

Improved Hybrid Behavior Ant Colony Algorithm to Solve the Vehicle Routing Problem

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Abstract: Vehicle routing problem is a NP-hard problem, with the expansion of problem solving more difficult. This paper proposes a hybrid behavior based on ant colony algorithm to solve the problem, ant to different objectives in the first place as the path selection according to the analysis of the impact on the algorithm, then define the ant behavior and design four concrete ant behavior by selecting different ways of ant behavior to form different improved algorithm. Finally, experimental results show that the improved algorithm can solve vehicle routing problems quickly and effectively.

Keywords: Vehicle Routing Optimization, Ant Colony Algorithm, Hybrid Behavior, Ant Behavior Mutation

I. Introduction

Particle Vehicle Routing Problem (VRP) was put forward by Dantzig and Ramser [1] in 1959. General meaning of VRP is that there are different demands of customers, from distribution centers according to the appropriate path to arrange supply to individual customers, total requirements under given conditions to meet the shortest path, minimum cost, least time consuming and other targets. The TSP is a special case of the VRP and have proved to be NP-hard problem, so the VRP also belongs to the NP problems. VRP commonly composed by customer demand point types, the number of distribution centers, vehicle model and quantity, customer supply time constraints, target types. Different constraints constitute different VRP problems. We study a non full-load vehicle scheduling problem on this article. The solution of the problem, an exact algorithm is viable when the problem size is small, such as branch and bound algorithm [2], k-Center-tree method [3], dynamic programming, etc. For relatively large VRP, traditional heuristic algorithm can be used to solve. Literature [4] first proposed using the saving algorithm solving the VRP. Song wei-gang [5] in China also tried saving algorithm to solve this problem. The traditional heuristic algorithm can solve the large-scale problem, but the accuracy can't be guaranteed. In recent years, modern intelligent algorithms have been successfully applied to solve this kind of problem, including Tabu Search algorithm [6], Genetic algorithm [7], Simulated Annealing algorithm [8], Neural Networks [9], Ant Colony algorithm [10], as well as between them or between them and the traditional heuristic algorithms combine to form hybrid algorithms [11]. Ant Colony algorithm for the earliest by Bullnheimer[12] applied to solve the VRP, after Bell[13] and others by using Ant Colony Optimization to solve VRP and proposed a multiple Ant Colony algorithm. Domestic scholars on applications of Ant Colony algorithm in VRP also did a lot of research. Liu zhi-shuo [14], and others applied Ant Colony algorithm to solving the VRP, by bringing in uniformity of solutions, selecting windows and attractive concept of transfer policy, such as improving and updating policies, structures with adaptive Ant Colony algorithm for function. Literature [15] aimed at the limitation of the traditional Ant Colony algorithm to improve the strategy of pheromone, heuristic factors, by comparing the experimental validity of the algorithm. Literature [16] combines the quantum computing, and proposed a quantum to solve vehicle routing problem with time windows Ant Colony algorithm. Literature [17] proposed a combination of improved Ant Colony optimization algorithm for hybrid heuristic algorithm for search and neighborhood decline. In view of basic optimization of Ant Colony algorithm in vehicle routing problem on the lack of an improved Ant Colony algorithm based on hybrid Ant behavior, experimental results show that it has significantly improved.

II. Problem Description and Mathematical Model

The non full-load vehicle scheduling problem can be described as: There is one distribution center and l customers. The distribution center have n vehicles with load capacity q . The demand of each customer is $g_i (i = 1, 2, \dots, l), q > g_i$. Vehicles set out from distribution center and finally go back to the center. Each customer point can only be supplied by one car. Customer demands can't exceed the sum weight of per car. In order to show the model conveniently, now define as follow: The number of distribution center is 0 and the number of each customer is $1, 2, \dots, l$. c_{ij} said the transportation cost from customer i to customer j . x_{ijk} indicates whether the vehicle k transport from point i to point j . y_{ik} indicates whether the point i distributed by

vehicle k and y_{0k} equal to 1. With this symbols to establish the mathematical model of vehicle scheduling optimization problems is as follow:

