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An Accuracy Assessment of A Braced Quadrilateral Figure Using Equal Shift and Least Squares Method of Adjustment

Faruk Ibrahim Amale[1] Abbas Ibrahim[2] Samuel Yohanna[3]and UsmanDahiru Bodejo[4]

[1-3] Department of Surveying and Geoinformatics, ModibboAdama University Yola, Nigeria
[4] Department of Urban and Regional Planning, ModibboAdama University Yola, Nigeria
Corresponding Author: Faruk Ibrahim Amale. E-mail: farukiamale@gmail.com

ABSTRACT

This project is concerned with an accuracy assessment of a braced quadrilateral figure using equal shift and least square methods of adjustment. It is also for determining among the two methods which is more accurate, more reliable and time saving in adjusting a braced quadrilateral, so as to know which method is best suited for the figure since Nigerian network is documented by triangulation scheme rather than any networks. The braced quadrilateral was established in the field, total station was used to observed the bearings of all the stations, from the observed bearings, the internal angles were deduced, adjustment of the internal angles of the braced quadrilateral was done by equal shift method and conditional equation method of least square adjustment. An independent samples test was carried out to test the degree of accuracy of the two adjustments. It is found from the statistical analysis that, both the equal shift and conditional equation methods of adjustments are almost equal in terms of degree of accuracy and are good for adjusting braced quadrilateral. Based on the outcome of the statistical analysis, the conditional equation method of least squares and equal shift method of adjustments can be used to adjust the triangulation network. It is recommended that further research should be conducted to compare different methods of adjustment using different programming languages to draw final conclusion on this.

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1. INTRODUCTION

Surveying being an essential element in the development of the human environment for so many centuries that its importance is often forgotten, has been defined as the art, science and technology of making precise measurements and observations so as to determine position of points on or below the earth surface with reference to a specified datum, and later these measurements can be stored, retrieved, processed, managed and presented in a digital format in order to form map, chart or plans for specified use.It is an imperative requirement in the planning and execution of nearly every form of construction. Surveying was essential at the dawn of history, and some of the most significant scientific discoveries could never have been implemented were it not for the contribution of surveying. Its principal modern uses are in the fields of transportation, building, apportionment of land, and communications.(Wright & Lyman 1982).

A traverse is a survey where you have occupied each station and measured each angle and distance between points.One of the common procedures for providing the rightframework is to create surveying networks which are a set ofconsecutive points or stations that are coordinated by successivemeasurements of lengths and angles, and form a network ofhorizontal canvas points. The main purpose of the survey is to fixspecified points to help link the subsidiary points together (Chrzanowski, 1965). Moreover, the accuracy of the surveypoints should be far greater than the accuracy of the subsidiarypoints. Depending on the geometry of the points, the surveynetworks are divided into two categories: open and closedtraverse networks. In open traverses, the startandendpointsofthe traverse are two separate points, while in closed ones, the startand end points coincide.

In triangulation project, it is necessary to carryout field evaluation as the preliminary stages so as to determine four braced quadrilateral figures for the network so as to ensure uniform accuracy throughout the network. The braced quadrilateral figure is used in determining the degree of precision of computation. While triangles are the basic components of triangulation network, they do not provide check on the measurement and subsequent computation. Triangles are therefore combined to form other figures that are geometric in nature. Projects are proposed in localities in which existing controls are either too sparse or too distant for an order of accuracy required by the project specification. In such circumstance, it may be necessary to extend the existing triangulation network or to provide a new local system based on an arbitrary origin.(Olson, 2014). In either case, the coordinates of points in the triangulation network has to be computed, random errors in the angular and linear observation have to be distributed so as to make the figures comprising the network geometrically consistent and strong, using two methods of adjustment that is Equal shift and Least squares methods so as to do assessment of an accuracy of the methods in braced quadrilateral figures.

2. REVIEW OF RELATED LITERATURE

The accuracy assessment of four braced quadrilateral figures using Bowditch, Transit and Least squares methods of adjustments. Because of its roles in triangulation evaluation, some researchers saw the need to investigate on the subject.

According to Banister and Raymond (1984), reconnaissance should be carried out carefully for every triangulation scheme so as to select suitable points for control stations.

Least squares adjustment is said to be the process of obtaining the least squares estimates. The need for a least squares adjustment arises when several measurements are made in excess of the minimum required for unique determination of the unknown quantities of network points. This lead to an over-determined system of equations. This procedure is meant to serve two purposes; firstly, it helps in the detection of blunders that need to be eliminated before any final least squares adjustment. Secondly, it is well known statistically that the standard error of the mean is smaller and thus a better estimate than the standard error of a single observation (Moka, *et al*., 2007).

