International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 5, Issue 5 [May. 2016] PP: 60-68

Derivative of Stress Strain,Deviatoric Undrainedcohesion Stress And Models Based on Stress

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ABSTRACT:- The deviatoric stress-strain, derivative of deviatoric stress-strain, and derivative of ratio of deviator stress to undrained cohesion behaviour of soils on three areas within Port Harcourt were carried out and their deformation trends established. The soils exhibited a nonlinear stress-strain deformation behaviour and the predicted soil modulus for the areas are generally in the range of E applicable for routine work in London clay; soils within Agip and Iriebe are identified as medium clay, except for UST soils that are within the range of medium to stiff clays. Stability magnitude of the soils increased in the order of Iriebe, Agip and UST soils, while deformation of soils response to loading were in decreasing order of Iriebe, Agip and UST soils. Soil modulus E, generally had a decreasing trend, with maximum values obtained at zero strain, but at 1.5 % strain, E values of the soils converged to about 22 MPa. For strains exceeding 1.5 %, E of soils within UST and Agip attained negative values for cell pressure of 100 kN/m^2, while soils within Iriebe have positive values. Under soil pressure of 300 kN/m^2, soil modulus generally reduced with increase in strain up to about 3 % strain, beyond which, E exhibited increase in value on the soils. Hence, input parameter of soil modulus, can easily be obtained from the predictive models, and can be adopted in preliminary analysis of shallow foundation deformation on cohesive soils.

Key words: *Deformation, Shallow Foundation, Stability and Young's Modulus*

I. INTRODUCTION

The Young's modulus (E), of a soil is commonly referred to as soil elastic modulus which is an elastic soil parameter that expresses its stiffness. Within the elastic soil behaviour, Young's modulus is defined as the ratio of the stress to strain along the axis of load application and is often used to estimate soil settlement and elastic deformation analysis. Soils are generally subjected to various loads; load bearing walls, columns, vehicular wheel loads, and machine foundations, etc, causing stresses in the soil mass that correspondingly experience varying levels of strain. However, the stress-strain deformation trend of soils generally depicts a non-linear relationship. The elastic modulus can be estimated from laboratory or in-situ tests or based on correlation with other soil properties. In the laboratory, the undrained soil modulus can be obtained from triaxial test results by determining the strain corresponding to 65 % of the maximum deviator stress and dividing this value into its corresponding stress or indirectly from oedometer test [1, 2]. In the field, it can be estimated from Standard penetration test, Cone penetration test , pressuremeter or indirectly from dilatometer test [3]. Evaluation of soil modulus, E, of clayey soils from undrained cohesion, c_u, using stress-strain models has been reported in some areas of Port Harcourt [4]. Results of Tangent modulus at 1% strain levels for cell pressures of 100 kN/m^2 and 300 kN/m^2 based on deviator stress-strain models and from derivatives of deviator stress-strain models generally identified the soils as soft to medium clays. Predicted E/c_u values were generally lower than reported field values frequently used for intact blue London clay, but were within values used for routine work in London clay. Previous study on soil modulus in six areas using oedeometer results identified them as having field values identified as very soft to soft clays [5, 6].Problems involving the application of stresses to soils may be divided into those in which deformation of the foundation soil control design and those in which failure of the foundation soil controls design. In deformation-controlled design, the shape of the stress-strain curve must be taken into account, but for failure-controlled design, the precise shape of the stress-strain curve need not be known if shear stress reaches a maximum and subsequently remains constant even at very large strain [7]. Available literatures in stability analysis reports stress-strain curves of soils attaining a maximum shear stress which then undergoes constant deformation under continuous strain. However, in the laboratory, compressive triaxial test are generally not continued to attain continuous deformation under shear stress. He highlighted four major factors considered to significantly control the shape of stress-strain curves of soil; soil type, initial structure, initial state and method of loading. Briaud [8] emphasised the nonlinear stress-strain behaviour exhibited by soils, hence, different soil moduli can be deduced from the slope of the curve; the secant modulus, tangent modulus, unloading modulus or reload modulus. He reported that the secant modulus is appropriate in predicting spread footing movement due to first load application, whereas, the tangent modulus is used in cases of evaluating incremental movement due to incremental load from additional storey in a high–rise building. Also, the unloading modulus is useful in calculating heave at bottom of excavation or rebound on pavement on removal of truck tyre load, while reload modulus is used in calculating bottom excavation movement on replacement of excavated soil or equivalent overburden. Typical values of Young's modulus of cohesive soils are presented in Table 1.

