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Derivation Of Raleigh- Ritz Based Peculiar Total Potential Energy Functional (TPEF) For Asymmetric Single (ASS) Cell Thin- Walled Box Column (TWBC) Cross- Section.

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ABSTRACT

Determination of the critical buckling load is an important exercise in the analysis of instability in Thin-Walled Structures (TWS) , especially Thin-Walled Columns(TWC). This research work is an attempt to evaluate, as well as to formulate the peculiar Total Potential Energy Functional (TPEF) for Asymmetric Single (ASS) cell Thin -walled Box Column (TWBC) cross sections in lieu of the eventual stability analysis. By using Thin-walled assumptions, the cross sectional properties of ASS were first evaluated and the cross sectional area, A^{ASS} and Moments of Inertia, I^{ASS} obtained. Then , based on the derived general/governing RRM based TPEF for the TWBC by Nwachukwu and others (2017), the peculiar Total Potential Energy Functional (TPEF) for Asymmetric Single (ASS) cell Thin -walled Box Column (TWBC) cross sections was now formulated for different boundary conditions. The formulated Energy Functional Equations support the stability analysis of a ASS cell thin-walled box (closed) column cross-section using Raleigh - Ritz Method (RRM). The derived expressions will now be used to formulate series of stability matrices in subsequent ASS research works where the critical buckling load for ASS TWBC cross –section will be determined..

KEYWORD: *Asymmetric Single (ASS) Cell cross section, Total Potential Energy Functional (TPEF), Thin - Walled Box Column (TWBC) or Thin-Walled Column (TWC), Raleigh- Ritz Method (RRM),Bulkling/ Stability Analysis*

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1.INTRODUCTION

Stability represents a fundamental problem in solid mechanics, which must be mastered to ensure the safety of structures against collapse (Srinath, 2009). The theory of stability is of crucial importance for structural engineering, aerospace engineering, nuclear engineering, offshore, and ocean engineering. According to Bazant (2000), the theory of stability plays an important role in certain problems of space structures, geotechnical structures, geophysics and material science.. The continued importance and vitality of research on structural stability problem is due to technical and economic developments that demand the use of ever stronger and ever higher structures in an increasingly wider range of applications. According to Mohri and others (2008), such an expansion of use is made possible by developments in manufacturing, fabrication technology, computer- aided-design, economic competition and construction efficiency. These developments continually do not only change the way in which traditional structures are designed and built, but they also make possible the economic use of materials in other areas of application, such as offshore structures, transportation vehicles, and outer- space structures. In all these applications, need for higher strength and lighter weight are considerable factors. This gives rise to Thin- walled structures.

In a nutshell, a thin-walled structure (TWS) , according to Murray (1984), is one which is made from thin plates joined along their edges. The plate thickness for the TWS however is small compared to other cross sectional dimensions which are in turn often small compared with the overall length of the member or structure. Thin-walled columns (TWC) as well as other TWS are very light compared with alternative structures and therefore, they are used extensively in long-span bridges and other structures where weight and cost are prime considerations.

ASS are common examples of TWC cross –sections .According to Simao and Simoes da silva (2004), the use of very slender thin-walled cross-sections members have become increasingly in demand due to their high stiffness/weight ratio, in recent years. In general, according to Ezeh and Osadebe (2010), thin-walled structures consist of a wide and growing field of engineering application which seek efficiency and effectiveness in strength and cost by minimizing material. For about a century many branches of the industry have sought stronger and at the same time lighter structural solutions which optimize the effectiveness and the cost of the structures (Andreassen, 2012). Such industries cut across civil, offshore, mechanical, naval, and aerospace industries.