(Amini&Mehrdad, 2020) concluded that the Least Squares methodin most cases gave the best solution among the other methods.When performing a traverse, the Bowditch method in many vertices gave a similar answer to the Least Squares method,which in fact indicates a good propagation of the observationerrors between the observations in this method. In addition, in theDouble-braced Quadrilateral method, the results were always thesame with the Least Squares with constant weight, which alsoresults from the proper error propagation in this method.

A least square adjustment technique was originated independently by Gauss and Legendre over 150 years ago, (Madkour, 1968). Its application to triangulation network of braced quadrilateral figure was scares probably because of the lengthy calculation involved. Nowadays, the emergence of electronic and digital computer has generated interest in the used of least square adjustment for braced quadrilateral of triangulation network and other survey problems. This is perhaps because of the ease in calculation nowadays. Computation which requires hours to be completed which can now be done by the electronic and digital computers in few seconds.

Surveyor have introduced new approach to the technique of least square for braced quadrilateral, triangulation adjustment in an effort to accommodate the complexity of present days' measurements. There has been some publication on the use of least square and non-least square techniques others discussed.

All the adjustment methods are equal, meaning that they lead to the same adjustment results regardless the specific method. The choice of one or another methoddepends on the type of the functional model. In the following, we will examine thethree basic adjustment methods, namely: the method of observation equations (themethod of parameters or the method of indirect observations), the method of condition equations (the method of direct observations) and the method of mixed orcompound equations. (Mikhail, 1976).

3. METHODOLOGY

Preamble

The methodology of this project involves the step by step procedure that was adopted to achieve the aim of the research. The general procedure is shown in the methodology flowchart in Figure 3.1.

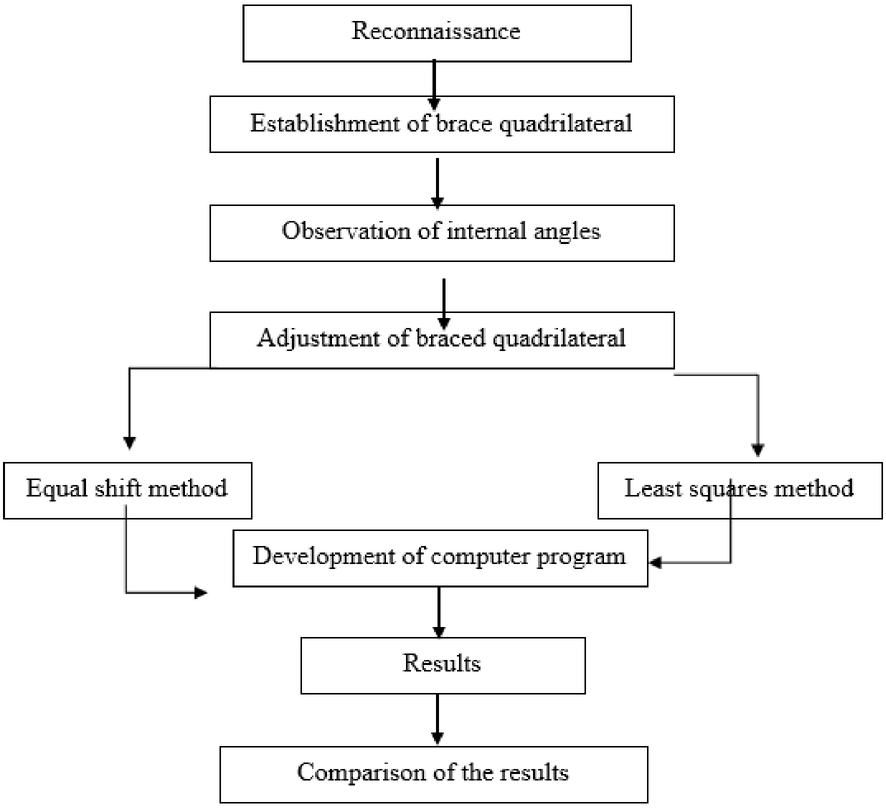

Figure 3.1: Methodology flowchart

3.2 Hardware and Software

The following hardware were used during the project.
i. HP EliteBook Folio 9470m Laptop Computer (Windows 10 Pro, 500GB HDD, 8GB RAM, Core™ i5 CPU @1.90GHz 2.40Ghz Processor Speed).
ii. Total Station
iii. Tripod stand
iv. Prism reflector
v. Steel tape (100m)
vi. Ranging poles
vii. Field book
viii. Wooden pegs

The following software was used during the project.
i. MATLAB
ii. SPSS 20

3.3 Data Acquisition

Data acquisition is the process of capturing data that will uniquely define a point and its position. In other words, it is the act of measuring the relative position of station and mark, overhead details and underground detail with reference to a particular datum with a view of representing them on paper map. For this to be achieved, the whole operation must start from an identified origin. The data used for this project work was obtained from two different sources; the primary data source and the secondary data source.

