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Effect of viscous dissipation on falkner-skin boundary layer flow past a Wedge through a porous medium with slips boundary condition

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Abstract: - The steady, two dimensional, Flakner-Skan boundary layer flow over a stationary Wedge with momentum and thermal slip boundary conditions and the temperature dependent thermal conductivity in the presence of porous medium and viscous dissipation. Using the similarity transformations, the governing equations have been transformed into a system of ordinary differential equations. These differential equations are highly nonlinear which cannot be solved analytically. Therefore, bvp4c MATLAB solver has been used for solving it. Numerical results are obtained for the skin-friction coefficient and the local Nusselt number as well as the velocity and temperature profiles for different values of the governing parameters, namely, Falkner-Skin parameter, thermal conductivity parameter, velocity slip parameter, thermal slip parameter and Eckert number.

Keywords: *Falkner-Skan, momentum slip, thermal slip, Wedge, Porous Medium, temperature dependent thermal conductivity, Viscous Dissipation.*

I. INTRODUCTION

Micropolar fluids are fluids of microstructure. They represent fluids consisting of rigid, randomly oriented, or spherical particles suspended in a viscous medium, where deformation of fluids particles is ignored. The dynamics of micropolar fluids, originated from the theory of Eringen [1-3], has been a popular area of research due to its application in a number of processes that occur in industry. Such applications include polymeric fluids, real fluids with suspensions, liquid crystal, animal blood, and exotic lubricants. Extensive reviews of theory of micropolar fluids and its applications can be found in review articles by Ariman et al. [4, 5] and recent books by Łukaszewicz [6] and Eringen [7].

According to most of the previous studies, the MHD flow has received the attention of many researchers due to its engineering applications. In metallurgy, for example, some processes involve the cooling of many continuous strips by drawing them through an electrically conducting fluid subject to a magnetic field (Kandasamy and Muhaimin [8]).This allows the rate of cooling to be controlled and final product with the desired characteristics to be obtained. Another important application of hydromagnetic flow in metallurgy is in the purification of molten metal’s from nonmetallic inclusions through the application of a magnetic field. Research has also been carried out by previous researchers on the flow and heat transfer effects of electrically conducting fluids such as liquid metals, water mixed with a little acid and other equivalent substance in the presence of a magnetic field. The studies have involved different geometries and different boundary conditions. Herdricha et al. [9] studied MHD flow control for plasma technology applications. They identified potential applications for magnetically controlled plasmas in the fields of space technology as well as in plasma technology. Seddeek et al. [10] investigated the similarity solution in MHD flow and heat transfer over a wedge taking into account variable viscosity and thermal conductivities. The magnetohydrodynamic (MHD) forced convection boundary layer flow of nanofluid over a horizontal stretching plate was investigated by Nourazar et al. [11] using homotopy perturbation method (HPM).

Unsteady free convection flows of dissipative fluids past an infinite plate have received a little attention because of non-linearity of the governing equations. Bhaskar Reddy and Bathaiah [12] studied the magnetohydrodynamic flow of a viscous incompressible fluid between a parallel flat wall and a long wavy wall. Neeraja and Bhaskar Reddy [13] investigated the MHD unsteady free convection flow past a vertical porous plate with viscous dissipation. Recently, El-Aziz [14] studied the mixed convection flow of a micropolar fluid from an unsteady stretching surface with viscous dissipation. Gangadhar [15] conclude that the local skin friction coefficient increases and local Nusselt number coefficient decreases in the presence of viscous dissipation. Aydin and Kaya [16] studied MHD mixed convection of a viscous dissipating fluid about a permeable vertical flat plate and found that the value of Richardson number determines the effect of the magnetic parameter on the momentum and heat transfer.

The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances (Yoshimura and Prudhomme [17]). It is a well-known fact that, a viscous fluid normally sticks to the boundary. But, there are many fluids, e.g. particulate fluids, rarefied gas etc., where there may be a slip between the fluid and the boundary (Shidlovskiy [18]). Beavers and Joseph [19] proposed a slip flow condition at the boundary. Andersson [20] considered the slip flow of a Newtonian fluid past a linearly stretching sheet. Ariel [21] investigated the laminar flow of an elastic-viscous fluid impinging normally upon a wall with partial slip of the fluid at the wall. Wang [22] undertook the study of the flow of a Newtonian fluid past a stretching sheet with partial slip and purportedly gave an exact solution. He reported that the partial slip between the fluid and the moving surface may occur in particulate fluid situations such as emulsions, suspensions, foams and polymer solutions. Fang et al [23] investigated the magnetohydrodynamic (MHD) flow under slip condition over a permeable stretching surface. Fang and Aziz [24] conclude that the combined effects of the two slips and mass transfer parameters greatly influence the fluid flow and shear stresses on the wall and in the fluid. Nandeppanavar et al. [25] analyze the second order slip flow and heat transfer over a stretching sheet. Sajid et al. [26] analyzed the stretching flow with general slip condition. Sahoo and Poncet [27] studied the Non-Newtonian boundary layer flow and heat transfer over an exponentially stretching sheet with partial slip boundary condition. Noghrehabadi *et al.* [28] analyzed the effect of partial slip on the flow and heat transfer of nanofluids past a stretching sheet. Zheng *et al.* [29] investigated the magnetohydrodynamic (MHD) flow and heat transfer over a stretching sheet with velocity slip and temperature jump. Sharma et al. [30] considered the velocity and temperature slip on the boundary. Sharma and Ishak [31] considered the Second order velocity slip flow model instead of no-slip at the boundary.

The present study investigates the steady, two dimensional, Flakner-Skan boundary layer flow over a stationary Wedge with momentum and thermal slip boundary conditions and the temperature dependent thermal conductivity in the presence of porous medium and viscous dissipation. Using the similarity transformations, the governing equations have been transformed into a set of ordinary differential equations, which are nonlinear and cannot be solved analytically, therefore, bvp4c MATLAB solver has been used for solving it. The results for velocity, microrotation and temperature functions are carried out for the wide range of important parameters namely; material parameter, magnetic parameter, Eckert number and first order slip velocity parameter and second order velocity slip parameter. The skin friction, the couple wall stress and the rate of heat transfer have also been computed.

