l} g_i \quad y_{ik} \le q_k,\ k = 1,2,\cdots,n \tag{8}$$

In above model, formula (1) is the objective function. Function require the total cost of vehicle dispatch scheme to be minimum. Formula (2) indicates whether the vehicle drive from point i to point j. Formula (3) indicates whether the customer i serviced by vehicles k. Formula (4) said that there are m cars start off from distribution center and the last also have m cars in return. Formula (5) represent that each customer has one and only one vehicle for it services. Formula (6) said the vehicle can only service for the needed customer. Formula (7) said the vehicle can only drive from the customer which have been served just now. Formula (8) said the sum of all customer demands served by vehicle k can't surpass the vehicle weight.

III. IMPROVED PARTICLE SWARM OPTIMIZATION ALGORITHM (IPSO)

3.1 Interaction between lepton

3.1.1 Lepton

Particle swarm algorithm which is a kind of swarm intelligent optimization algorithms, is put forward by foraging act of research and observation about groups of birds. Each alternative solution is known as a "particle", which update speed and position according to their own "experience" and the best "experience". Improved algorithm in this paper, the influence by individual extreme and group extreme is defined as weak interaction and particles within the scope of neighborhood always affected by the weak interaction. So the particles within the neighborhood be called lepton.

3.1.2 Neighborhood range

Adding neighborhood topology into the algorithm means that dividing all the particles into several neighborhood by a certain topological structure. Particles in neighborhood can share information with each other. Though the convergence speed is slow, it is hard to fall into local optimum. In order to transfer global information effectively and better maintain the independent searching ability of neighborhood, we take a simple scheme based on literature [8]. According to the particle index number to divide neighborhood, if there have been divided into L neighborhoods, the amount of former $L-1$ neighborhoods' particle is just like this: $h = (m\, div\, L) + 1$. The total number of particles divide number of neighborhood exactly and plus one. The last neighborhood's particle number equal to $m \bmod h$. So the number j neighborhood's express is:

$$N_j = \begin{cases} \{i \mid (j-1)\times h + 1 \le i \le j \times h\}, 1 \le j < L \\ \{i \mid (L-1)\times h + 1 \le i \le m\}, j = L \end{cases} \tag{9}$$

3.1.3 The change of speed and position

In a n dimensional space, the current neighborhood X is consist of m particles, $X = \{x_1, \cdots x_i, \cdots x_m\}$. The number i particle's position $x_i = (x_{i1}, x_{i2}, \cdots, x_{in})^T$ and the speed $V_i = (v_{i1}, v_{i2}, \cdots v_{in})^T$. The individual extreme $P_i = (p_{i1}, p_{i2}, \cdots p_{in})^T$, the global extreme of current neighborhood $P_g = (p_{g1}, p_{g2}, \cdots p_{gn})^T$. x_i changes speed and position according to formula (10) and (11).

$$v_{id}^{(t+1)} = \omega^{(t)} v_{id}^{(t)} + c_1 r_1 (p_{id}^{(t)} - x_{id}^{(t)}) + c_2 r_2 (p_{gd}^{(t)} - x_{id}^{(t)}) \tag{10}$$

$$x_{id}^{(t+1)} = x_{id}^{(t)} + v_{id}^{(t+1)} \tag{11}$$

Where $d=1,2,\ldots,n$, $i=1,2,\ldots,m$, m as the population size, t for the current number of iteration, r_1 and r_2 are random number between $[0,1]$. c_1 and c_2 are acceleration constant. The inertia weight ω describe the last generation's velocity influence on the current particles. With large value of ω, it have strong global optimization ability, but weak local search ability. On the other hand, it will get a opposite effect. Shi[9] put forward the linear inertia weight and the fuzzy system, used to adjust the weight, but it still have some shortcomings. So we adopt the strategy of random inertia weight value, namely ω range between $[0,1]$ randomly[10]. In addition, for the particle velocity not so big, can set a upper limit $V_{\max}$.When $v_{id} > V_{\max}$, taking $v_{id} = V_{\max}$. When $v_{id} < -V_{\max}$,taking $v_{id} = -V_{\max}$. Particle swarm generate initial position and speed randomly, and then transform to find the optimal solution in the field according to (10) and (11).

3.2 The interaction between hadron

3.2.1 Hadron

There is strong interaction between hadron, which express as collision with each other. In this paper, the local optimal particles be defined as hadron and make it crash with the global optimal particle to change speed and position, so as to increase the population diversity and avoid fall into local optimum.