3.3.1 Primary data source

Primary source of data including existing maps of the area concerned, coordinates of existing controls and benchmarks was obtained from the physical planning unit of MAU Yola.

3.3.2 Secondary data source
This is the set of data obtained from the site by means of direct field observation. In this project, secondary data was obtained by means of observation to determine the coordinates of the various points.

3.4 Establishment of a braced quadrilateral figure
Braced quadrilateral figure is a triangulation figure in which the individual triangle overlap each other. A four braced quadrilateral figure as shown in Figure 3.2 was established in the field in order to achieve the aim of the project.

3.4.1Reconnaissance
The most important state of triangulation project is the preliminary reconnaissance which involves office and field reconnaissance.

3.4.1.1 Office reconnaissance
The office reconnaissance may include collection of information on cost of the survey work, access of various triangulation stations, transport facilities, availability of food and water, availability of labour, camping ground, examination of terrain to be surveyed, e.t.c.
3.4.1.2 Field reconnaissance
Field reconnaissance consists of selection of stations to be occupied and the number of angles or directions to be observed, the intervisibility and accessibility of the stations, the selection of suitable sites for measurement of baselines, selection of well-defined natural points to be used as intersected points, the usefulness of the station in later works and the convenience of the base line measurements.

3.4.1.3 Factors to consider during station selection
i. The stations should be mutually visible.
ii. Main principle of chain survey should strictly be observed.
iii. If possible, line through the whole length of the area be drawn.
iv. All triangles should be well-defined.
v. A check line should be provided in each triangle.
vi. Survey lines should be as few as possible.
vii. Position of survey lines should be such that to avoid obstacles during observation.

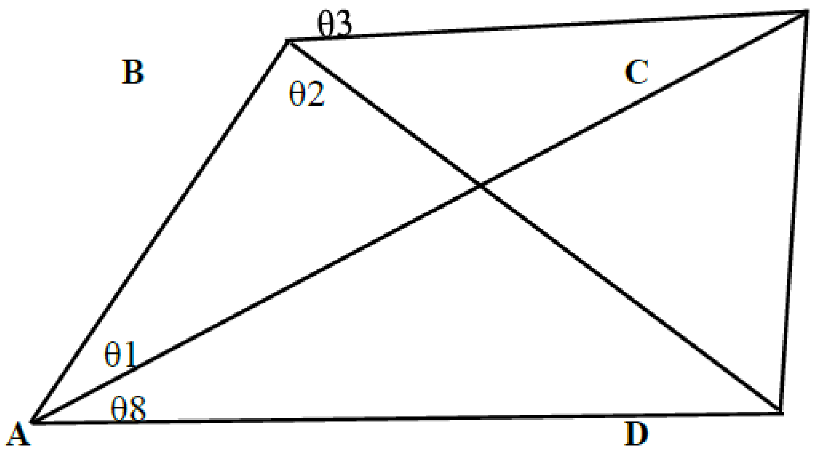

Figure 3.2:A Braced Quadrilateral Figure.

3.5 Data observation and acquisition
Before commencement of any triangulation project, there are certain survey operations that are needed to be carried out in order obtain a required degree of accuracy. These operations include: reconnaissance, in-situ check, baseline measurement e.t.c.

3.5.1 In-situ check
Before any controls to be used in survey work it has to be checked to ensure that they are in their proper and accurate position in terms of angular measurement and distance from the time of establishment to present day.

The processes involved in achieving these tasks are called in-situ check were carried out on the following stations.

3.5.2 Data observation

During the in-situ check, total station instrument was set on the existing control (EJS 005) and centered and leveled. The angular mode was first displayed by total station and orientation also set by inputting the back bearing of the back station (EJS 001). Later the mode was changed from angular mode to coordinate mode by pressing the coordinate mode icon. Then f4 was pressed to change the observation page (page 1) to inputting page (page 2) and coordinates of the occupied station were inputted in the instrument after pressing f3. Then f2 was pressed to input the height of reflector and later press f1 and input the height of the instrument.
A reflector was set on the next control station to be checked and bisected with the telescope of the instrument, automatically the coordinates of that station were displayed, the observation was recorded. After checking, then the reflector was moved to the next station and was bisected with the telescope of the instrument, automatically the coordinate of that station was displayed and was recorded. This procedure was repeated until the all the stations were covered and also recorded.