II. MATHEMATICAL FORMULATION

Consider a two dimensional steady Falkner-Skan boundary layer laminar flow past a static wedge in the moving free stream. The physical model is depicted in Figure 1. We consider the effects of the momentum and thermal slip boundary conditions and temperature dependent thermal conductivity. It is further assumed that the velocity of the free stream is of the form $\bar{u}_e = U_\infty \left(\frac{\bar{x}}{L}\right)^m$ (Yacob et al. [32]). A Cartesian coordinate system $\left(\bar{x}, \bar{y}\right)$, where $\bar{x}$ and $\bar{y}$ are the coordinates along the surface of the wedge and normal to it. Under the above assumptions, the partial differential equations and the corresponding boundary conditions govern the problem are given by (White [33]):

Continuity equation

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2.1}$$

Linear momentum equation

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \bar{u}_e \frac{d\bar{u}_e}{d\bar{x}} + \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\upsilon}{k}\left(\bar{u} - \bar{u}_e\right) \tag{2.2}$$

Energy equation

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{1}{\rho c_p}\frac{\partial T}{\partial \bar{y}}\left[k\left(T\right)\frac{\partial T}{\partial \bar{y}}\right] + \frac{\mu}{\rho c_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \tag{2.3}$$

The boundary conditions for the velocity, Angular Velocity and temperature fields are

$$\bar{u} = N\left(\bar{x}\right)\upsilon\frac{\partial \bar{u}}{\partial \bar{y}}, \bar{v} = 0, T = T_w + D_1\left(\bar{x}\right)\frac{\partial T}{\partial \bar{y}} \quad \text{at} \quad \bar{y} = 0$$

$$\bar{u} = \bar{u}_e\left(\bar{x}\right), T = T_\infty \quad \text{as} \quad \bar{y} \to \infty \tag{2.4}$$

Where $\bar{u}$ and $\bar{v}$ are the velocity components in the $\bar{x}$ - and $\bar{y}$ - directions, respectively, *T* is the fluid temperature inside boundary layer, is the fluid density, is the specific heat, is the kinematic viscosity, is the variable slip factor with dimension (velocity)$^{-1}$, $D_1\left(\bar{x}\right)$ is the variable thermal slip factor with dimension length, T_∞ is the free stream temperature, T_w is the wall temperature and k is the thermal conductivity, c_p is the heat capacity pressure.

The following relations for *k* are introduced (Aziz et al. [34]),

$$k\left(T\right) = k_\infty\left[1 + c\left(T - T_\infty\right)\right] \tag{2.5}$$

where *c* and k_∞ are constants.

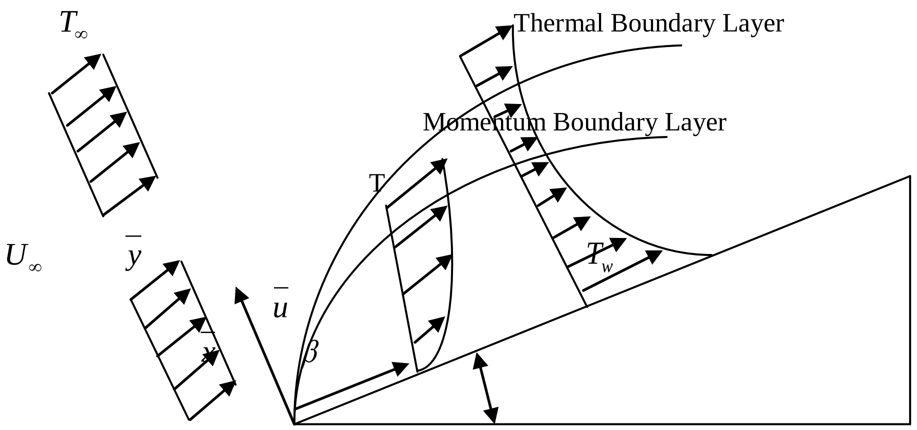

Figure 1. Physical model and coordinate system

For lubricating fluids, heat is generated by internal friction and the increase in the temperature affects the viscosity and thermal conductivity of the fluid and hence the fluid properties should no longer be assumed to be constant (Prasad et al. [35]). Therefore, in order to predict the flow characteristics in an accurate and reliable manner it is necessary to consider the variation of thermal conductivity with the temperature.

We now introduce the following dimensionless variables to reduce the number of independent variables and the number of equations,

$$x = \frac{\bar{x}}{L}, y = \frac{\bar{y}\sqrt{\text{Re}}}{L}, u = \frac{\bar{u}}{U_\infty}, v = \frac{\bar{v}\sqrt{\text{Re}}}{U_\infty}, u_e = \frac{\bar{u}_e}{U_\infty}, \theta = \frac{T - T_\infty}{T_f - T_\infty} \tag{2.6}$$

Here Re is the Reynolds number, *L* is the characteristics length and U_∞ is some reference velocity.

The dimensionless forms of the governing equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} - \gamma\left(u - u_e\right) \tag{2.8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \text{Pr}^{-1}\frac{\partial}{\partial y}\left[\left(1 + A\theta\right)\frac{\partial \theta}{\partial y}\right] + Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.9}$$

here $A = c\left(T_w - T_\infty\right)$ is the parameter of thermal conductivity, α is the thermal diffusivity, $\gamma = \frac{\upsilon L}{U_\infty k}$ is the permeability parameter, $\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, and $Ec = \frac{\mu U_\infty}{\rho c_p L(T_w - T_\infty)}$ is the Eckert number.

The boundary conditions become,

$$u = N(x)\upsilon \frac{\partial u}{\partial y}\frac{\sqrt{\mathrm{Re}}}{L}, v = 0, \theta = 1 + \frac{D_1(x)\sqrt{\mathrm{Re}}}{L}\frac{\partial \theta}{\partial y} \quad at \quad y = 0$$

$$u = u_e\left(x\right), \theta = 0 \qquad \text{as} \quad y \to \infty \tag{2.10}$$

We introduce stream function ψ which is defined as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ to reduce the number of equations and number of dependent variables. Then (2.7)- (2.9) with the boundary conditions in (2.10) transform as follows:

$$\psi_y \psi_{xy} - \psi_x \psi_{yy} = u_e \frac{du_e}{dx} + \psi_{yyy} - \gamma\left(\psi_y - u_e\right) \tag{2.11}$$

$$\psi_y \theta_x - \psi_x \theta_y = \Pr^{-1}\frac{\partial}{\partial y}\left[\left(1 + A\theta\right)\frac{\partial \theta}{\partial y}\right] + Ec\psi_{yy} \tag{2.12}$$

with the boundary conditions,

$$\psi_y = N\upsilon \frac{\sqrt{\mathrm{Re}}}{L}\psi_{yy}, \psi_x = 0, \theta = 1 + \frac{D_1\sqrt{\mathrm{Re}}}{L}\theta_y \quad at \quad y = 0$$

$$\psi_y = u_e\left(x\right), \theta = 0 \qquad \text{as} \quad y \to \infty \tag{2.13}$$

A closed-form solution of the set of partial differential (2.11) - (2.13) may not exist. So we transform this system to an ordinary system using scaling group transformations (Aziz et al. [34]; Khan et al. [36]; Uddin et al. [37]; Mutlag et al. [38]),

$$\Gamma : x^* = e^{\varepsilon c_1}x, y^* = e^{\varepsilon c_2}y, \psi^* = e^{\varepsilon c_3}\psi, \theta^* = e^{\varepsilon c_4}\theta \tag{2.14}$$

Here ε is the parameter of the group Γ and c_i 's, (i = 1,2,3,4) are arbitrary real numbers. The system of (2.11)-(2.13) will remain invariant under the group transformations in (2.14) if the following relationships hold,

$$c_2 = \frac{1}{2}\left(1 - m\right)c_1, c_3 = \frac{1}{2}\left(1 + m\right)c_1, c_4 = 0 \tag{2.15}$$

In terms of differential,

$$dx = c_1 x, dy = \frac{1}{2}\left(1 - m\right)c_1 y, d\psi = \frac{1}{2}\left(1 + m\right)c_1 \psi, d\theta = 0 \tag{2.16}$$

Solving (16) we obtain,

$$\eta = x^{\frac{m-1}{2}}y, \psi = x^{\frac{m+1}{2}}f\left(\eta\right), \theta = \theta\left(\eta\right) \tag{2.17}$$

Here $\eta, f\left(\eta\right), \theta\left(\eta\right)$ are similarity independent and dependent variables.