Table 1 Young's modulus of cohesive soils [9]

Type of Soil	Es (MPa)
Very Soft clay	0.5 - 5
Soft clay	5 - 21
Medium clay	21 - 53
Stiff clay, Silty clay	53 - 107

Under axi-symmetric loading, the classical elasticity model assumes elastic soil behaviour, being described by the following stress-strain matrix equation [10];

σ, , (1)

, =, , , , , ,
σ

, , , , , −μ, 1, −μ, , σ, ,
, , , −μ, −μ, 1, , , ,
Where ,

and are three dimensional stresses, and ∈,
∈

, ∈ are three-dimensional strains, μ is poison ratio and E is soil modulus. Under uniaxial loading,

= = 0,
∈

=∈= 0. Consequently, equation (1) becomes;

(2)

Assuming that represents the stress-strain curve, then the derivative of the stress-strain curve can be expressed as;

, , , 1, −μ, −μ, ,
= ∈ = E

Given that the deviator stress is − , then the slope of
(strain, can be represented as follows;
! −/"#% &
∈ =

Where & = ( ) and c_u = undrained cohesion.

(3)

−)/c_u versus axial

4)

In evaluating immediate settlement of shallow foundations placed on cohesive soils, the undrained modulus, E_u, of the supporting soil is an input parameter, but its determination is faced with constrains. Jamiolkowski et al. [11] proposed ratio of undrained modulus to undrained cohesion (E_u/c_u) depending on over consolidation ratio and plasticity index. An E_u/c_u ratio of 400 that is frequently used for intact blue over consolidated London clay has been proposed [12], while Padfield and Sharrock [13] proposed E_u/c_u ratio of 140 for routine work in London clay. In many literatures, E_u for various soils is presented in a wide range of values with little emphasis as to whether they are secant modulus, tangent modulus, unload modulus or reload modulus [14]. However, the initial tangent modulus is mostly used to represent the stress-strain modulus of a soil because of the elastic response of soils generally observed near the origin. However, the re-load modulus has been emphasised as a better choice.Based on the difficulty of evaluating soil modulus needed in the analysis and design of foundation, an attempt is made in this paper to develop predictive models for evaluation of tangent modulus on the studied areas.

II. 2. MATERIALS AND METHODS

2.1. Acquisition and Analysis of Data

A total of eighty (80) unconsolidated undrained triaxial test results were analysed from each of the four areas studied in Port Harcourt: Agip, Iriebe and UST**.** The deviator stresses, induced strains, cross-sectional area, major (and minor principal stresses were evaluated. For instance, the deviator stress (* , is evaluated noting that the average cross-sectional area (A) of the specimen does not remain constant throughout the test. When the original cross-sectional area of the specimen is A_o and the original volume is V_o, then, for a decrease in volume of the specimen during the test, the average cross-sectional area (A) is expressed as;

$$A = A_o \frac{1 - \varepsilon_v}{1 - \varepsilon_a} \qquad (5)$$

If the volume of specimen increases during the test, then Equation (5) becomes;

$$A = A_o \frac{1 + \varepsilon_v}{1 - \varepsilon_a} \qquad (6)$$

Where ε_v is the volumetric strain (v/v_o), and ε_a is the axial strain (l/l_o). Deviator stresses were evaluated by dividing the load with corresponding cross-sectional area of the sample.