The objective function:

$$\min Z = \sum_{k=1}^m \sum_{i=0}^l \sum_{j=0}^l c_{ij} x_{ijk} \quad (1)$$

Constraints:

$$x_{ijk} = 0 \text{ or } 1, \quad i, j = 0, 1, \dots, l; \quad k = 1, 2, \dots, n \quad (2)$$

$$y_{ik} = 0 \text{ or } 1, \quad i = 1, 2, \dots, l; \quad k = 1, 2, \dots, n \quad (3)$$

$$y_{0k} = 1, \quad k = 1, 2, \dots, n \quad (4)$$

$$\sum_{k=1}^n y_{ik} = 1, \quad i = 1, 2, \dots, l \quad (5)$$

$$\sum_{i=0}^l x_{ijk} = y_{jk}, \quad j = 0, 1, \dots, l; \quad k = 1, 2, \dots, n \quad (6)$$

$$\sum_{j=0}^l x_{ijk} = y_{ik}, \quad i = 0, 1, \dots, l; \quad k = 1, 2, \dots, n \quad (7)$$

$$\sum_{i=1}^l g_i \quad y_{ik} \leq q_k, \quad k = 1, 2, \dots, n \quad (8)$$

In the above model, formula (1) is the objective function. The function requires vehicle dispatch project arising out of total costs to a minimum. Formula (2) indicates whether the vehicle k drives from point i to point j . Formula (3) indicates whether the customer i serviced by vehicles k . Formula (4) said that there are m cars start from distribution centers and the last also have m cars in return. Formula (5) represents that each customer i can be served by only one vehicle. Formula (6) said the vehicle can only serve for the needed customer. Formula (7) said the vehicle can only drive into the customers who have been served just now. Formula (8) said the sum of all customer demands served by vehicle k can't surpass the vehicle weight.

III. The Principle and Implementation of Hybrid Behavior Ant Colony Algorithm

3.1 Ant behavior impacts on Ant Colony algorithm

Ant Colony algorithm actually is a positive feedback mechanism and the product of some combination of heuristic. In early iterations of algorithm, due to the amount of pheromone on the path difference isn't big, ants, primarily on the basis of the distance between two points (heuristic) to find a better solution, then equivalent to the greedy algorithm of Ant Colony algorithm. After the iteration algorithm to a certain algebraic, better path pheromone significantly higher than the other side of the pheromone, now the ant colony is mainly through the pheromone interaction and communication to find a better solution, namely the stronger the pheromone path to become the greater the chance of the optimal path. Algorithm to search process at this stage is mainly using the principle of positive feedback, the process in enhancing performance solution at the same time, however, can easily lead to stagnation. This article is based on ant path selection in several different ways, and Ant Colony optimization algorithm in solving the VRP, except the distance between two points and pheromone concentration can be used as a judgment, other two factors should also be consider: First, the distance between the customer point and the distribution center that means the customer points' geographic position to the distribution center is also very important. Second, try to reduce the number of vehicles can reduce the cost, so the premise of meeting the vehicle load, vehicle loading solutions of higher income or better. Therefore, this article designed an improved ant behavior, contains paths to save values and vehicle load by two factors.

3.2 The design of ant behavior

Ant behavior is the rule set which guide ants' moving direction. Usually the set can be expressed as: $action = \{\Omega_i \mid i = 1, 2, \dots, n\}$, and Ω_i is the i factor which affect the behavior of ants. Based on the above analysis towards solving the VRP, we defined the following four kinds of ant behavior.

action1: Ants choose the next point in a random way.

action2: Ants choose the next point in a greedy way. The transition probability is as follow:

$$p_{ij}^k = \begin{cases} \eta_{ij} / \sum_{s \in J_k(i)} \eta_{is}, & j \in J_k(i) \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

$\eta_{ij} = 1/d_{ij}$, d_{ij} is the distance between the two clients. $J_{k(i)}$ is a collection which ant k can choose the next customer from it.

action3: Ants choose the next point according to the pheromone concentration. The transition probability is as follow:

$$p_{ij}^k = \begin{cases} \tau_{ij} / \sum_{s \in J_k(i)} \tau_{is}, & j \in J_k(i) \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