Substituting (2.17) into (2.11)- (2.13), we get,

$$f''' + ff'' + \frac{2m}{m+1}\left(1 - f'^2\right) - \gamma\frac{2}{m+1}\left(f' - 1\right) = 0 \tag{2.18}$$

$$\left(1 + A\theta\right)\theta'' + \frac{m+1}{2}\Pr f\theta' + A\theta'^2 + \Pr Ecf''^2 = 0 \tag{2.19}$$

The boundary conditions become,

$$f\left(0\right) = 0, f'\left(0\right) = Sf''\left(0\right), \theta\left(0\right) = 1 + b\theta'\left(0\right)$$

$$f\left(\infty\right) = 1, \theta\left(\infty\right) = 0 \tag{2.20}$$

It is worth noting that if at this stage of our analysis we put $Ec=\gamma=0$, then our problem reduces to Mutlag et al. [38]. This supports the validity of our group analysis.

For further investigation, we use the following minor modification:

$$\eta=\sqrt{\frac{m+1}{2}}x^{\frac{m-1}{2}}y,\psi=\sqrt{\frac{2}{m+1}}x^{\frac{m+1}{2}}f\left(\eta\right),\theta=\theta\left(\eta\right) \tag{2.21}$$

Substituting (21) into (18) - (19), we get,

$$f'''+ff''+\frac{2m}{m+1}\left(1-f'^{2}\right)-\frac{2}{m+1}\left(f'-1\right)=0 \tag{2.22}$$

$$\left(1+A\theta\right)\theta''+\Pr f\theta'+A\theta'^{2}+\Pr Ecf''^{2}=0 \tag{2.23}$$

The boundary conditions become,

$$\begin{aligned}&f\left(0\right)=0,\ f'\left(0\right)=Sf''\left(0\right),\theta\left(0\right)=1+b\theta'\left(0\right)\\&f\left(\infty\right)=1,\theta\left(\infty\right)=0\end{aligned} \tag{2.24}$$

where prime is the derivative with respect to $\eta, S=\frac{\upsilon\sqrt{\mathrm{Re}}}{L}N(x)x^{\frac{(m-1)}{2}}$ is the velocity slip parameter and $b=\frac{D_1\sqrt{\mathrm{Re}}}{L}\sqrt{\frac{m+1}{2}}x^{\frac{(m-1)}{2}}$ is the thermal slip parameter. Note that for true similarity solutions we have $N(x)\alpha x^{\frac{1-m}{2}}, D_1(x)\alpha x^{\frac{1-m}{2}}$.

Expressions for the quantities of physical interests, the skin friction factor and the rate of heat transfer can be found from the following definitions:

$$C_{fx}=\frac{\mu}{\rho\bar{u}_e}\left(\frac{\partial\bar{u}}{\partial\bar{y}}\right)_{\bar{y}=0},Nu_{\bar{x}}=\frac{-\bar{x}}{T_w-T_\infty}\left(\frac{\partial T}{\partial\bar{y}}\right)_{\bar{y}=0} \tag{2.25}$$

Using (2.6) and (2.21) into (2.25) we get,

$$\left(\frac{2\mathrm{Re}_{\bar{x}}}{m+1}\right)^{\frac{1}{2}}C_{f\bar{x}}=f''(0),\left(\frac{\mathrm{Re}_{\bar{x}}(m+1)}{2}\right)^{\frac{-1}{2}}Nu_{\bar{x}}=-\theta'(0) \tag{2.26}$$

where $\mathrm{Re}_{\bar{x}}=\frac{\bar{u}_e\bar{x}}{\upsilon}$ is the local Reynolds number.

III. SOLUTION OF THE PROBLEM

The set of equations (2.12) to (2.24) were reduced to a system of first-order differential equations and solved using a MATLAB boundary value problem solver called **bvp4c**. This program solves boundary value problems for ordinary differential equations of the form $y'=f\left(x,y,p\right),a\le x\le b$, by implementing a collocation method subject to general nonlinear, two-point boundary conditions $g\left(y(a),y(b),p\right)$. Here ***p*** is a vector of unknown parameters. Boundary value problems (BVPs) arise in most diverse forms. Just about any BVP can be formulated for solution with **bvp4c**. The first step is to write the *ODEs* as a system of first order ordinary differential equations. The details of the solution method are presented in Shampine and Kierzenka[39].

IV. RESULTS AND DISCUSSION

The governing equations (2.13) - (2.14) subject to the boundary conditions (2.15) are integrated as described in section 3. In order to get a clear insight of the physical problem, the velocity and temperature have been discussed by assigning numerical values to the parameters encountered in the problem.

Figure 1 shows the effect of the power law index parameter (*m*) on the non-dimensional velocity profiles. We observe that the velocity increases with the influence of *m*. These findings are similar to the results reported by Mutlag et al. [12]. The results are quite different in the case of stretching sheet. Figure 2 illustrate the effect of permeability parameter (γ) on the velocity. We observed that the velocity increases with increasing γ. The variation of the velocity profiles with the velocity slip parameter (*S*) is shown in Figure 3. It is observed that the velocity increases with an increasing *S*.

Figure 4 illustrate the effect of power law index parameter on the temperature. We observed that the temperature decreases with increasing *m*. Moreover, the boundary layer thickness decreases, these results are similar to the findings by Mutlag et al. [38]. Figures 5, 6, 7 & 8 illustrate the effects of the thermal conductivity parameter (*A*), thermal slip parameter (*b*), Prandtl number (*Pr*) and Eckert number on the temperature. It is observed that temperature of the fluid reduces with a rising the parameters *A*, *b*, *Pr* and *Ec*.

Figure 9 shows the effects of *S* and *m* on skin friction. From Figure 9 it is seen that the skin friction decreases with an increase *S* and increases with an increase *m*. The variation of S and γ on skin friction is shown in Figure.10. It is observed that the skin friction increases with an increase γ. The effect of *A* and *m* on local Nusselt number is shown in fig.11. It is found that the local Nusselt number enhances with an increase in the parameters *A* and *m*. The variations of *b* and *Ec* on local Nusselt number are shown in fig.12. It is observed that the local Nusselt number decrease with an increasing the parameter *b* whereas local Nusselt number increases with an raising *Ec*. Tables.1, 2 & 3 shows that the present results perfect agreement to the previously published data.

V. CONCLUSIONS

In the present prater, the steady, two dimensional, Flakner-Skan boundary layer flow over a stationary Wedge with momentum and thermal slip boundary conditions and the temperature dependent thermal conductivity by taking porous medium and viscous dissipation into account. The governing equations are approximated to a system of non-linear ordinary differential equations by similarity transformation. Numerical calculations are carried out for various values of the dimensionless parameters of the problem. It has been found that

1. The velocity increases with an increase in the permeability parameter and velocity slip parameter.
2. The thermal slip parameter and Eckert number reduces the temperature.
3. The skin friction enhances the permeability parameter or power law index parameter and decreases the velocity slip parameter.
4. The local Nusselt number enhances the Eckert number or power law index parameter and decreases the thermal slip parameter.