III. 3. RESULTS AND DISCUSSION

3.1. Deviator stress - strain curve

The various nonlinear stress-strain curves of clayey soils from the three studied areas are shown in Figures 1 and 2. Stress-strain curves magnitude, increased in the order of Iriebe soils, Agip soils and UST soils respectively. Consequently, soils within UST exhibited higher stability and lower deformation as against the response of loading to stability and deformation on soils within the two other areas. Soils within Agip area showed middle bound deformation, while soils within UST have higher stability compared to those within Iriebe. The stress-strain deformation trend of these soils for cell pressure of 100 kN/m^2 and 300 kN/m^2 are given by equations (7–9) and (10–12) respectively. The failure curves is typical of the non-linear behaviour of soils under deformation and the stress magnitudes sustained by the soils are highest on soils in UST and lowest on soils within Iriebe. Soils within Agip had middle bound values.

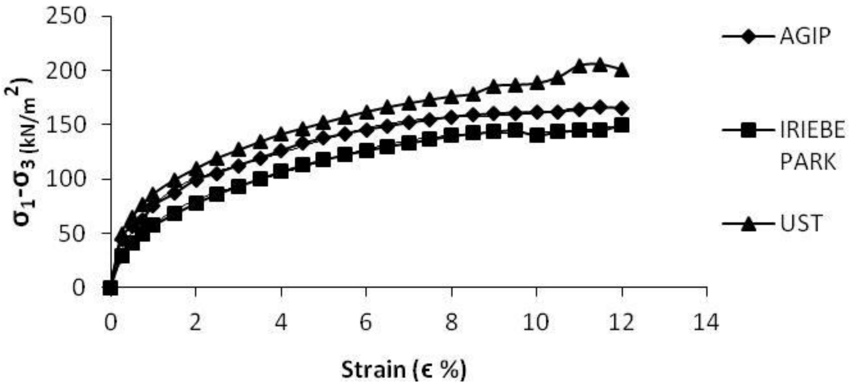
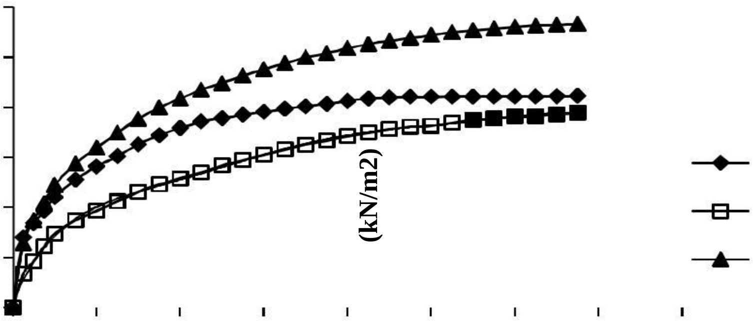


Figure 1 Deviator stress and strain (σ_3=100kN/m^2)

$$(\sigma_1-\sigma_3)_{UST} = -0.004\,\varepsilon^6 + 0.1522\,\varepsilon^5 - 2.270\,\varepsilon^4 + 16.53\,\varepsilon^3 - 61.90\,\varepsilon^2 + 125.3\,\varepsilon + 10.40;\ R^2 = 0.994 \quad (7)$$

$$(\sigma_1-\sigma_3)_{Agip} = -0.002\,\varepsilon^6 + 0.102\,\varepsilon^5 - 1.582\,\varepsilon^4 + 11.98\,\varepsilon^3 - 46.93\,\varepsilon^2 + 102.1\,\varepsilon + 10.25;\ R^2 = 0.993 \quad (8)$$

$$(\sigma_1-\sigma_3)_{Iriebe} = -0.001\,\varepsilon^6 + 0.051\,\varepsilon^5 - 0.868\,\varepsilon^4 + 7.085\,\varepsilon^3 - 30.0\,\varepsilon^2 + 74.9\,\varepsilon + 66.349;\ R^2 = 0.996 \quad (9)$$