3.5.3 Observation of internal angles

For every station of the four points of the braced quadrilateral, angles were observed. On station 1, the instrument was set and leveled and all the necessary adjustments were made on the face left. The reflector was set on another station and bisected by the telescope of the instrument and automatically it displayed the bearings and were recorded. The reflector was moved to another station and bisected and bearings are also recorded, the reflector was again moved on all the stations until the bearings of all the stations of the braced quadrilateral were recorded. From the bearings observed, the internal angles of the braced quadrilateral were deduced using the following formulae:

$$\theta 1 = \text{BearingAC} - \text{BearingAB} \quad (3.1)$$
$$\theta 2 = \text{BearingBA} - \text{BearingBD} \quad (3.2)$$
$$\theta 3 = \text{BearingBD} - \text{BearingBC} \quad (3.3)$$
$$\theta 4 = \text{BearingCB} - \text{BearingCA} \quad (3.4)$$
$$\theta 5 = \text{BearingCA} - \text{BearingCD} \quad (3.5)$$
$$\theta 6 = \text{BearingDC} - \text{BearingDB} \quad (3.6)$$
$$\theta 7 = \text{BearingDB} - \text{BearingDA} \quad (3.7)$$
$$\theta 8 = \text{BearingAD} - \text{BearingAC} \quad (3.8)$$

3.6 Adjustment of the braced quadrilateral

For angle condition, it is expected that the sum of the internal angles of each of the braced quadrilateral figure should sum up to 360^0

$$(\theta 1 + \theta 2 + \theta 3 + \theta 4 + \theta 5 + \theta 6 + \theta 7 + \theta 8) = 360° \quad (3.9)$$

The adjustment of the quadrilateral was made through the following methods:

i. Equal shift
ii. Least squares method of adjustment

3.6.1 Adjustment by equal shift

The method of equal shifts indicates that any shift which is necessary to satisfy the local equation should be the same for each triangle of the polygon. Similarly, any shift necessary to satisfy the side equation should be the same for each triangle. In equal shift, the misclosure that arises from the first angle is distributed equally among the total number of angles observed. In equal shift;

i. The misclosure arising from the first and second angles condition were distributed by dividing the error by four, then subtracting the value from those angles whose sum is greater and adding the same value to those angles whose sum is less than the other.
ii. The misclosure that arises from the third angle condition is distributed equally among the total number of angles observed. And this does not have effect on the first adjustment.
iii. These corrections are applied to observed angles to give preliminary adjusted angles.

iv. The log sin of these preliminary adjusted angles are found and their log sin difference 1" the sum of log sine odd numbered angles (i.e. 1, 3, 5 & 7) were summed. Those of even numbered angles (i.e. 2, 4, 6 & 8) were also added up.

v. The sum of log sine difference 1" was also obtained. The difference (Σodd – Σeven) was obtained. This value was divided by the adjustment to that which was less will have positive correction. The sum of the log sine that is greater was applied negative correction as well.

3.6.3 Adjustment by conditional equation method of least square

This method is applicable only when the equation can be expressed as a relation between the adjust value of the observed quantities alone. For the triangulation network adjustment of four braced quadrilateral figures, below is the method to be applied for each of the braced quadrilateral.

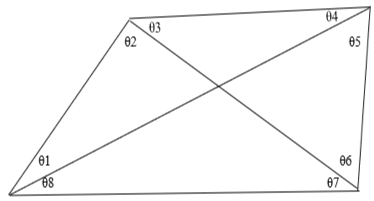

Figure 3.3: Established braced quadrilateral in the field

From the above figure 3.3, we obtained the following condition equations;

$(\theta 7 + \theta 8) = (\theta 3 + \theta 4)$ $\quad$ $(\theta 7 + \theta 8) - (\theta 3 + \theta 4) = 0$ (3.10)

$(\theta 5 + \theta 6) = (\theta 1 + \theta 2)$ $\quad$ $(\theta 5 + \theta 6) - (\theta 1 + \theta 2) = 0$ (3.11)

$(\theta 1 + \theta 2 + \theta 3 + \theta 4 + \theta 5 + \theta 6 + \theta 7 + \theta 8) = 360°$

$360° - (\theta 1 + \theta 2 + \theta 3 + \theta 4 + \theta 5 + \theta 6 + \theta 7 + \theta 8) = 0$ (3.12)

$\delta 1v1 - \delta 2v2 + \delta 3v3 - \delta 4v4 + \delta 5v5 - \delta 6v6 + \delta 7v7 - \delta 8v8 = 0$ (3.13)

Standard Deviation (σ) = $\sqrt{\frac{\sum(x_i - \mu)^2}{N}}$ (3.14)

Standard Error = $\frac{\sigma}{\sqrt{N}}$ (3.15)

IV. RESULTS AND DISCUSSION

4.1 Presentation of Results

The results are presented in tables, charts and figures. Table 4.1 presented the observed bearings from the field and the internal angles deduced from the bearings. Table 4.2 presented the final adjusted internal angles by equal shift method. Table 4.3 presented deduced internal angles and computed standard error for conditional equation method of adjustment. Table 4.4 presented Log Sin of deduced internal angles. Table 4.5 presented Final adjusted internal angles by condition equation method. Table 5.6 presented Adjusted internal angles by both equal shift and condition equation. Table 4.7 presented statistical analysis result using independent samples test. Figure 4.1 presented a graph of final adjusted internal angles by equal shift method plotted against final adjusted internal angles by conditional equation method.