τ_{ij} is the pheromone concentration between point i and point j .

action4: Ants choose the next point by the following way:

$$j = \arg \max \left\{ [\tau_{is}]^\alpha [\eta_{is}]^\beta [\mu_{is}]^\varphi [\omega_{is}]^\lambda \right\}, s \in J_k(i) \quad (11)$$

$\tau_{is}, \eta_{is}, J_k(i)$ with the same type. $\mu_{is} = d_{i0} + d_{0s} - d_{is}$, this is a new variable which consider the distance between customer point and distribution center, known as the saving value. It absorbs the saving method. μ_{is} reflects the saving value of connecting two customer points to the distribution center respectively than connecting the two points directly. $\omega_{is} = (q_i + g_s)/q$, it be introduced after considering the vehicle load capacity. With the condition of $\omega_{is} \leq 1$, the bigger of ω_{is} , the higher vehicle utilization rate. $\alpha, \beta, \varphi, \lambda$ refer to the weighting factor of the pheromone concentration, stimulating factor, saving value, vehicle load rate in ant path level of importance in the process. This article will select one or several kinds of ant behavior combination to solve VRP, comparison and analysis to find the optimal results of algorithm, the following 4 types of behavior matrix to explain the algorithm implementation process.

3.3 The algorithm's implementation process

After all the ants complete the construction process solutions, calculating the optimal solution of this iteration will be compared to the iterative optimal solutions and compare current optimal solutions, if better you can replace the current optimum, then updating the path pheromone according to the following formula:

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \frac{Q}{L^{gb}} \quad (12)$$

L^{gb} is the current optimal solution. ρ is pheromone volatilization factor, $\rho \in (0,1)$. Using the global optimal can accelerate the algorithm's convergence speed, but it makes the best path's pheromone many times higher than the other path. This will lead to stagnate in local, so limit the pheromone in $[\tau_{\min}, \tau_{\max}]$. When $\tau_{ij}(t) > \tau_{\max}$, setting $\tau_{ij}(t) = \tau_{\max}$. When $\tau_{ij}(t) < \tau_{\min}$, setting $\tau_{ij}(t) = \tau_{\min}$.

Algorithm description:

Step1: Initializing parameters: $r_1 : r_2 : r_3 : r_4$, ρ , Q , $\tau_{ij}(0)$, α , β , φ , λ , the largest iteration number NC_{\max} , and the algorithm basis data.

Step2: Generating m ants, $\frac{mr_1}{r_1+r_2+r_3+r_4}$ ants act according to action1. $\frac{mr_2}{r_1+r_2+r_3+r_4}$ ants act according to action2. $\frac{mr_3}{r_1+r_2+r_3+r_4}$ ants act according to action3. $\frac{mr_4}{r_1+r_2+r_3+r_4}$ ants act according to action4.

Step3: Putting all the ants on the distribution center.

Step4: For each ant
Repeat

Choosing the next customer according to it's own rule;
 Moving to the next customer point;
 Putting this customer point number into their own taboo table;
 Until exceeding the vehicle load capacity;
 Back to the center;
 End for

Step5: Calculating each ant's path length and finding out the current optimal solution L^b . If $L^b < L^{gb}$, using L^b to exchange L^{gb} ;

Step6: Updating the path pheromone according to (12), and judging whether it beyond the pheromone scope $[\tau_{\min}, \tau_{\max}]$;

Step7: $NC = NC + 1$,

If $NC > NC_{\max}$

Output the optimal solution;
 Exit the algorithm;

Else

Clean all ants' taboo table;
 Go to Step4;

End if

IV. Simulation Experiments and Analysis

Taking Chinese 31 cities coordinate data and different goods demands as an example to solve the VRP. The hybrid of action 1,2,3,4 as algorithm I, the hybrid of action 2,3,4 as algorithm II, and the algorithm based on action 4 called algorithm III. We will compare the tree kinds of algorithms with the basic ant colony algorithm (algorithm IV).