This present study is an attempt to evaluate and formulate the TPEF for ASS cross-sections. It is the follow up of the works by Nwachukwu and others (2017) and Nwachukwu and others (2021a) where the governing equation for the TPEF for a TWBC applicable to RRM and peculiar TPEF for DSS cross – section were derived respectively .Recently , many researchers have carried out one form of analysis or the other on thin- walled box columns and related topics. For instant, Krolak and others (2009) presented a theoretical, numerical and experimental analysis of the stability and ultimate load of multi-cell thin-walled columns of rectangular and square cross-sections subjected to axial compression. Shanmugam and others (1989) presented a numerical method to investigate the ultimate strength behavior of thin-walled steel box columns subjected to axial loads and biaxial end moments. The work of Ezeh (2009) involved a theoretical formulation based on Vlasov's theory as modified by Varbanov, in analyzing flexural, flexural-torsional, and flexural-torsional-distortional buckling modes of thin-walled closed columns. Chidolue and Osadebe (2012), also used Vlasov's theory to carryout Torsional- Distortional analysis of thin- walled box girder bridges. Chidolue and Aginam (2012) investigated the effects of shape factor on the Flexural- Torsional-Distortional behavior of thin- walled box girder structures using Vlasov's Theory. Ezeh (2010) also investigated the buckling behavior of axially compressed multi- cell doubly symmetric thin- walled column using Vlasov's theory. The works of Osadebe and Chidolue (2012a), Osadebe and Chidolue (2012b), Osadebe and Ezeh (2009a), Osadebe and Ezeh (2009b) were also based on Vlasov's method. Nwachukwu and others (2017) and Nwachukwu and others (2021a) derived the RRM based governing TPEF equation for the TWBC applicable to RRM and evaluate and formulate the peculiar TPEF for DSS cross – section respectively . Nwachukwu and others (2021b) evaluated and formulated the TPEF for DSM and MSM cross section.. Finally, Nwachukwu and others (2022) have also evaluated and formulated the peculiar TPEF for MSS TWC cross section.

Thus in the area of stability analysis of thin-walled box (closed) columns, little or no effort has been done to use RRM to evaluate and formulate the peculiar TPEF for ASS cross section. Henceforth, the need for this recent research work. The derived energy functional will now be used to analyze a ASS thin- walled box (closed) columns of different boundary conditions in subsequent works.

2. BACKGROUND TO RRM BASED TWBC STABILTY ANALYSIS

Following the work of Nwachukwu and others (2017), the TPEF based on RRM has been formulated as

$$\pi = k_1 \int_L v^2\, x^2 (2L - x)^2 dx + k_2 \int_L (v^I)^2\, dx + k_3 \int_L (v^{II})^2\, dx - k_4 \int_L (v^I)^2\, dx. \quad \textbf{(1)}$$

where, $k_1 = \frac{Ap^2}{8EI^2}$; $k_2 = \frac{AG}{2}$; $k_3 = \frac{EI}{2}$; $and\ k_4 = \frac{P}{2}$ **2(a-d)**

Where P is critical buckling load, A is Cross sectional area, E is young modulus of elasticity, G is shear modulus, I is moment of inertia, and L is length of the column.

Here, v = the displacement function, which is a function of polynomial shape function, ϕ

According to Raleigh- Ritz Theory

$$v = \sum_i^n c_i \phi_i = c_1\phi_1 + c_2\phi_2 + c_3\phi_3 + \ldots\ldots + c_n\phi_n$$

(3)

Where c = undetermined coefficient / unknown constant and ϕ = Polynomial shape function which has been generated by Nwachukwu and others (2021a). Again Nwachukwu and others (2021a) has demonstrated the efficacy of Eqn. (1) by using it to formulate the peculiar TPEF for Doubly Symmetric Single (DSS) cell TWBC for different boundary conditions. For instance, the peculiar TPEF for DSS formulated by Nwachukwu and others (2021a) for Fixed-Fixed or Clamped- Clamped (C-C) boundary condition is as given under:

$$\pi_{DSS}^{C-C} = k_1{}^{DSS} \varphi_1{}^{C-C} + k_2{}^{DSS} \varphi_2{}^{C-C} + k_3{}^{DSS} \varphi_3{}^{C-C} - k_4{}^{DSS} \varphi_4{}^{C-C}. \quad \textbf{(4)}$$

$$= k_1{}^{DSS} [360c_1{}^2 L^{12} - 1575c_1{}^2 L^{13} + 2870c_1{}^2 L^{14} - 2772c_1{}^2 L^{15} + \frac{16380c_1{}^2 L^{16}}{11}$$

$$- 420c_1{}^2 L^{17} + \frac{630c_1{}^2 L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} +$$

$$\frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} +$$

$$\frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960c_2{}^2 L^{12} - 31185c_2{}^2 L^{13} + 105490c_2{}^2 L^{14} - 1995840c_2{}^2 L^{15} +$$

$$230580c_2{}^2 L^{16} - 166320c_2{}^2 L^{17} +$$

$$\frac{949410{c_1}^2L^{18}}{13} - 17820{c_2}^2L^{19} + 1848{c_2}^2L^{20}]$$

$$+{k_2}^{DSS}\,[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6$$
$$-24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 -$$
$$360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 -$$
$$600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8]$$