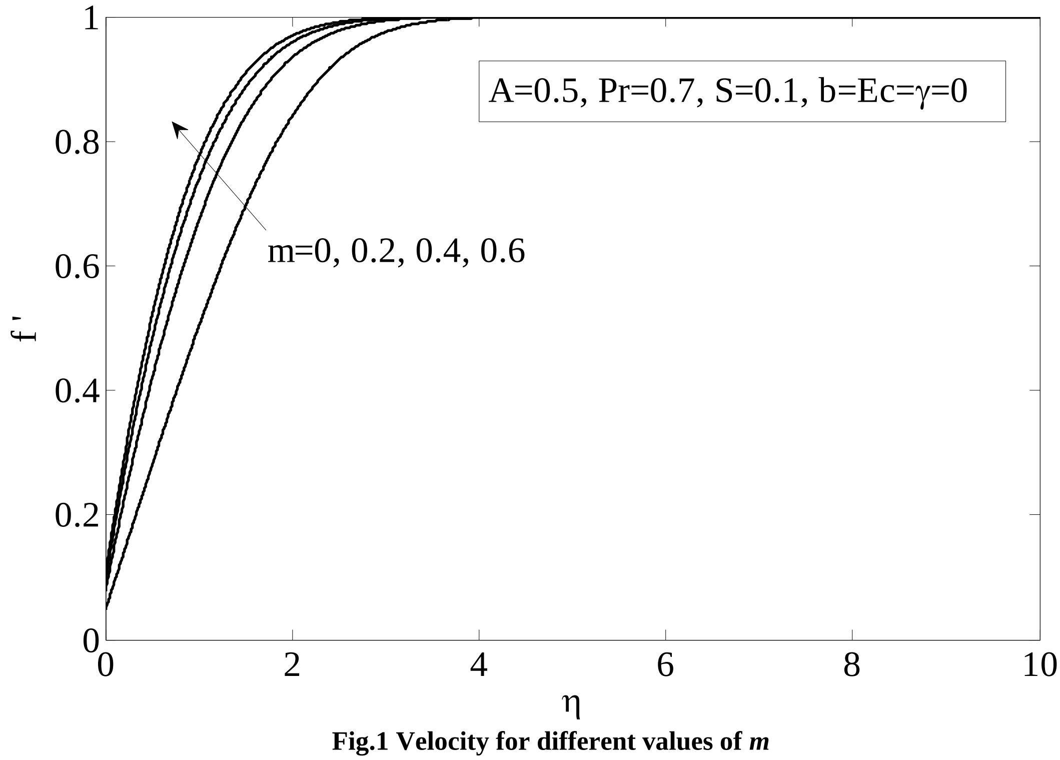

Fig.1 Velocity for different values of *m*

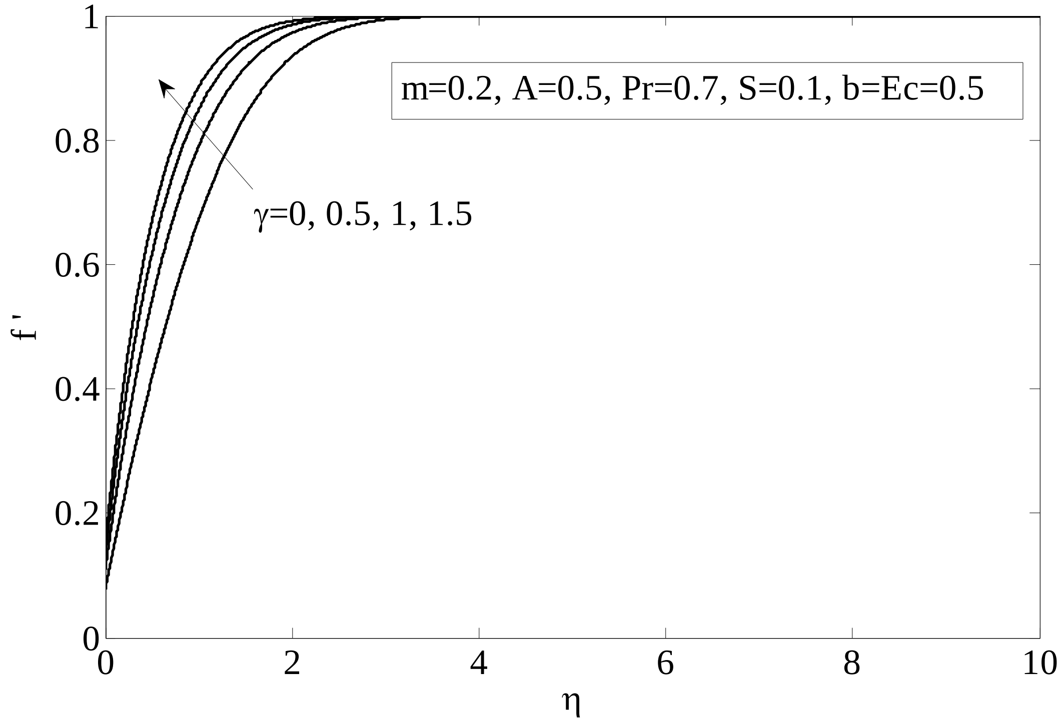

Fig.2 Velocity for different values of *γ*

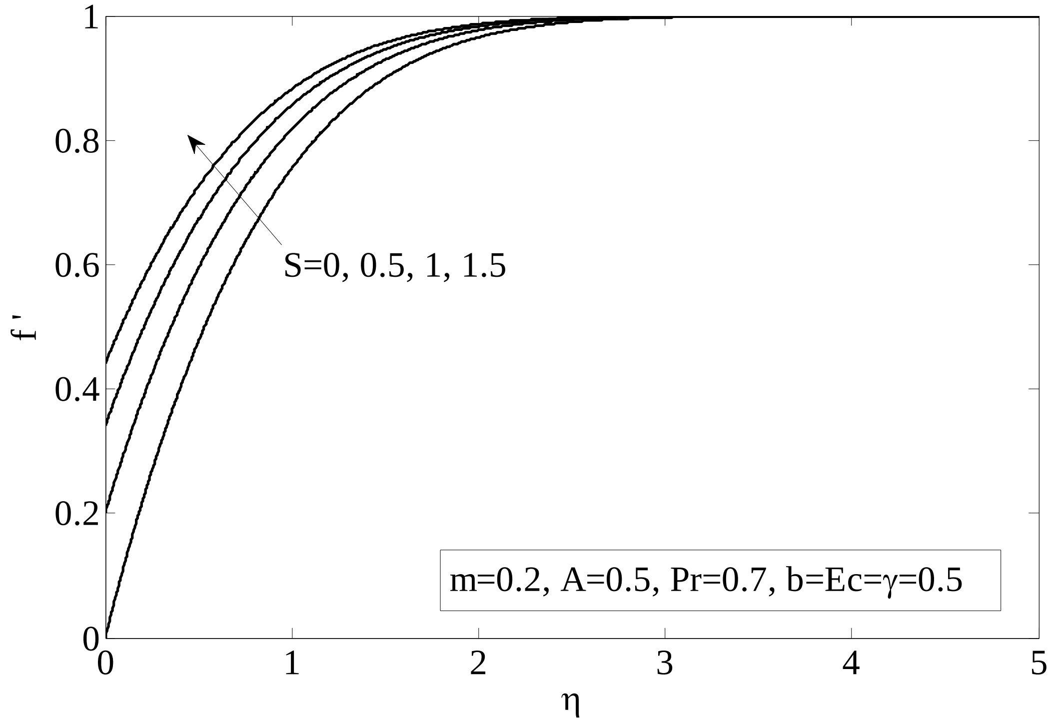

Fig.3 Velocity for different values of *S*

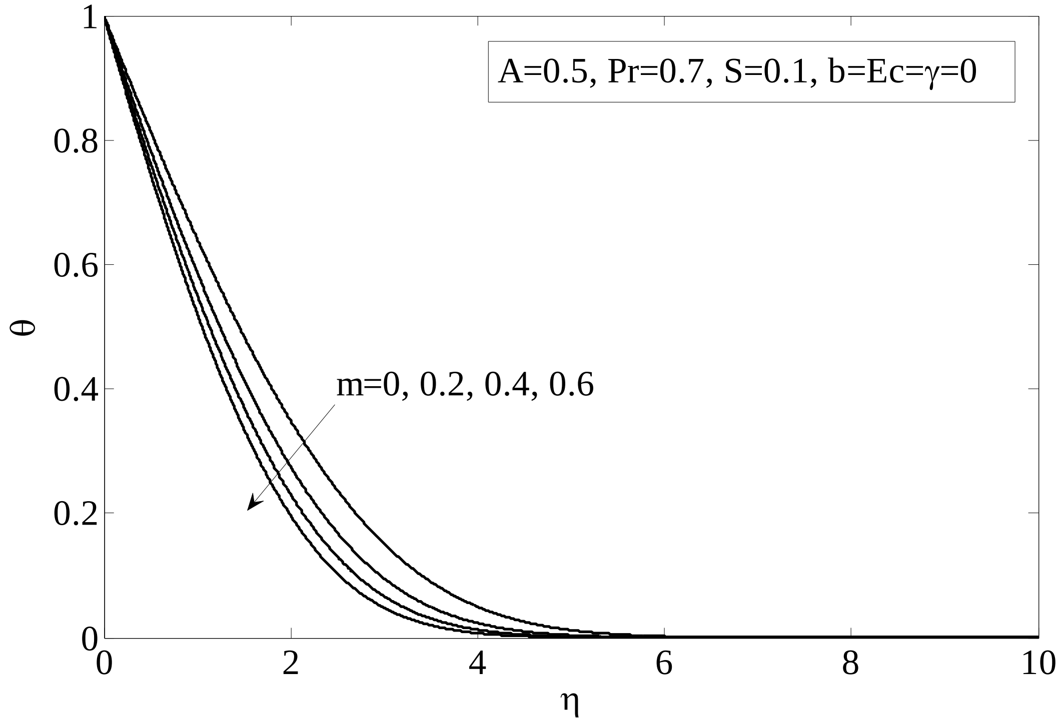

Fig.4 Temperature for different values of *m*

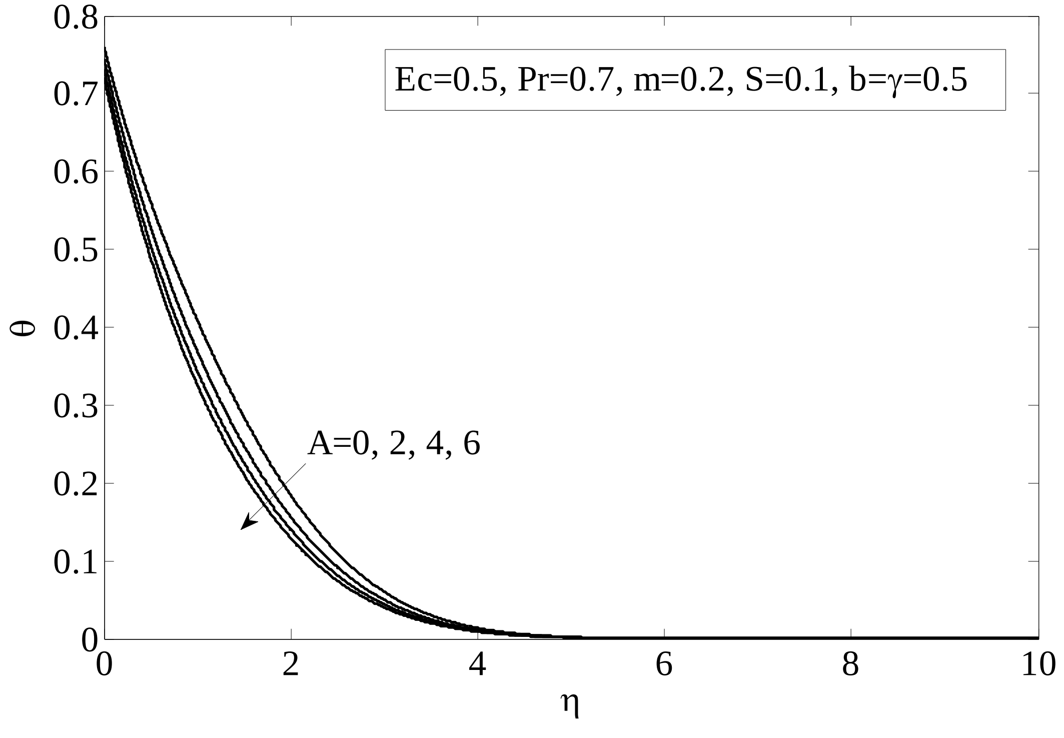

Fig.5 Temperature for different values of A

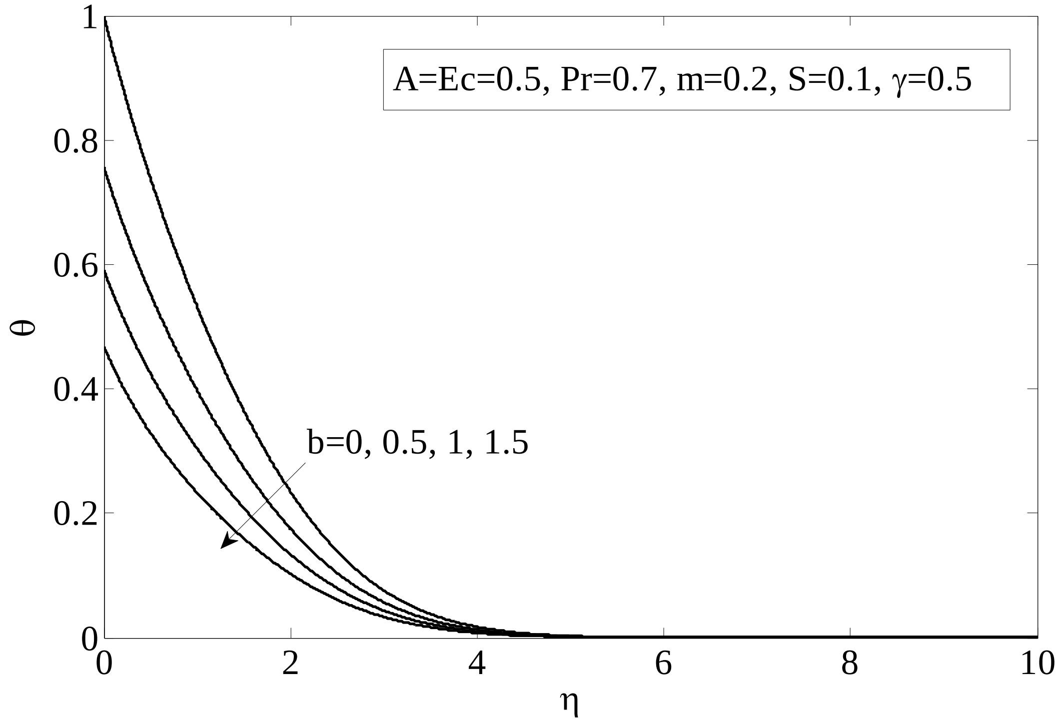

Fig.6 Temperature for different values of *b*

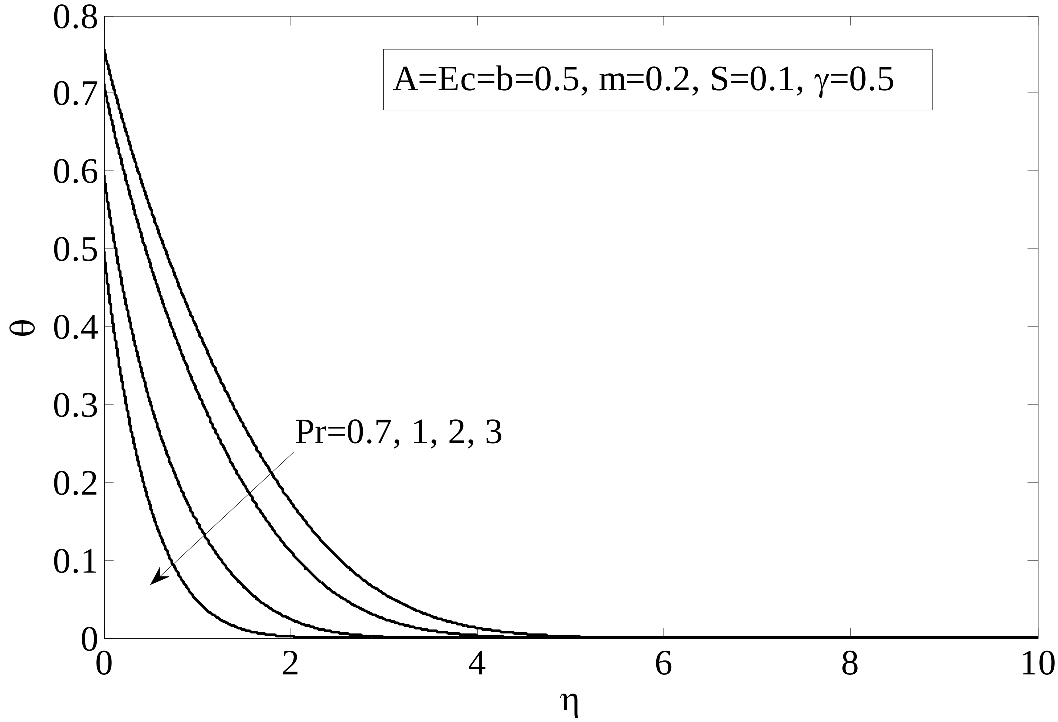

Fig.7 Temperature for different values of *Pr*

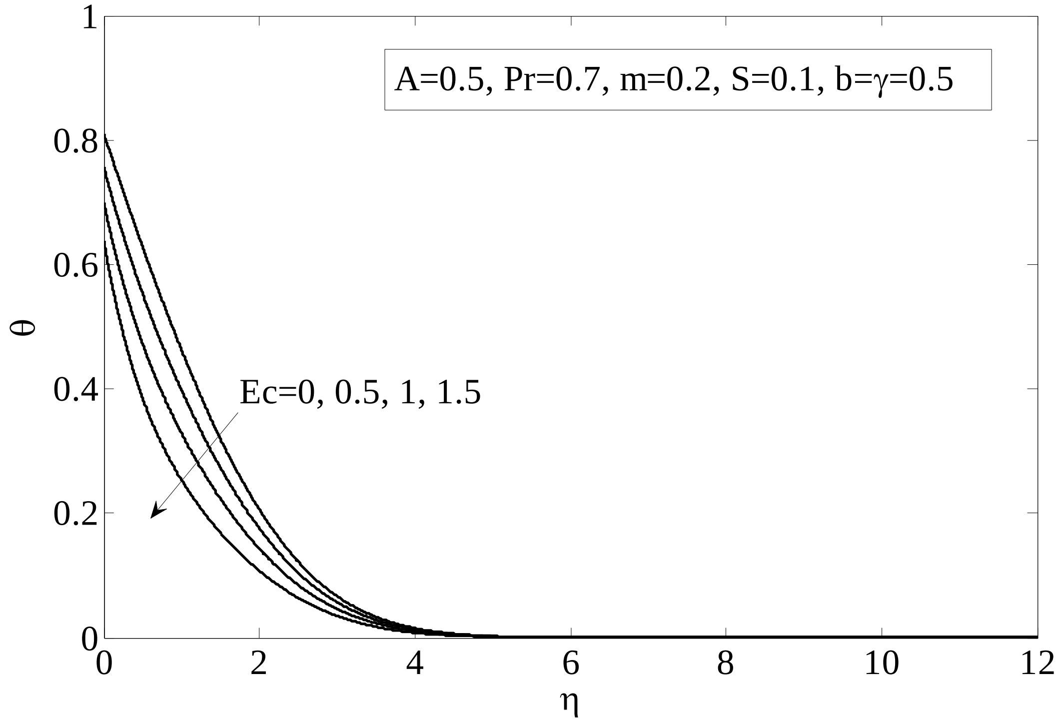

Fig.8 Temperature for different values of *Ec*

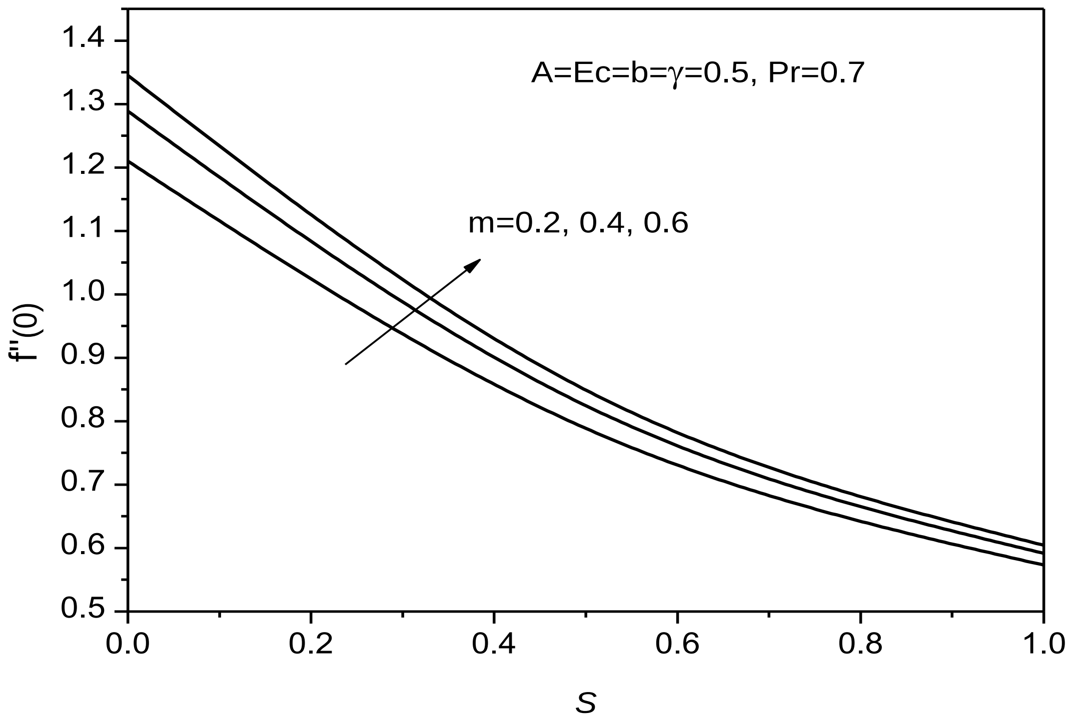

Fig.9 Local Skin friction for different values of *S* and *m*

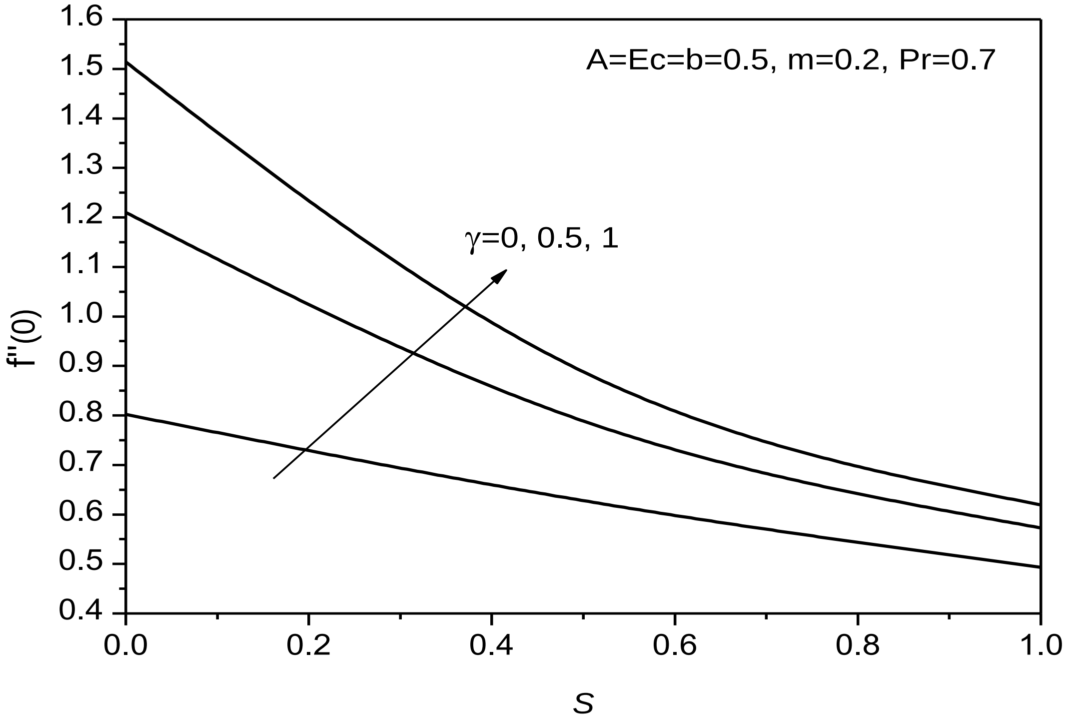