300

250

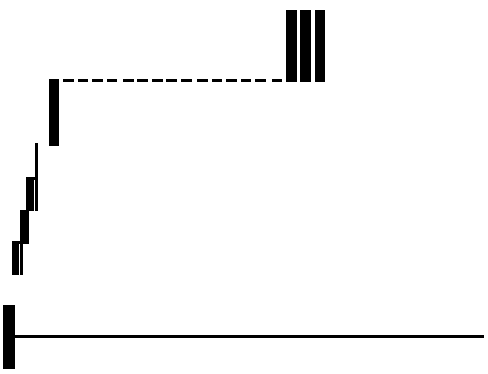
200

150

100

50

0



AGIP
IRIE
BE

UST

0 2 4 6 8 10 12 14 16

Strain (%)

Figure 2 Deviator Stress and Strain ($\sigma_3 = 300$ kN/m^2)

σ

$\sigma = -0.001\,\varepsilon^6 + 0.084\,\varepsilon^5 - 1.492\,\varepsilon^4 + 13.00\,\varepsilon^3 - 59.52\,\varepsilon^2 + 153.2\,\varepsilon + 15.60$; $R^2 =$ 0.995 UST (10)

σ

$\sigma = -0.001\,\varepsilon^6 + 0.075\,\varepsilon^5 - 1.322\,\varepsilon^4 + 11.52\,\varepsilon^3 - 52.85\,\varepsilon^2 + 132.8\,\varepsilon + 20.28$; $R^2 =$ 0.986 Agip (11)

σ

$\sigma = -0.001\,\varepsilon^6 + 0.05\,\varepsilon^5 - 0.891\,\varepsilon^4 + 7.834\,\varepsilon^3 - 35.87\,\varepsilon^2 + 93.88\,\varepsilon + 6.213$; $R^2 = 0.998$ Iriebe (12)

3.2. Deviator stress to Undrained cohesion, (σ_1 - σ_3)/c_u, and Strain

Soils response in terms of the ratio of deviator stress to undrained cohesion**,** $(\sigma_1\text{-}\sigma_3)/c_u$, and strain for cell pressure of 100 kN/m^2 and 300 kN/m^2 are presented in Figures 3 and 4 respectively. The deformation trend is generally non-linear with highest values found on soils within UST, and lowest on Iriebe soils, while intermediate values are found on soils within Agip. The response curves are given by equations (13–15) for cell pressures of 100 kN/m^2, while equations (16–18) represent response curves of 300 kN/m^2 cell pressure.

$(\sigma_1 - \sigma_3)/c_u = -6E\text{-}05\,\varepsilon^6 + 0.003\,\varepsilon^5 - 0.049\,\varepsilon^4 + 0.374\,\varepsilon^3 - 1.447\,\varepsilon^2 + 2.924\,\varepsilon$; $R^2 = 0.990$ UST (13)

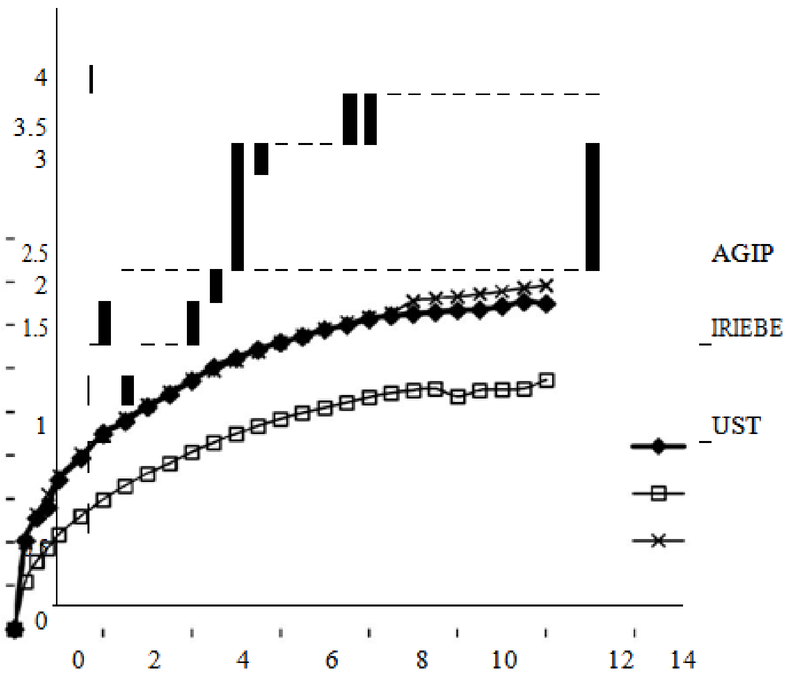
$(\sigma_1 - \sigma_3)/c_u = -6E\text{-}05\,\varepsilon^6 + 0.002\,\varepsilon^5 - 0.036\,\varepsilon^4 + 0.272\,\varepsilon^3 - 1.065\,\varepsilon^2 + 2.318\,\varepsilon + 0.231$; $R^2 =$, , , , , 0.993, Agip, (14)

