

## **Modeling and Analysis of Vibration Controlled Cantilever Beam Bounded by PZT Patch**

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**Abstract:** - This paper presents active vibration control of mechanical structure beam and systems using smart material. The goal of vibration control was to suppress unwanted vibration of various dynamic structures. A smart structure would be able to sense the vibration and generate a controlled actuation to it, so the vibration can be minimized. The piezoelectric sensors are used to detect the vibration. Simultaneously, feedback controller sends correction information to the actuator that minimizes the vibration. In present works deals with the active vibration control of cantilever beam bonded with Lead Zirconate Titanate (PZT) patches as piezoelectric sensor and actuator. Rectangular aluminum beam modeled in cantilever configuration with PZT patch consider as smart structure. The study uses ANSYS software to derive the finite element model of the smart structure. The effect of PZT sensor/actuator pair is investigated at different locations of beam in vibration control. It can be concluded from the work that best result is obtained when the PZT patches are bonded near the fixed end. In this experiment we find an optimal position of PZT patches to get more effective vibration control method.

**Keywords:** active vibration control, actuator/sensor, piezoelectric material, PZT patch, smart structure beam.

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### **I. INTRODUCTION**

The smart material as materials, which possess ability to change their physical property in specific manner in response to stimulus input. These smart structures have some important component like structural material, distributed actuators and sensors, control system, and power conditioning electronics. Piezoelectric material Lead Zirconate Titanate (PZT) have got more importance in vibration control of structure in recent year, because these material have light weight, small volume, mechanical simplicity and easy conversion of mechanical energy into electrical energy (sensor) and electrical energy into mechanical energy (actuator) as shown in Fig.1.

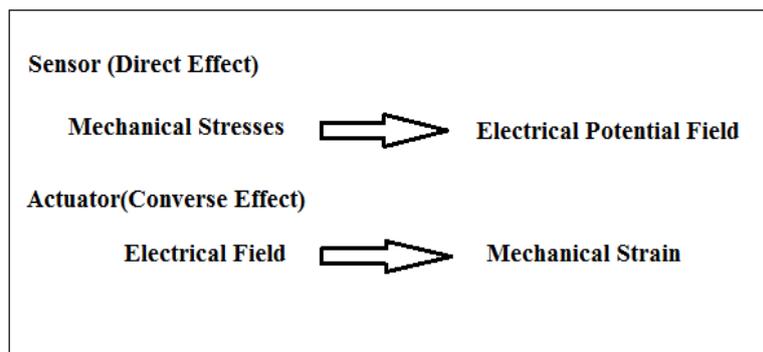


Fig.1 Principle of piezoelectric effect

With these components a smart structure has the ability to respond to changing environmental and operational conditions such as vibrations and deformation. These materials also have very high strain rate. Piezoelectric materials are devices used to control vibration in flexible structures. These materials have the characteristic that they show both sensor and actuator effects. The development of piezoelectric material has been used as sensors and actuators

K. Ramesh Kumar and S Narayanan [1] used a finite element method based on Euler–Bernoulli beam theory. The LQR performance is taken as the objective for finding the optimal location of sensor–actuator pairs. Lucy Edery-Azulay and Haim Abramovich [2] described that the active damping is obtained by using an actuator and a sensor piezoceramic layer acting in closed-loop. Ken Susanto [3] studied vibration analysis of Piezoelectric Laminated Slightly Curved Beams using Distributed Transfer Function Method. Dias Rodrigues [4] used an analysis and comparison of the classical control strategies, constant amplitude and constant gain

velocity feedback (CAVF and CGVF), and optimal control strategies, linear quadratic regulator (LQR) and in order to investigate their effectiveness to suppress vibrations in beams with piezoelectric patches acting as sensors or actuators. T.C Manjunath and B Bandyopadhyay [5] used multi-rate output feedback based discrete sliding mode control for SISO systems in vibration control of Timoshenko smart structure. The beam structure is modeled in the State Space form using the concept of piezoelectric theory. The adaptive shape control for vibration suppression of a cantilever beam using piezoelectric damping modal actuator/sensor has been presented [6].

The objective of this work is to design and analysis piezoelectric smart beam with commonly used control method. The Proportional Integral Derivative (PID) when Piezoelectric Sensor/Actuator pair is placed at different locations 30mm, 100mm and 250mm from fixed end on the beam. The paper is organized four parts i.e. Experimental set up, FE formulation of Smart beam, modeling and analysis of smart beam and Results.