Table 4.1: Observed bearings and deduced internal angles.

From	To	Bearing	Deduced Internal Angle (θ)	Angle (θ)
A	B	40° 50' 24"	19° 11' 23.0"	1
A	C	62° 22' 32"	64° 39' 21.0"	2
A	D	92° 20' 45"		
B	C	105° 34' 08"	52° 57' 40.0"	3
B	D	158° 31' 48"	43° 11' 26.5"	4
B	A	223° 11' 09"		
C	D	194° 01' 10.5"	48° 21' 31.0	5
C	A	242° 22' 41.5"	35° 29' 12.5"	6
C	B	285° 34' 08"		
D	A	272° 21' 02.5"	66° 10' 55.5"	7

D	B	338° 31' 58"	29° 58' 13.0"	8
D	C	14° 01' 10.5"		

4.1.1 Adjustment by Equal Shift

(θ7 + θ8) = (θ3 + θ4)   (θ7 + θ8) - (θ3 + θ4) = 0   (4.1)

(θ5 + θ6) = (θ1 + θ2)   (θ5 + θ6) - (θ1 + θ2) = 0   (4.2)

(θ1 + θ2 + θ3 + θ4 + θ5 + θ6 + θ7 + θ8) = 360°   (4.3)

The adjustment by equal shift must satisfy the above equations.

i. The misclosure arising from the first and second angles condition were distributed by dividing the error by four, then subtracting the value from those angles whose sum is greater and adding the same value to those angles whose sum is less than the other.

For the first angle condition;

(66° 10' 55.5" + 29° 58' 13.0") – (52° 57' 40.0" + 43° 11' 26.5") = 2”

2”/4 = 0.5”

Therefore, 0.5” was subtracted to those angles whose sum is greater and added to those angles whose sum is less than the other.

The sum of θ7 and θ8 is greater, therefore 0.5” was subtracted from each;

θ7 = 66° 10' 55.5" – 0.5” = 66° 10' 55"

θ8 = 29° 58' 13.0" – 0.5” = 29° 58' 12.5"

The sum of θ3 and θ4 is less than, therefore 0.5” was added to each;

θ3 = 52° 57' 40.0" + 0.5” = 52° 57' 40.5"

θ4 = 43° 11' 26.5" + 0.5” = 43° 11' 27"

For the second angle condition;

(48° 21' 31.0 + 35° 29' 12.5") – (19° 11' 23.0" + 64° 39' 21.0") = -0.5”

0.5”/4 = 0.125”

Therefore, 0.125” will be subtracted to those angles whose sum is greater and added to those angles whose sum is less than the other.

The sum of θ5 and θ6 is less than, therefore 0.125” was added to each;

θ5 = 48° 21' 31.0” + 0.125” = 48° 21' 31.125”

θ6 = 35° 29' 12.5" + 0.125” = 35° 29' 12.625”

The sum of θ1 and θ2 is greater, therefore 0.125” was subtracted from each;

θ1 = 19° 11' 23.0" – 0.125” = 19° 11' 22.875”

θ2 = 64° 39' 21.0" – 0.125” = 64° 39' 20.875”

To check;

(θ7 + θ8) - (θ3 + θ4) = 0

(66° 10' 55" + 29° 58' 12.5") – (52° 57' 40.5" + 43° 11' 27") = 0

(θ5 + θ6) - (θ1 + θ2) = 0

(48° 21' 31.125” + 35° 29' 12.625”) – (19° 11' 22.875” + 64° 39' 20.875”) = 0

ii. The misclosure that arises from the third angle condition is distributed equally among the total number of angles observed. And this does not have effect on the first adjustment.