3.2.2 The change of speed and position

When the current local optimal particle P_g crash with the global optimal particle G_b, according to the momentum conservation theorem, the formula is as follow:

$$m_g v_g + m_b v_b = m_g v_g^{'} + m_b v_b^{'} \tag{12}$$

Omit the quality of the particles:

$$v_g + v_b = v_g^{'} + v_b^{'} \tag{13}$$

Where v_g is the optimal speed of current neighborhood. v_b is the global optimal velocity of previous generation. Two particles' speed has changed after collision, taking the better velocity to update the local particle. At the same time, updating the global optimal particle's velocity. Particles' motion equation is as follow:

$$v_{t+1} = \begin{cases} v_g, if & f(P_g(t+1)) < f(P_g(t)) \\ v_g^{'}, if & f(P_g(t+1)) > f(P_g(t)) \end{cases} \tag{14}$$

$$x_{t+1} = x + v_{t+1} \tag{15}$$

3.3 Particle decay

Particle decay is a spontaneous process. Improved algorithm defined particle decay into a process of variation. Variation several consecutive generation, when the fitness of optimal particle have little change or no change, there may be some particle of certain dimensions have premature convergence. The algorithm will fall into the local optimum, at this time make the optimal particle decay or variation. This article use Cauchy mutation operator to act on G_b. Cauchy mutation operator are defined as follow:

For the global optimal particle $G_b = (G_{b1}, \cdots, G_{bk}, \cdots G_{bn})$, performing mutation on the number K component, $G_{bk}^{'} = G_{bk} + 0.618 \times \xi(G_{bk})$. $\xi(G_{bk})$ is the Cauchy distribution density function, specific defined as:

$$\xi(G_{bk}) = \frac{\alpha}{\pi \times (G_{bk}^2 + \alpha^2)} \qquad (\alpha = 0.2) \tag{16}$$

IV. IMPROVED ALGORITHM TO SOLVE VRP

4.1 Constructing the particle expression

The distribution schemes can use one dimensional array to express, which must include all the customer point number and different symbols to express different cars. In this paper, we use the encode mode like this[6]: putting $(K-1)$ breakpoints in the array with L numbers, making the array break into K parts, then constructing a $(L+K-1)$ dimensional space for K vehicles service L customer points' VRP. For example, with customer demand point 14 and vehicle number 4, each particle's position vector element represent 14 demand points and 3 "break" points. The three "break" points in the role of make the 14 demand points into 4 portions, corresponding to the four cars. So the element of each distribution scheme composed by $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,0,0,0\}$.

Making $A=[1,2,3,4,5,6,7,8,9,10,11,12,13,14,0,0,0]$ as the initial feasible solution. Based on the particle position vector X to get the distribution scheme vector A. The specific process is like this: establish a one to one correspondence relationship with X and the initial distribution vector A, order the components of X from small to large, then get the new delivery scheme A'. For example, in the process of operation a particle's position vector: X =[15.6561 10.5431 10.0870 0.8137 3.4068 9.5864 14.5864 2.0402 10.2328 4.3162 0.8195 14.7793 13.8601 1.3014 4.5167 11.9475 3.0926], then getting the new delivery scheme A'=[4 11 14 8 0 5 10 0 6 3 9 2 0 13 7 12 1]. A' represent the distribution scheme: vehicle 1:0—4—11—14—8—0(0 represent the starting point and ending point); vehicle 2:0—5—10—0; vehicle 3:0—6—3—9—2—0; vehicle 4:0—13—7—12—1—0. The advantage of this notation is that every customer point can get vehicle service and also limit each point can only be accomplished by one vehicle.

4.2 Algorithm implementation steps

Step1: Initialization, setting speed constant c_1 and c_2, inertia weight ω, the maximum iterations $T_{\max}$, and the current evolution algebra $t=1$; generating m particles in the defined space randomly $x_1, x_2, \cdots, x_m$, to form the initial population $X(t)$, and generating each particle initial displacement change $v_1, v_2, \cdots, v_s$, to construct the displacement change matrix $V(t)$;

Step2: Evaluating the population $X(t)$, calculating the adaptive value of each particle in every dimension, choosing Q particle with high adaptive value to generate the initial neighborhood.

Step3: Particles within the neighborhood update status according to formula (10) and (11). Comparing the particle's fitness with its optimal value P_b, if the current value better than P_b, then assign the current value for P_b and set the location of P_b as the current position of the n dimensional space. Comparing the particle fitness with the optimal value P_g within the neighborhood, if the current value better than P_g, then assign the current particle matrix subscript and fitness for P_g, and generate the global optimal particle G_b.

Step4: Taking the local optimal particle crash with the global optimal particle according to formula (13), (14) and (15), changing speed and position. We take the OX crossover operation.

Step5: Updating the local optimum particle, the neighborhood and the global optimal particle to generate new population $X(t+1)$, repeating step (3) and (4).