## II. EXPERIMENTAL SET UP FOR VIBRATION CONTROL OF CANTILEVER BEAM

The aluminum cantilever beam is fixed at one end on the set table and other end is hanging freely hence is a cantilever beam, from the end we will give under control vibration and this is accomplished by using an exciter as shown in Fig.2. The function of the exciter is to produce under control vibration on the beam and the nature of the vibration will depend upon the input signal from the function generator, whatever will be the nature of the waveform similar kind of vibration will be produced in the beam. The function generator is used to generate the desired wave form which can be either of sinusoidal, triangle, Square the range of the frequency can be adjusted and can be set anywhere between 1Hz to 1000 KHz but as our exciter has limitations so we can only set the frequency between 1Hz to 1KHz [7]. The frequency is high but the amplitude of the wave form is very low to produce any notable vibration in the beam hence an amplifier is used to amplify the signal the range of amplification can be varied using the knob provider at the amplifier but we should not amplify more than the safe limit of the exciter and also the quality of the vibration will be degraded and also the PZT patches may be damaged [8]. Vibration produces deflection in the beam which is maximum at the free end, and to measure this deflection scanning laser Doppler vibrometer is used, it is very accurate and can record even the smallest deflection which is produced in the beam. The details of the smart beam along with the details of the PZT patches considered in this work are given in Table 1.

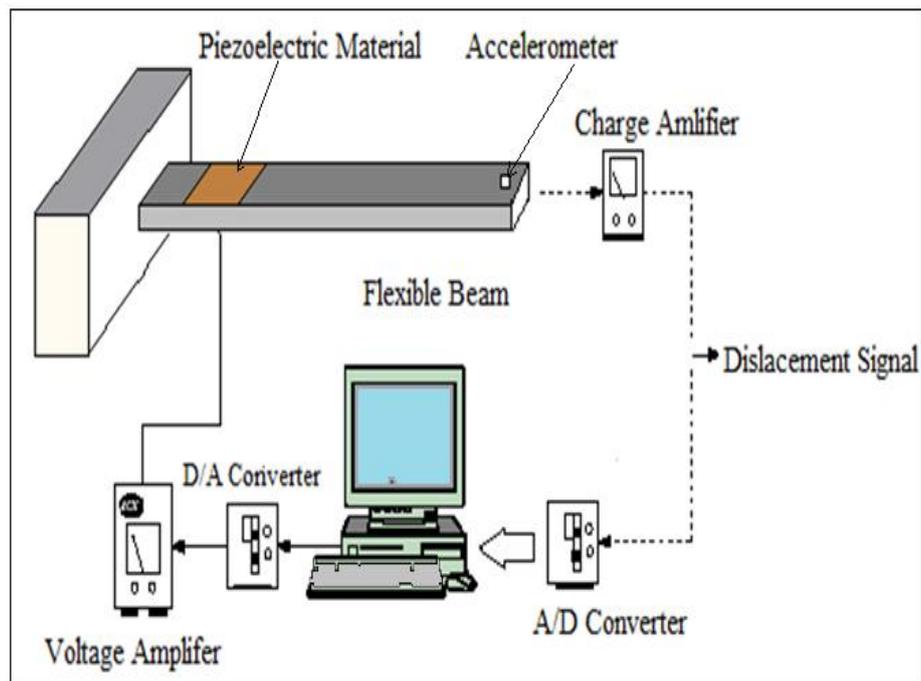


Fig.2. Experimental set up for vibration control of cantilever beam

Table.1 Specification of beam with PZT patches

Parameter	Cantilever Aluminum Beam	Piezoelectric Sensor/ Actuator
Length (L)	0.45m	0.015m
Width (b)	0.025m	0.015m
Thickness (d)	0.0025m	0.0005m
Young's Modulus(E)	$70 \times 10^9 \text{ N/m}^2$	$E_p = E_a = E_s = 6.66 \times 10^{10} \text{ N/m}^2$
Density( $\rho$ )	$2700 \text{ kg/m}^3$	$\rho_p = \rho_a = \rho_s = 7400 \text{ kg/m}^3$
Piezoelectric Stress Constant (PZT)		$g_{31} = 8.5 \times 10^{-3} \text{ Vm/N}$
Piezoelectric Strain Constant (PZT)		$d_{31} = 265 \times 10^{-12} \text{ C/N}$
Damping Constant	$\alpha = 0.001$ and $\beta = 0.0001$	