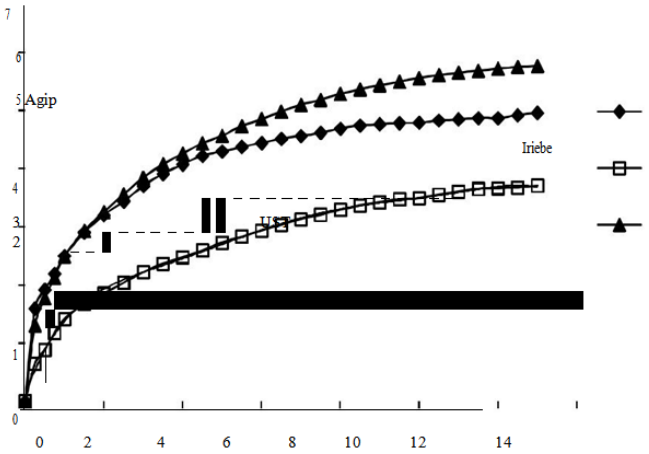
$(\sigma_1 - \sigma_3)/c_u = -2E\text{-}05,\,\varepsilon^{6,} + 0.001\,\varepsilon^{5,} - 0.016\,\varepsilon^{4,} + 0.134\,\varepsilon^{3,} - 0.572\,\varepsilon^2 + 1.431\,\varepsilon + 0.117$; $R^2 =$, , , , , 0.996, Iriebe, , (15)

$(\sigma_1 - \sigma_3)/c_u = -5E\text{-}05,\,_{\varepsilon}6, + 0.002\,\varepsilon^{5,} - 0.034\,\varepsilon^{4,} + 0.292\,\varepsilon^{3,} - 1.291\,\varepsilon^{2,} + 3.206\,\varepsilon + 0.303$; $R^2 =$, , , , 0.995, UST, , , (16)

$(\sigma_1 - \sigma_3)/c_u = -5E\text{-}05,\,\varepsilon^{6,} + 0.002\,\varepsilon^{5,} - 0.035\,\varepsilon^{4,} + 0.294\,\varepsilon^{3,} - 1.290\,\varepsilon^{2,} + 3.109\,\varepsilon + 0.443$; $R^2 =$, , , , 0.986, Agip, , , (17)

$(\sigma_1 - \sigma_3)/c_u = -3E\text{-}05,\,\varepsilon^{6,} + 0.001\,\varepsilon^{5,} - 0.019\,\varepsilon^{4,} + 0.166\,\varepsilon^{3,} - 0.733\,\varepsilon^{2,} + 1.849\,\varepsilon + 0.110$; $R^2 =$, , , , 0.998, Iriebe, , , (18)





Strain (□ %)

Figure 4 Deviator Stress to Undrained cohesion and Strain (σ_3 = 300 kN/m^2)