(θ1 + θ2 + θ3 + θ4 + θ5 + θ6 + θ7 + θ8) = 360°

360° - (θ1 + θ2 + θ3 + θ4 + θ5 + θ6 + θ7 + θ8)

360° - (19° 11' 22.875” + 64° 39' 20.875” + 52° 57' 40.5" + 43° 11' 27" + 48° 21' 31.125” + 35° 29' 12.625” + 66° 10' 55" + 29° 58' 12.5") = 17.5”

17.5”/8 = 2.1875”

Therefore, 2.1875” was added to all the angles since the misclosure is positive;

θ1 = 19° 11' 22.875” + 2.1875” = 19° 11' 25.0625”

θ2 = 64° 39' 20.875” + 2.1875” = 64° 39' 23.0625”

θ3 = 52° 57' 40.5" + 2.1875” = 52° 57' 42.6875”

θ4 = 43° 11' 27" + 2.1875” = 43° 11' 29.1875”

θ5 = 48° 21' 31.125” + 2.1875” = 48° 21' 33.3125”

θ6 = 35° 29' 12.625” + 2.1875” = 35° 29' 14.8125”

θ7 = 66° 10' 55" + 2.1875” = 66° 10' 57.1875”

θ8 = 29° 58' 12.5" + 2.1875 = 29° 58' 14.6875”

Therefore, the final adjusted observations are;

Table 4.2: Final adjusted internal angles by equal shift method.

Angle (θ)	Deduced Included Angle (θ)	Adjusted Observations
1	19° 11' 23.0"	19° 11' 25.0625"
2	64° 39' 21.0"	64° 39' 23.0625"
3	52° 57' 40.0"	52° 57' 42.6875"
4	43° 11' 26.5"	43° 11' 29.1875"
5	48° 21' 31.0	48° 21' 33.3125"
6	35° 29' 12.5"	35° 29' 14.8125"
7	66° 10' 55.5"	66° 10' 57.1875"
8	29° 58' 13.0"	29° 58' 14.6875"

To check;
$360° - (\theta 1 + \theta 2 + \theta 3 + \theta 4 + \theta 5 + \theta 6 + \theta 7 + \theta 8)$
360° - (19° 11' 25.0625" + 64° 39' 23.0625" + 52° 57' 42.6875" + 43° 11' 29.1875" + 48° 21' 33.3125" + 35° 29' 14.8125" + 66° 10' 57.1875" + 29° 58' 14.6875") = 0
$(\theta 7 + \theta 8) - (\theta 3 + \theta 4) = 0$
(66° 10' 57.1875" + 29° 58' 14.6875") – (52° 57' 42.6875" + 43° 11' 29.1875") = 0
$(\theta 5 + \theta 6) - (\theta 1 + \theta 2) = 0$
(48° 21' 33.3125" + 35° 29' 14.8125") – (19° 11' 25.0625" + 64° 39' 23.0625") = 0

4.1.2 Adjustment by Conditional Equation of Least Square

Table 4.3: Deduced internal angles and standard errors.

Angle (θ)	Internal Angle (θ)	Standard Error (σ^2)	Station
1	19° 11' 23.0"	4.2	A
2	64° 39' 21.0"	2.8	B
3	52° 57' 40.0"	2.8	B
4	43° 11' 26.5"	6.4	C
5	48° 21' 31.0	2.8	C
6	35° 29' 12.5"	3.5	D
7	66° 10' 55.5"	2.1	D
8	29° 58' 13.0"	2.8	A

Condition equations:
$(\theta 7 + \theta 8) = (\theta 3 + \theta 4) \qquad (\theta 7 + \theta 8) - (\theta 3 + \theta 4) = 0 \qquad (4.4)$
$(\theta 5 + \theta 6) = (\theta 1 + \theta 2) \qquad (\theta 5 + \theta 6) - (\theta 1 + \theta 2) = 0 \qquad (4.5)$
$(\theta 1 + \theta 2 + \theta 3 + \theta 4 + \theta 5 + \theta 6 + \theta 7 + \theta 8) = 360°$
$360° - (\theta 1 + \theta 2 + \theta 3 + \theta 4 + \theta 5 + \theta 6 + \theta 7 + \theta 8) = 0 \qquad (4.6)$
$\delta 1v1 – \delta 2v2 + \delta 3v3 – \delta 4v4 + \delta 5v5 – \delta 6v6 + \delta 7v7 – \delta 8v8 = 0 \qquad (4.7)$
Log sin (θ1) + Log sin (θ3) + Log sin (θ5) + Log sin (θ7) – Logsin (θ2) – Log sin (θ4) – Log sin (θ6) – Log sin (θ8)
Where;

$\delta i = [\text{Log Sin } (\theta+1'') - \text{Log Sin } (\theta)]$

Table 4.4: Log Sin of deduced internal angles.