### III. FINITE ELEMENT FORMULATION OF SMART BEAM ELEMENT

When PZT patches are assumed as Euler-Bernoulli beam elements the elemental mass and stiffness matrices of PZT beam element can be computed as shown in equation (1) and (2),

$$M = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & -4l^2 \end{bmatrix} \quad (1)$$

$$K = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (2)$$

In this work, common parameters of the system were determined theoretically and were validated in through the experimental process. From the concepts of machine vibrations, the natural frequency of the vibrating beam was determined by the following equation (3) and (4).

$$\omega_n = \sqrt{\frac{K}{m}} \quad (3)$$

$$\text{Beam Inertia } (I) = I = \frac{bd^3}{12} \quad (4)$$

Where,  $\omega_n$  = natural frequency of the beam (radian/sec) =  $2\pi n f$ ; 'K' = the beam stiffness ( $N/m^2$ );  $m$  = the modal mass of the beam (kg); 'b' is width of beam and 'd' is thickness of beam.

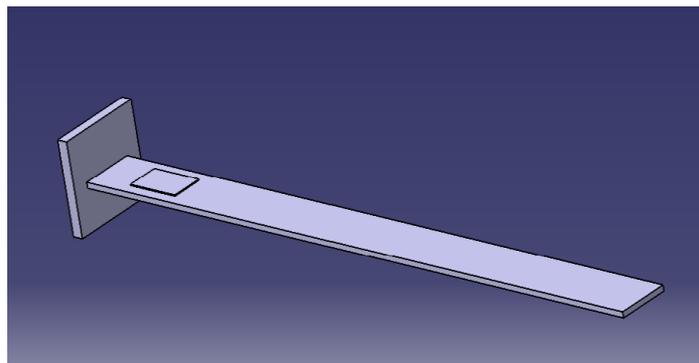
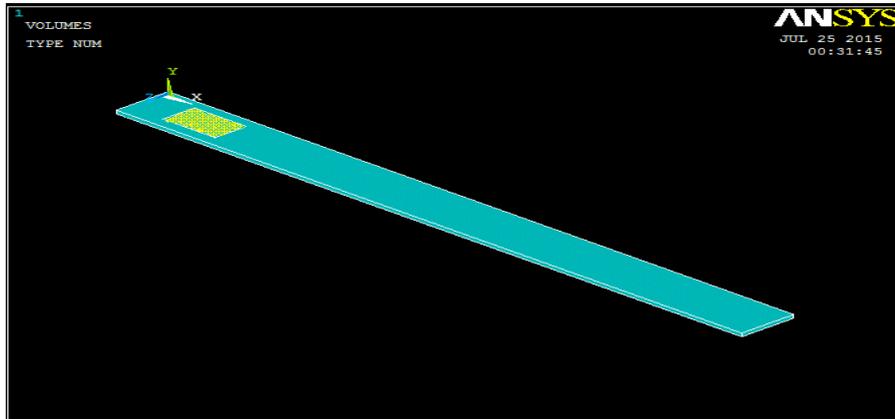


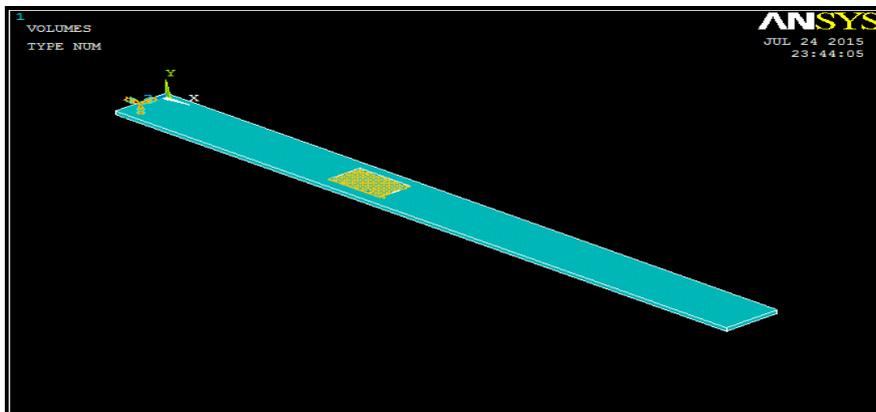
Fig.3 Smart cantilever beam with PZT patches modeling in CATIA software