4.1 Algorithm parameters

Firstly, analysis the value of pheromone volatilization factor ρ and the different ratio of $\alpha, \beta, \varphi, \lambda$, based on algorithm III. In the case of fixed the pheromone volatilization factor ρ , taking different ratio of $\alpha, \beta, \varphi, \lambda$ to solve the problem. The results are shown in table 1. "Best", "avg", "worst" represent the best solution, average and worst solution after running 20 times.

Table 1. The influence of different parameter proportions on algorithm III

$\alpha : \beta : \varphi : \lambda$	1:1:1:1	1:2:1:1	1:2:2:1	1:2:2:2	2:2:2:1	1:3:2:1
best	103360	104860	103520	104390	105120	104350
avg	106108	105794	105439	105215	106362	106031
worst	107290	106670	106970	105860	107260	106910

It can be seen from table 1, when $\alpha : \beta : \varphi : \lambda = 1:1:1:1$, the algorithm get the optimal value. When $\alpha : \beta : \varphi : \lambda = 1:2:2:1$, the effect is also good. Then fixed the proportion of $\alpha : \beta : \varphi : \lambda$, taking different values of ρ , the results are shown in table 2.

Table 2. The influence of different values of ρ on the algorithm III

ρ	0.02	0.05	0.1	0.2	0.3	0.4	0.5
best	105530	104180	103170	104220	105230	105740	104590
avg	106559	105821	105238	106087	105944	106372	105659
worst	107040	107010	106610	106830	106620	107340	106820

It can be seen from table 2, when the value of ρ equal to 0.1, the algorithm's "best", "avg" both obtain the best performance.

4.2 Contrast experiments

Firstly, analysis the influence of different proportions of ant behavior on the algorithm I and algorithm II. The results are shown in table 3 and table 4, setting the number of ants as 100.

Table 3 The influence of different ant behavior proportions on algorithm I

$r_1 : r_2 : r_3 : r_4$	1:2:5:12	1:2:8:9	1:1:2:6	1:1:4:4	2:1:2:5	2:2:3:3
best	104370	105180	105250	105760	104220	103230
avg	105927	106476	106952	106854	107010	107030
worst	107450	107900	107970	107950	108100	108010

From table 3, it can be seen, when $r_1 : r_2 : r_3 : r_4 = 2 : 2 : 3$, algorithm I obtain the best value, but the average value is not very ideal. But when $r_1 : r_2 : r_3 : r_4 = 1 : 2 : 5 : 12$, it have a better average value. The overall effect is good.

Table 4 The influence of different ant behavior proportions on algorithm II

$r_2 : r_3 : r_4$	1:2:7	2: 2:6	2:3:5	3:3:4
best	104000	104550	104510	105080
avg	105978	105932	106050	107212
worst	107660	107100	107170	108120

It can be seen from table 4, when $r_2 : r_3 : r_4 = 1:2:7$, algorithm II obtain the optimal value. That means action 4 in a large proportion is effective.

And then compare the performance of algorithm I, II, III and basic ant colony algorithm (algorithm IV). The parameter of algorithm IV: $\alpha = 1.5, \beta = 4, \rho = 0.5, Q = 15$. Algorithm I: $\alpha = 1, \beta = 2, \varphi = 2, \lambda = 1, \rho = 0.1, r_1 : r_2 : r_3 : r_4 = 1 : 2 : 5 : 12, Q = 15$. Algorithm II: $\alpha = 1, \beta = 2, \varphi = 2, \lambda = 1, \rho = 0.1, r_2 : r_3 : r_4 = 1 : 2 : 7, Q = 15$. Algorithm III: $\alpha = 1, \beta = 2, \varphi = 2, \lambda = 1, \rho = 0.1, Q = 15$. Ants number with the same value 100, each algorithm run 20 times. The results are shown in table 5.

Table 5. Algorithm comparing

	Algorithm IV	Algorithm I	Algorithm II	Algorithm III
best	107070	103230	104130	103060
avg	107534	106708	105816	104949
worst	108070	108100	107040	105910
Average number of vehicle	13	12	12	12
Average vehicle load rate	86.54%	93.75%	93.75%	93.75%

It can be seen from table 5, the improved algorithm I, II, III are superior than the algorithm IV. Improved algorithm's average number of vehicles is 12 and vehicle load rate increase significantly. What's more, the path length of improved algorithm also has greatly improved, algorithm III obtain the optimal value. Taking action 4 as ants' rout selecting principle has obtained the best improvement.