Fig.10 Local Skin friction for different values of *S* and *γ*

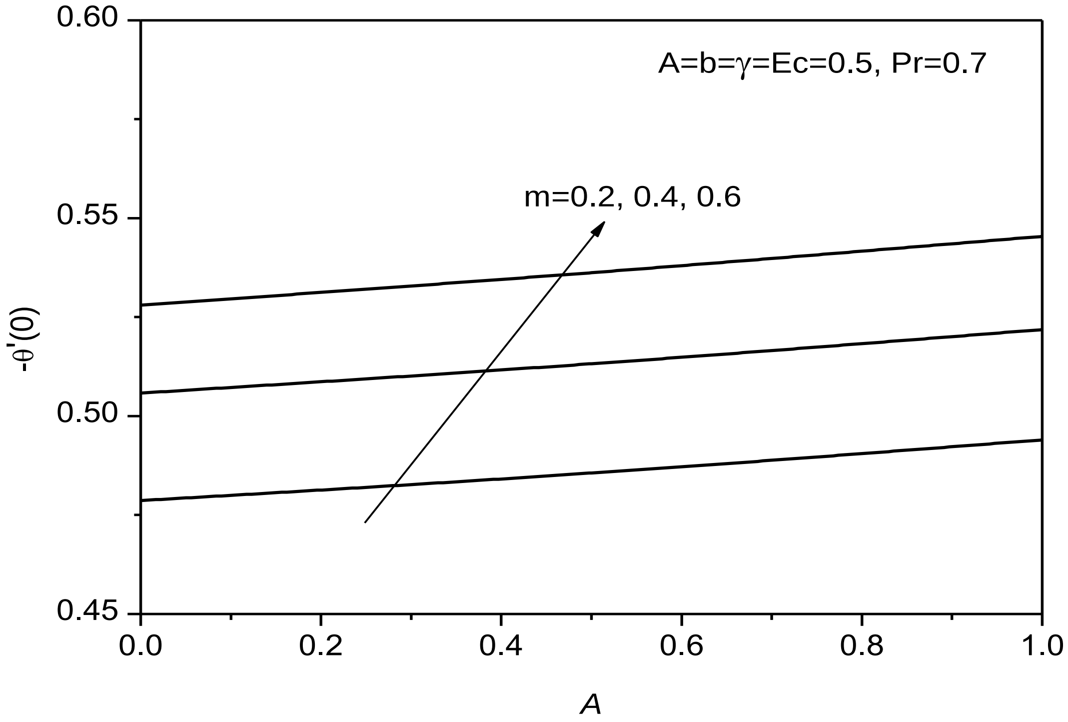

Fig.11 Local Nusselt number for different values of *A* and *m*

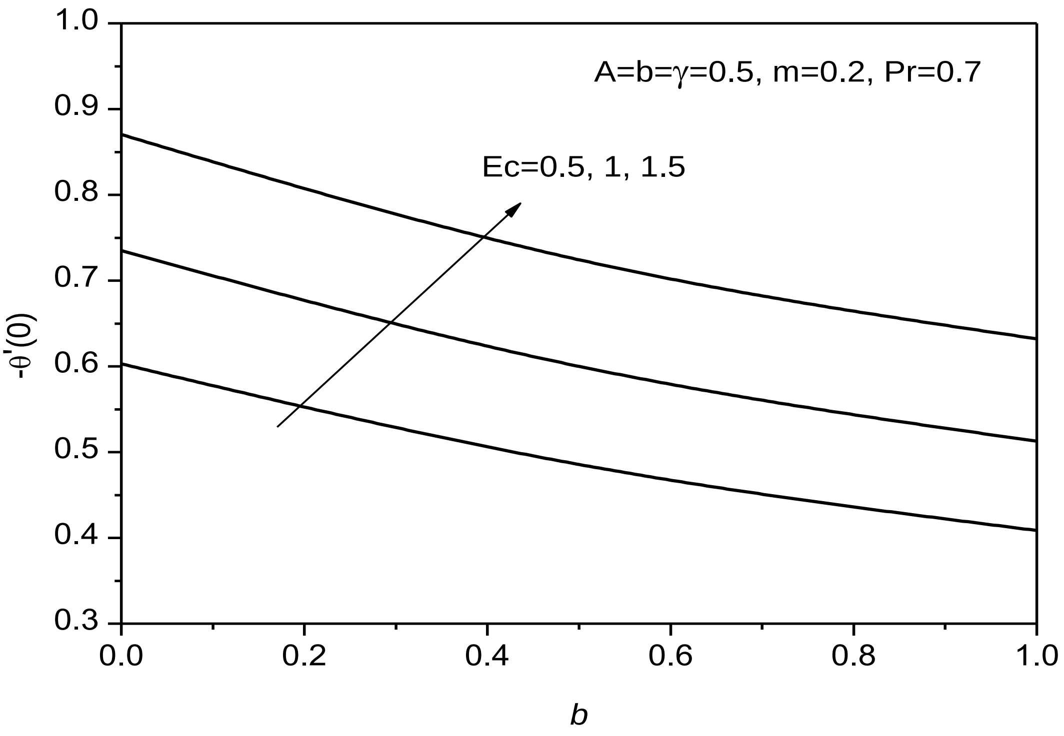

Fig.12 Local Nusselt number for different values of *b* and *Ec*

Table.1 Comparison for the values of $f''(0)$ for *A=S=b=Ec=γ=0, Pr=1* and for various values of *m*.

m	Present results	Mutlag et al. [38]	Yacob [32]	Yih [40]	Watanabe [41]
0	0.469600	0.4696452	0.4696	0.649600	0.46960
1/11	0.654994	0.6550006	0.6550	0.654979	0.65498
0.2	0.802126	0.8021272	0.8021	0.802125	0.80213
1/3	0.927680	0.9276804	0.9277	0.927653	0.92765
0.5	1.038903	1.0389035	1.0389	……….	………
1	1.232588	1.2325876	1.2326	1.232588	………

Table.2 Comparison for the values of $-\theta'(0)$ for *A=S=b=Ec=γ=0* and for various values of $\beta = \dfrac{2m}{m+1}$ and *Pr.*