#### IV. MODELING OF SMART CANTILEVER BEAM

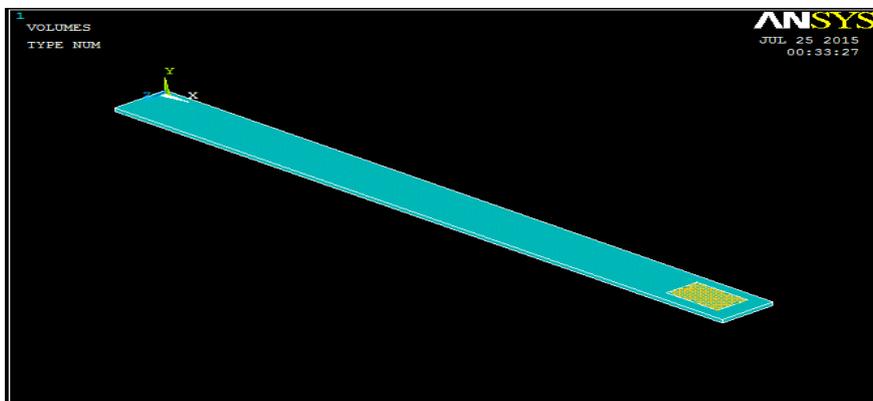
Cantilever beam modeled using ANSYS as shown in Fig.4. The PZT patches have been placed at various positions along the length of the beam such as 30 mm, 100 mm and 250 mm from the fixed end. The influences of these positions over the settling time of the beams have been studied.



(a) 30mm from fixed end



(b) 100mm from fixed end



(c) 250mm from fixed end

Fig.4 Position of PZT patch from the fixed end of the beam

The typical finite element used in the modeling and analysis of piezoelectric patch was (SOLID5), which has piezoelectric capacity in three dimensional couple field problem. Like other structural solid elements, this element has three displacement degrees of freedom per node. In addition to this degree of freedom, the element has also potential degree for analyzing of the electromechanical coupling problems.

Piezoelectric actuator inherently exhibits anisotropic and yield three-dimensional spatial vibration in their response to the piezoelectric actuation. The models developed for the passive portion includes consistent degree of freedom at the location where these elements interface. For modeling the passive portion of the smart structure solid element used is (SOLID45). The passive portion is made of aluminum.

**V. MODEL ANALYSIS AND DEVELOPMENT OF CONTROL LAWS**

Modal analysis was performed on the aluminum beam to find out the natural frequency of the structure. The analysis was further carried out for both passive and active structures. Table 2 presents the first four natural frequencies of aluminum and composite beams or structures. It can be inferred that the addition of PZT patch increases both the mass and stiffness of the system. But the increase was not proportional, causing the natural frequency to increase. If they had proportionally increased, the natural frequency would have remained constant. The natural frequency of the beams can be validated analytically by using the equation (5) and (6) [9].

$$\omega_n = (\beta l)^2 \sqrt{\frac{EI}{\rho A l^4}} \tag{5}$$

$$f = \frac{\omega_n}{2\pi} \tag{6}$$

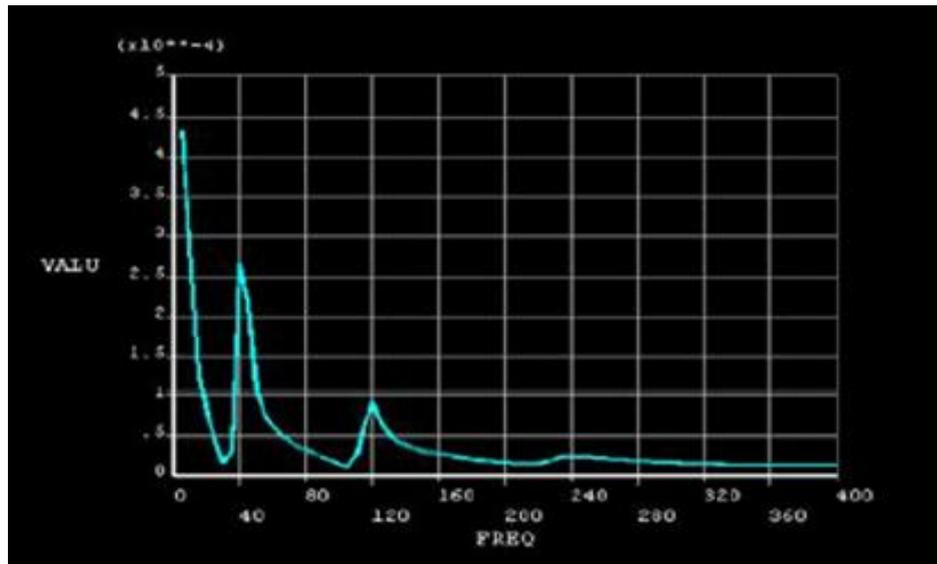


Fig.5 Harmonic response of cantilever beam

Table 2 Natural frequencies of beam

Modes	Natural Frequency of Aluminum (Hz)	
	Passive Beam	Active Beam
First Mode	6.64	6.62
Second Mode	41.20	41.14
Third Mode	115.5	114.4
Fourth Mode	226.26	225.2