V. Conclusions

At first, this paper analyzes the mechanism of ant colony searching path in the process of solving VRP, then puts forward several different ways of ant behavior accordingly and brings in the saving value in the ant behavior and vehicle loading rate, so that ant behavior has made a great improvement. Finally, selecting one or more of the behaviors constitute different algorithms, analyzing the performance of the algorithm is found to improve distance and load rate of the algorithm are improved obviously. The algorithm III obtain the best effect.

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REFERENCES

- [1] Dantzig G, Ramser J. The truck dispatching problem[J]. Management Science, 1959,6(1),pp. 80-91.
- [2] Bard JF, Kontoravdis G and Gang Y. A branch-and-cut procedure for the vehicle routing problem with time windows[J]. Transportation Science, 2002, 36(3),pp. 260-269.
- [3] Christofides N, Mingozzi A, Toth P. Exact algorithms for the vehicle routing problem based on spanning tree and shortest path relaxations[J]. Mathematical Programming, 1981, 20(1),pp. 255-282.

- [4] Clarke G, Wright J. Scheduling of vehicles from a central depot to a number of delivery points[J]. *Operations Research*, 1964, pp.568-581.
- [5] Song wei-gang, Zhang hong-xia, Tong ling. C-W Algorithm for Vehicle Routing Problem of Non-Full Loads with Time Windows[J]. *Journal of Northeastern University(Natural Science)*, 2006, 27(1)pp. 65-68.
- [6] Cordeau JF, Laporte G. A tabu search heuristic for the static multi-vehicle dial-a-ride problem. *Transportation Research*, 2003, 37(6):.pp.579-594.
- [7] Baker BM, Ayechew MA. A genetic algorithm for the vehicle routing problem. *Computer&Operations Research*, 2003, 30(5),pp.787-800.
- [8] Czech ZJ, Czarnsa P. parallel simulated annealing for the vehicle problem with time windows[C]. 10th Euromicro Workshop on routing Parallel, Distributed and Network-based Processing.
- [9] Storkey AJ, Valabregue R. The basin of attraction of a new Hopfield learning rule[J]. *Neural Networks*, 1999, 12(6),pp. 869-876.
- [10] Dorigo M, Gambardella LM. Ant colony system: A cooperative learning approach to the traveling salesman problem. *IEEE Transactions on Evolutionary Computation*, 1997, 1(1),pp. 53-56.
- [11] Bullnheimer B, Hartl RF and Strauss C. Applying the Ant System to the Vehicle Routing Problem[C]. 2nd International Conference on Meta-heuristics, Sophia-Antipolis, France, 1997.
- [12] Bullnheimer B, Hartl R and Strauss C. Applying the ant system to the vehicle routing problem. *proc of Meta-heuristics: Advances and trends in local search paradigms for Optimization*, 1999, pp.285-296.
- [13] Bell J, McMullen P. Ant colony optimization techniques for the vehicle routing problem[J]. *Advanced Engineering Informatics*, 2004, 18(1),pp. 41-48.
- [14] Liu zhi-shuo, Shen jin-sheng. An Adaptive Ant Colony Algorithm for Vehicle Routing Problem Based on the Evenness Solution[J]. *Journal of System Simulation*, 2005, 17(15) .pp.1079-1083.
- [15] Chen ying-xin. Study on VRP based on improved ant colony optimization[J]. *Application Research of Computers*, 2012, 29(6),pp.2031-2034.
- [16] He xiao-feng, Ma liang. Quantum-inspired ant colony algorithm for vehicle routing problem with time windows[J]. *Systems Engineering—Theory&Practice*, 2013, 33(5),pp. 1255-1261.
- [17] Wang ren-min, Bi ying-zhou, Liu a ning,etc. Improved variable neighbourhood search algorithm for DVRP[J]. *Computer Engineering and Applications*, 2014, 50(2),pp. 237-241.