Pr	$-\theta'(0)$								
	$\beta = 0$			$\beta = 0.3$			$\beta = 2$		
	Present results	Mutlag et al. [38]	White [33]	Present results	Mutlag et al. [38]	White [33]	Present results	Mutlag et al. [38]	White [33]
0.1	0.198031	0.198033	0.1980	0.209075	0.20907	0.209	0.22600	0.22600	0.2260
0.3	0.303718	0.303717	0.3037	0.327829	6	0	2	2	0.3668
0.6	0.391675	0.391675	0.3916	0.428924	0.32782	0.327	0.36680	0.36680	0.4913
0.72	0.418091	0.418091	0.4178	0.459551	9	8	9	8	0.5292
1.0	0.469600	0.469600	0.4690	0.519518	0.42892	0.428	0.49130	0.49130	0.6052
2.0	0.597234	0.597233	0.5972	0.669045	4	9	3	2	0.7959
3.0	0.685961	0.685961	0.6859	0.773436	0.45955	0.459	0.52960	0.52960	0.9303
6.0	0.867278	0.867277	0.8672	0.987268	1	2	8	7	1.2069
10.0	1.029747	1.029747	1.0297	1.179130	0.51951	0.519	0.60519	0.60519	1.4557

30.0 60.0 100.0 400.0 1000.0 4000.0 10000. 0	1.487319 1.874595 2.222906 3.529230 4.790061 7.603875 10.32008 1	1.487319 1.874594 2.222905 3.529230 4.790063 7.604410 10.36639 1	1.4873 1.8746 2.2229 3.5292 4.7901 7.6039 10.320	1.719842 2.177566 2.589234 4.133069 5.623036 8.948064 12.15767 2	8 0.66904 4 0.77343 6 0.98726 7 1.17912 9 1.71984 1 2.17756 5 2.58923 3 4.13306 9 5.62303 6 8.94808 4 12.1617 47	5 0.669 0 0.773 9 0.987 2 1.179 1 1.719 8 2.177 0 2.589 2 4.133 1 5.623 0 8.948 1 12.15 7	7 0.79599 1 0.93035 2 1.20692 4 1.45575 0 2.15773 7 2.75196 3 3.28625 0 5.28901 7 7.22117 3 11.5320 34 15.6927 70	7 0.79599 1 0.93035 1 1.20692 4 1.45574 9 2.15773 7 2.75196 2 3.28624 9 5.28901 7 7.22117 2 11.5320 3 15.6927 7	2.1577 2.7520 3.2863 5.2890 7.2212 11.532 15.692

Table.3 Comparison for the values of $f''(0)$ for *A=S=b=Ec=γ=0, Pr=1* for various values of $\beta = \frac{2m}{m+1}$.

β	$f''(0)$			
	Present results (BVP4c)	Mutlag et al. [38] (Runge–Kutta–Fehlberg fourth- fifth)	Bararnia et al. [42] (Homotopy perturbation method)	Rajagopal et al. [43] (Block-tridiagonal factorization technique)
0.0	0.469600	0.469600	0.46964	0.4696
0.05	0.531130	0.531129	0.53119	0.5311
0.1	0.587035	0.587035	0.58716	0.5871
0.2	0.686708	0.686708	0.68672	0.6867
0.3	0.774755	0.774754	0.77475	0.7747
0.4	0.854421	0.854421	.854420	.85440
0.6	0.995836	0.995836	0.99589	0.9958
0.7	1.059808	1.059807	1.05985	……..
0.8	1.120268	1.120267	1.12020	1.1202
0.9	1.177728	1.177727	1.17699	……..
1.0	1.232588	1.232587	1.23150	1.2325
1.2	1.335721	1.335720	1.33559	1.3357
1.6	1.521514	1.521513	1.52141	1.5215
2.0	1.687218	1.687218	1.68462	……..

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