The harmonic response analysis was used to determine the steady response of the linear structure under the harmonic loads. Under normal circumstances, the PZT patches were actuated by a sine-wave power from the power supply. This kind of PZT structure coupled analysis accorded with the conditions of the harmonic response analysis. Fig.5 shows the response of harmonic analysis of the aluminum and composite beams. It can be noticed that the peak occurs in the frequencies corresponding to the frequencies found by using modal analysis.

From these figures, it can be inferred that only the vibration modes corresponding to first, second and fourth modes have been obtained. This is due to the fact that they correspond to the bending loads, since bending load is only applied. Vibration modes corresponding to the third and fifth natural frequencies would rise while applying the torsion loads. Only, when bending loads are applied, their corresponding natural frequencies are validated.

### VI. DETERMINATION OF OPTIMUM POSITION OF PZT

The position of PZT over the aluminum is simulated by using finite element code to find out optimum location of PZT, resulting in improved vibration control. Fig.6, Fig.7 and Fig.8 show the settling time of aluminum beam for the positions of piezoelectric 30 mm, 100 mm and 250 mm respectively. From these, it can be clearly noticed that the settling of the beam is minimum when piezoelectric patch is located at the distance of 30 mm from the fixed end of the beam.

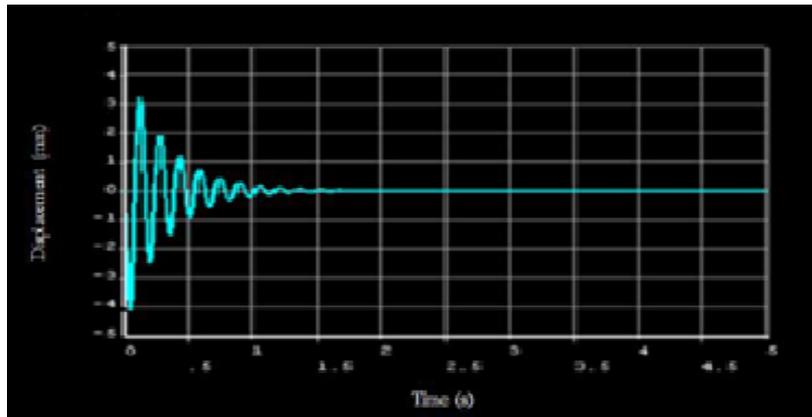


Fig. 6 Settling time of aluminum beam when PZT at 30 mm

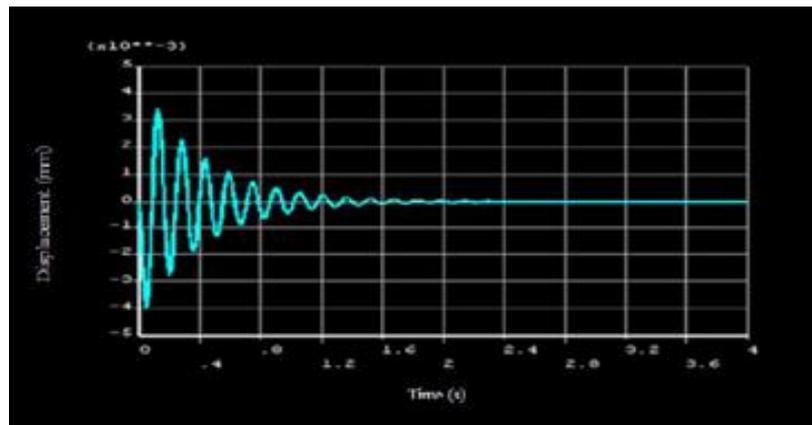


Fig.7 Settling time of aluminum beam when PZT at 100 mm

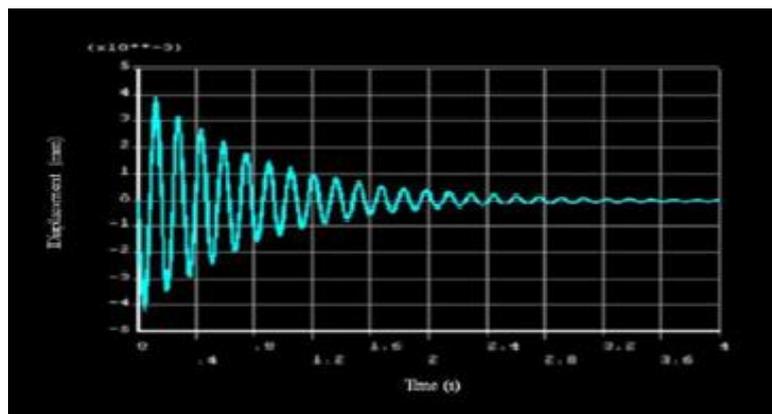


Fig.8 Settling time of aluminum beam when PZT at 250 mm

This is due to the position of piezoelectric in the high strain region resulting in more controlled reduction in the tip displacement of the structure. Similarly, the positions of piezoelectric at 100 mm and 250 mm have more settling time. Settling time values of aluminum structure for the various locations from the fixed end of the beam is shown in Table 3 and Fig. 9. Beam with PZT patch at 30mm from fixed end shows less settling time than patches at distance 100mm and 250mm from fixed end as shown in Fig.9. Hence optimum position of PZT patch for maximum vibration control of beam is 30mm from fixed end.