Angle (θ)	Obs. Angle (θ)	Log Sin (θ+1")	Log Sin (θ)	δi (x 10^{-6})
1	19° 11' 23.0"	-0.483197895	-0.483203945	6.050
2	64° 39' 21.0"	-0.043949323	-0.04395032	0.997
3	52° 57' 40.0"	-0.097872079	-0.097873668	1.589
4	43° 11' 26.5"	-0.164669543	-0.164671786	2.243
5	48° 21' 31.0	-0.126492515	-0.126494387	1.872
6	35° 29' 12.5"	-0.236183256	-0.236186209	2.953
7	66° 10' 55.5"	-0.038657043	-0.038657972	0.929
8	29° 58' 13.0"	-0.301416793	-0.301420444	3.651

Designing the matrices needed for the adjustment

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 6.05\times10^{-6} & 0.997\times10^{-6} & 1.589\times10^{-6} & 2.243\times10^{-6} & 1.872\times10^{-6} & 2.953\times10^{-6} & 0.929\times10^{-6} & 3.651\times10^{-6} \end{bmatrix}$$

$$W = \begin{bmatrix} 2 \\ -0.5 \\ 17.5 \end{bmatrix}$$

$-1.213\text{x}10^{-6}$

$P = \text{diag.}\ [1/\sigma^2_1\ \ 1/\sigma^2_2\ \ 1/\sigma^2_3\ \ 1/\sigma^2_4\ \ 1/\sigma^2_5\ \ 1/\sigma^2_6\ \ 1/\sigma^2_7\ \ 1/\sigma^2_8]$

$P^{-1} = \text{diag.}\ [\sigma^2_1\ \ \sigma^2_2\ \ \sigma^2_3\ \ \sigma^2_4\ \ \sigma^2_5\ \ \sigma^2_6\ \ \sigma^2_7\ \ \sigma^2_8]$

$$P^{-1} = \begin{bmatrix} 17.64 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7.84 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7.84 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 40.96 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.84 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 12.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.41 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.84 \end{bmatrix}$$

To compute for matrix M;

$M = B\ P^{-1}\ B^T$

$$M = \begin{bmatrix} 74.2800 & 0 & -23.3200 & 0.0000 \\ 0 & 32.3400 & 0.9800 & 0.0000 \\ -23.3200 & 0.9800 & 106.6200 & 0.0003 \\ 0.0000 & 0.0000 & 0.0003 & 0.0000 \end{bmatrix}$$

To compute for matrix K;

$K = -\ M^{-1}\ W$

$$K = \begin{bmatrix} 74.2800 & 0 & -23.3200 & 0.0000 \\ 0 & 32.3400 & 0.9800 & 0.0000 \\ -23.3200 & 0.9800 & 106.6200 & 0.0003 \\ 0.0000 & 0.0000 & 0.0003 & 0.0000 \end{bmatrix}^{-1} \text{x} \ -0.5 \begin{bmatrix} 2 \\ 17.5 \\ -1.213\text{x}10^{-6} \end{bmatrix}$$

$$K = \begin{bmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ -4.6851 \end{bmatrix}$$

To compute for matrix V;

$V = P^{-1}\ B^T\ K$

$$V = \begin{bmatrix} -8.0783'' \\ 7.5157'' \\ 5.3412'' \\ -5.2204'' \\ 0.3635'' \\ 6.0111'' \\ 6.3458'' \\ 5.2214'' \end{bmatrix}$$

Finally, adding the residuals to the given observations, weobtain the adjusted observations:

$L_a = L_b + V$.

Table 4.5: Final adjusted internal angles by condition equation.

Angle (θ)	Observed Angle (θ)	Residuals (V)	Corrected Angle (θ)
1	19° 11' 23.0"	-8.0783"	19° 11' 14.9217"
2	64° 39' 21.0"	7.5157"	64° 39' 28.5157"
3	52° 57' 40.0"	5.3412"	52° 57' 45.3412"
4	43° 11' 26.5"	-5.2204"	43° 11' 21.276"
5	48° 21' 31.0	0.3635"	48° 21' 31.3635"
6	35° 29' 12.5"	6.0111"	35° 29' 18.5111"
7	66° 10' 55.5"	6.3458"	66° 11' 01.8458"
8	29° 58' 13.0"	5.2214"	29° 58' 18.2214"
Σ	359 59' 42.5"	17.5"	360° 00' 0.00"