3.3. Soil Modulus

The slope of stress-strain curves of equations (7- 9) are presented in equations (19– 21) while the variation of soil modulus E, with strain for cell pressure of 100 kN/m^2 is presented in Figure 5. Soil modulus E, generally had a decreasing trend, with maximum values obtained at zero strain; soils within UST had highest values, for cell pressure of 100 kN/m^2 while soils within Iriebe had lowest E. But at 1.5 % strain, E values of the soils converged to about 22 MPa. For strains exceeding 1.5 %, soil modulus of soils within UST and Agip attained negative values for cell pressure of 100 kN/m^2, while soils within Iriebe have positive values. Under soil pressure of 300 kN/m^2, the rate of change of stress with strain, expressed by soil modulus is presented in equations (22–24) and depicted in Figure 6. It is noticed that soil modulus generally decreased with increase in strain up to about 3 % strain, beyond which E exhibited increase in value on the soils.

$$E = -0.0024\varepsilon^5 + 0.26\varepsilon^4 - 9.08\varepsilon^3 + 49.59\varepsilon^2 - 123.8\varepsilon + 125.3; \quad R^2 = 0.994 \text{ UST} \quad (19)$$

$$E = -0.012\varepsilon^5 + 0.051\varepsilon^4 - 6.328\varepsilon^3 + 35.94\varepsilon^2 - 93.86\varepsilon + 102.1; \quad R^2 = 0.993 \text{ Agip} \quad (20)$$

$$E = -0.006\varepsilon^5 + 0.255\varepsilon^4 - 3.472\varepsilon^3 + 21.255\varepsilon^2 - 60\varepsilon + 74.9; \quad R^2 = 0.993 \text{ Iriebe} \quad (21)$$

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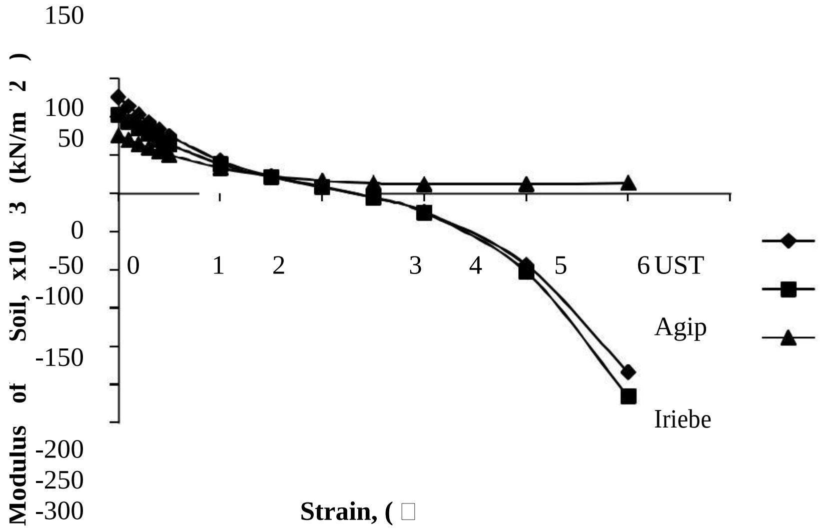

Strain, (□ %)

Figure 5 Soil Modulus and Strain (σ_3 = 100 kN/m^2)

$+ 39.0\varepsilon^2 - 119.0\varepsilon + 153.2; \; R^2 = 0.995$ UST

$$E = -0.006\varepsilon^5 + 0.420\varepsilon^4 - 5.968\varepsilon^3 \quad (22)$$

$$E = -0.006\varepsilon^5 + 0.375\varepsilon^4 - 5.288\varepsilon^3 + 34.56\varepsilon^2 - 105.7\varepsilon + 132.8;\ R^2 = 0.986 \quad \text{Agip} \quad (23)$$

$$E = -0.006\varepsilon^5 + 0.2\ 5\varepsilon^4 - 3.56\varepsilon^3 + 23.50\varepsilon^2 - 71.74\varepsilon + 93.88;\ R^2 = 0.998 \quad \text{Iriebe} \quad (24)$$