**VII. RESULT**

Present work deals with modeling and time response of the beam structure is studied after bonding the sensor/actuator pair at different locations on the beam says near the fixed end, at the middle and at the free end. It has been observed that without control the transient response is predominant and with control laws, sufficient vibrations attenuation can be achieved. The results are shown in Fig.6, 7 and 8 respectively. Settling time of beam was less for patch at 30mm from fixed end than other. It was clearly show the uncontrolled and controlled vibration of beam without and with PZT patches respectively in Fig. 9.

Table 3.Settling time of aluminum beam with various positions of PZT

Patch Distances (mm)	Beam with PZT patch (Controlled) Settling Time (sec)	Beam without Patch (Uncontrolled) Settling Time (sec)
30	1.6	1.9
100	2	2.2
250	3.4	3.6

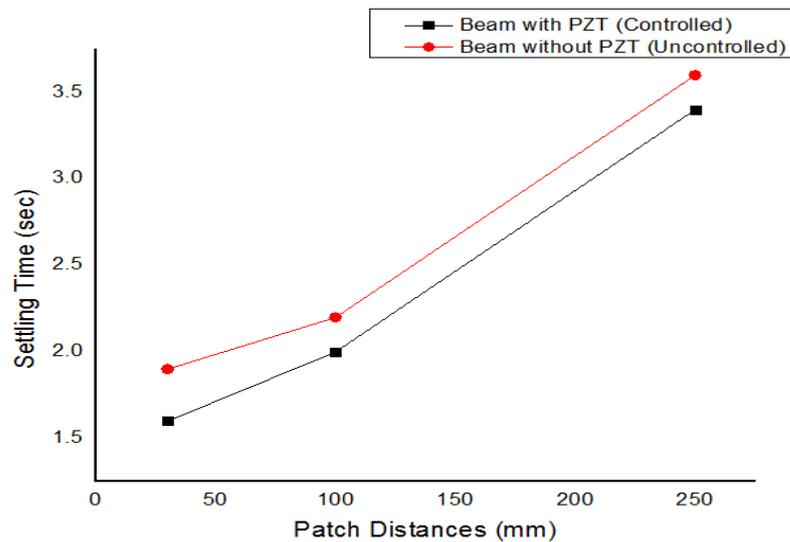


Fig.9 Settling time vs PZT patch distances from fixed end of beam

**VIII. CONCLUSION**

Finite element modeling for the closed loop control system has been developed by using ANSYS. Modal analysis and harmonic analysis have been carried out to find the undamped natural frequencies of the system and it is compared with the analytical results. From this, the optimal location of the sensor and actuator was found by taking into consideration the clamping distance and settling time of beam. From this study optimal position of patch was found near fixed end. Settling time was reduced 18.75%, 10% and 5.8% when PZT patches bounded 30mm, 100mm and 250mm from fixed end of beam as shown in Table 3. So effective vibration control of beam was achieved when PZT patch at 30 mm from fixed end. With the modal frequencies as input, time step is calculated for closed loop transient analysis. This experiment helps well into many applications of aerospace like airplane wings, helicopter propellers and structural engineering.

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