Table 4.6: Adjusted internal angles by both equal shift and condition equation

Angle (θ)	Observed Angle (θ)	Adjusted Observations by Equal Shift	Adjusted Observations by Condition Equation
1	19° 11' 23.0"	19° 11' 25.0625”	19° 11' 14.9217"
2	64° 39' 21.0"	64° 39' 23.0625”	64° 39' 28.5157"
3	52° 57' 40.0"	52° 57' 42.6875”	52° 57' 45.3412"
4	43° 11' 26.5"	43° 11' 29.1875”	43° 11' 21.276"
5	48° 21' 31.0"	48° 21' 33.3125”	48° 21' 31.3635"
6	35° 29' 12.5"	35° 29' 14.8125”	35° 29' 18.5111"
7	66° 10' 55.5"	66° 10' 57.1875”	66° 11' 01.8458"
8	29° 58' 13.0"	29° 58' 14.6875”	29° 58' 18.2214"
Σ	359 59' 42.5"	360° 00’ 0.00"	360° 00’ 0.00"

4.1.3 Statistical Analysis of the Results

Independent samples test was carried out in order to measure the level of accuracy of the adjusted angle observations from the two method of adjustment (i.e Equal Shift and Conditional Method of Least Square Adjustment) used in the project.

Table 4.7: Independent Samples Test(Group Statistics)

	Groups	N	Mean	Std. Deviation	Std. Error Mean
observations	Equal shift	8	4.500000002E1	1.6455695669E1	5.8179669982E0
	Condition Equation	8	4.499999988E1	1.6456680049E1	5.8183150292E0

Table 4.8: Independent Samples Test (Inferential Statistics)

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Diff.
observations	Equal variances assumed	.000	1.000	.000	14	1.000	.0000001425	8.2280939331E0
	Equal variances not assumed			.000	14.000	1.000	.0000001425	8.2280939331E0

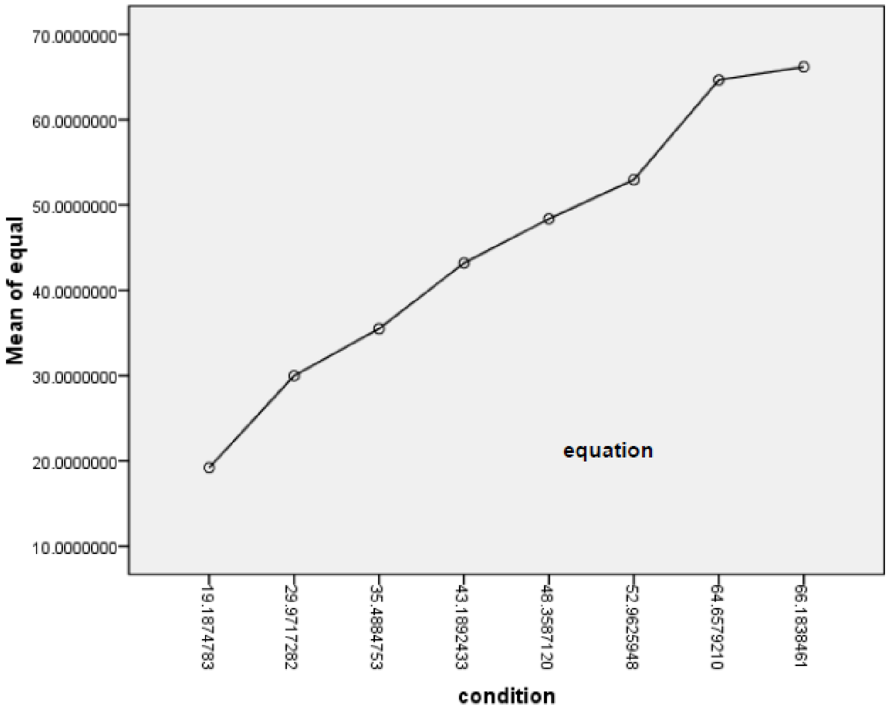

Figure 4.1: Graph of Equal Shift plotted against Condition Equation.

4.2 Discussion of Results

It can be seen from the Table 4.6 above that the adjusted observations obtained by both the equal shift and conditional equation method of least square are good for adjusting the internal angles of braced quadrilateral.
From Table 4.7 above, it can be seen that the mean, the standard deviation and the standard error of both the adjustment methods are almost equal, which means the level of accuracy of the two methods are almost equal and both are good for triangulation network scheme. From Table 4.8, it can be seen that the F-value is 1 which means the groups are not dispersed that is, equal variances are assumed to be equal. Also the value of t (t-test), df (degrees of freedom) and the sig. (2-tailed) are all equal which shows that both the methods of adjustment are equal in terms of degree of accuracy when adjusting a braced quadrilateral or triangulation network scheme.
Also considering the computing time, conditional equation method of least square method is slower than that of equal shift method of adjustment; therefore, equal shift method saves time.

V. RECOMMENDATIONS

It is recommended that in the adjustment of a triangulation network scheme of braced quadrilateral figures, the conditional equation method of least square and equal shift method of adjustment can be used. But when considering computing time, it is recommended that equal shift method of adjustment should be used in adjusting a braced quadrilateral

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