Step6: Estimating the rate of adaptive value change of G_b, if less than a certain threshold criterion, then take Cauchy mutation for the global optimal particle G_b.

Step7: Checking the end condition, if satisfied, end the process, otherwise turn to step (2). Finding out the delivery scheme according to the particle expression.

V. THE EXPERIMENTAL SIMULATION RESULTS

Adopted in this paper, the improved particle swarm algorithm for the classic example of VRP has carried on the simulation research[11].

Example 1: Delivery from 1 distribution center to 8 demand points, the demand of each point is $q_i(i=1,2,3,\cdots,8)$ (the unit is ton). The distribution center has two vehicle for delivery, and each vehicle have a capacity of 8 tons. The distance between the distribution center and each demand point is shown in table 5-1 (where 0 represent distribution center).

Table5-1 The distance between the demand points

d_{ij}	0	1	2	3	4	5	6	7	8
0	0	4	6	7.5	9	20	10	16	8
1	4	0	6.5	4	10	5	7.5	11	10
2	6	6.5	0	7.5	10	10	7.5	7.5	7.5
3	7.5	4	7.5	0	10	5	9	9	15
4	9	10	10	10	0	10	7	9	7.5
5	20	5	10	5	10	0	7	9	7.5
6	10	7.5	7.5	9	7.5	7	0	7	10

7	16	11	7.5	9	7.5	9	7	0	10
8	8	10	7.5	15	10	7.5	10	10	0

Table5-2 Demand of each point

No.	1	2	3	4	5	6	7	8
Demand	1	2	1	2	1	4	2	2

Setting the population size of 500, maximum number of iteration 200, acceleration constant $c_1 = c_2 = 1.49618$, inertia weight random variation between $[0,1]$. The calculated optimal path length is 66.5 and the corresponding distribution path are:

0—2—7—4—8—0; 0—1—3—5—6—0.

Example 2: Delivery from 1 distribution center to 8 demand points, the vehicle capacity is 8 unit. The data of each point shown in table 5-3.

Table 5-3 Coordinate and demand of each point

	0	1	2	3	4	5	6	7	8
coordinate	(31,9)	(76,38)	(77,16)	(90,82)	(60,74)	(76,86)	(11,31)	(25,90)	(10,60)
Demand	0	2.46	0.41	2.16	2.27	1.83	3.76	2.54	2.39

Setting parameters the same as example 1, the result of optimal path length is 416.83 and the corresponding distribution path is:

0—8—7—4—0; 0—5—3—1—2—0; 0—6—0.

Example 3: Delivery from 1 distribution center to 20 demand points, the coordinate and demand are shown in table 5-4. The vehicle load is 8 tons.

Table 5-4 Coordinate and demand of each point

points	0	1	2	3	4	5	6
coordinates	(52,4)	(15,49)	(0,61)	(51,15)	(25,71)	(38,62)	(35,45)
demand	0	1.64	1.31	0.43	3.38	1.13	3.77
points	7	8	9	10	11	12	13
coordinates	(100,4)	(10,52)	(26,79)	(87,7)	(24,89)	(19,25)	(20,99)
demand	3.84	0.39	0.24	1.03	2.35	2.60	1.00
points	14	15	16	17	18	19	20
coordinates	(73,91)	(100,95)	(7,73)	(69,86)	(24,3)	(66,14)	(9,30)
demand	0.65	0.85	2.56	1.27	2.69	3.26	2.97

Setting parameters the same as example 1, the calculated optimal path length is 924.80 and the corresponding distribution path is:

0—9—11—13—17—15—14—10—0; 0—20—12—0; 0—4—16—2—8—0;
0—1—5—6—0; 0—7—19—3—0; 0—18—0.

For comparison purposes, taking the improved particle swarm algorithm(IPSO), improved algorithm of literature[10](GA), basic particle swarm algorithm(PSO) to operate above three example ten times respectively. It is concluded that the optimal solution in the following table 5-5. Parameters selection is the same as IPSO.

Table 5-5 Optimal result

	Example 1	Example 2	Example 3
IPSO	66.5	416.83	924.80
GA	67.5	476.29	964.48
PSO	79.5	492.57	996.11

VI. CONCLUSION

From the experiment contrast, the IPSO for classic example of VRP has certain improvement, especially for example 2 and 3, so the algorithm is proved good with obvious effect. By introducing topology structure, it effectively enhance the capacity of global information transmission between particles. At the same time, defining different way of speed and position updating, it's up to the different kind of particle lepton and hadron. Iteration to a certain time, adopting particle decay to avoid particle trapped in local optimal solution. In solving the problem of vehicle routing optimization, the algorithm convergence speed and stability, has a certain practicality.

VII. ACKNOWLEDGEMENTS

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