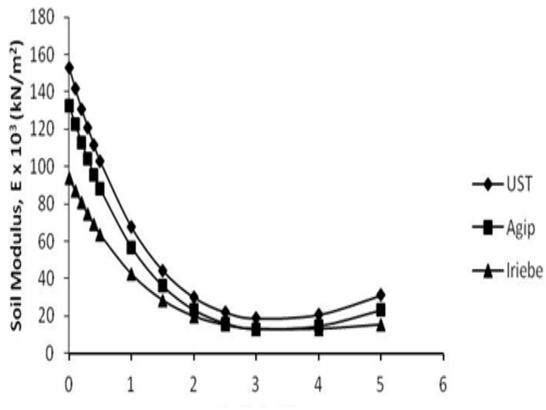

Figure 6 Variation of Modulus with Strain (σ_3 = 300 kN/m^2)

3.4. Model Verification

3.4.1. SOIL MODULUS

Evaluation of tangent mo dulus of the soils at 1 % strain level for cell p ressures of 100 kN/m^2 and 300 kN/m^2 based on deviator stress-strain models of equations (7–9) and (10–12) respectively are shown in Table 2. The soil modulus are g enerally in the range of E, identified as medium clay soils, except for UST soils that are in the range of medium to stiff clay. From the derivatives of the deviator stress-strain models of equations (19–21) and (22–24), the soil modulus at 1% strain level are presented in Table 3. Agip and Iriebe soils gave E values commonly described as medium clays, while those within UST had E associated with medium to stiff clays.

Table 2 Model verification of soil modulus (Stress-strain models)

LOCATION	Predicted Tangent Modulus E (MPa)	E from field values (MPa)	
		Clay soil	Range
UST	50–69	Very soft	0.5 - 5
Agip	38–65	Soft	5 - 21
Iriebe	33–51	Medium	21 - 53
		Stiff clay, Silty clay	53 - 107

Table 3 Model verification of soil modulus)

LOCATION	Predicted Tangent Modulus E (MPa)	E from field values (MPa)	
		Clay soil	Range

UST	53–85	Very soft	0.5 - 5
Agip	24–25	Soft	5 - 21
Iriebe	42–105	Medium	21 - 53
		Stiff clay, Silty clay	53 - 107

3.4.2. STRESS TO UNDRAINED COHESION, $(\sigma_1$-$\sigma_3)/C_U$ AND STRAIN

The predicted values of E/c_u based on the ratio of the derivative of deviator stress to undrained cohesion**,** $(\sigma_1 - \sigma_3)/c_u$, to strain for cell pressure of 100 kN/m^2 and 300 kN/m^2 at strain level of 1% is presented in Table 4. Generally the E/c_u values for the areas, is representative of the value used for routine work in London clay, but lower than reported field value frequently used for over consolidated intact blue London clay.

Table 4 Model verification of $(\sigma_1 - \sigma_3)/c_u$) and strain

Location	Predicted E/c_u	Field values of E/c_u	
		Clay soil	**Application**
UST	130	140	Routine work in London clay
Agip	130		
Iriebe	100	400	Frequently used for intact blue over consolidated London clay

IV. CONCLUSION

The soils exhibited a nonlinear stress-strain deformation behaviour and the predicted soil modulus for the areas are generally in the range of E applicable for routine work in London clay. Soils within Agip and Iriebe are identified as medium clay soils, except for UST soils that are within the range of medium to stiff clays. Based on the derivatives of the deviator stress-strain models, Agip and Iriebe soils gave E values commonly described as medium clays, while those within UST had E associated withmedium to stiff clays. Results of E obtained from the stress-strain models generally gave E values similar to those obtained from the derivatives of the deviator stress-stress to undrained cohesion**,** $(\sigma_1 - \sigma_3)/c_u$), at strain level of 1 %, generally gavevalues used for routine work in London clay. Hence, input parameter of soil modulus, can easily be obtained within the study area from their respective predictive models, which can easily be adopted in preliminary analysis of shallow foundation deformation on cohesive soils.

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