$$+ {k_3}^{DSS}\,[\frac{2520{c_1}^2}{L^2} - \frac{8820{c_1}^2}{L} + 40320{c_1}^2 - 45360{c_1}^2L + 18144{c_1}^2L^2 - \frac{72c_1c_2\sqrt{53900}}{L^2} + \frac{648c_1c_2\sqrt{53900}}{L}$$
$$-2592c_1c_2\sqrt{53900} + 4896c_1c_2\sqrt{53900}\,L - 4320c_1c_2\sqrt{53900}L^2 +$$
$$1440c_1c_2\sqrt{53900}L^3 + \frac{27720{c_2}^2}{L^2} - \frac{332640{c_2}^2}{L} + 1884960{c_2}^2 - 5266800{c_2}^2L +$$
$$7650720{c_2}^2L^2 - 5544000{c_2}^2L^3 + 1584000{c_2}^2L^4\,]$$

$$-{k_4}^{DSS}\,[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6$$
$$-24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 -$$
$$360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 -$$
$$600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8\,] \qquad \textbf{(5)}$$

Where k_1, k_2, k_3 and k_4 are all defined in Eqns.2 (a-d) respectively, but in terms of DSS cross-section. The next section will now examine the evaluation and derivation of peculiar TPEF of ASS TWBC cross-section.

3. EVALUATION AND DERIVATION OF TPEF FOR ASS TWBC CROSS-SECTION

3.1. EVALUATION OF ASS CROSS- SECTIONAL PROPERTIES

Evaluat ion of the cross sectional properties here implies determining the Cross- Sectional Area for ASS, A^{MSS} and its Moment of Inertia, I^{ASS}. An ASS thin walled box column cross section is shown in Fig.1.

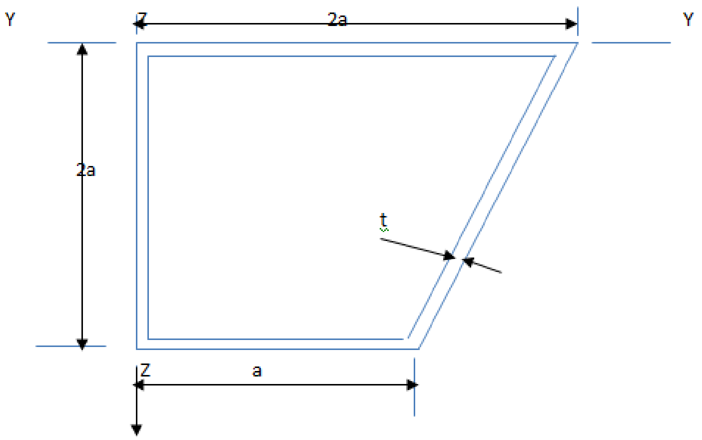

Fig.1: An ASS thin-walled box column cross section

(a).CROSS- SECTIONAL AREA, A^{ASS} AND CENTROID.

Applying the same thin- walled assumptions we have in Fig.2 as follows:

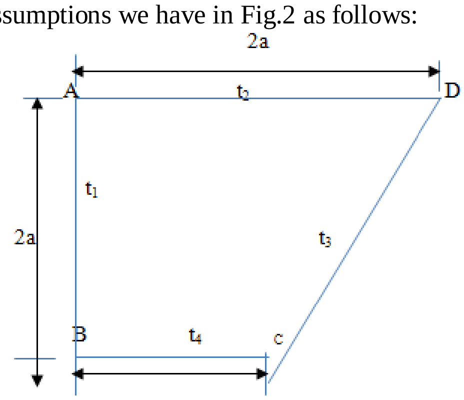

Fig.2 : Thin – Walled Assumption for ASS

From Fig 2 ,

$$CD = \sqrt{a^2 + (3a)^2} = \sqrt{5a^2} = a\sqrt{5} \qquad \textbf{(6)}$$

The centroid, cross- sectional area, and moment of inertia are evaluated in Table 1.

Table 1: Centroid, Cross- Sectional Area and moment of inertia for ASS

Section	$\bar{y}_i$	$\bar{z}_i$	A_i	I_{Yci}	I_{Zci}
1	0	$-\frac{2a}{2}$	2at	$\frac{(2a)^3 t}{12}$	$\frac{(t)^3 2a}{12}$
2	$\frac{2a}{2}$	0	2at	$\frac{(2a)^3 t}{12}$	$\frac{(t)^3 2a}{12}$
3	0	$\frac{-a\sqrt{5}}{2}$	$at\sqrt{5}$	$\frac{(a\sqrt{5})^3 t}{12}$	$\frac{(t)^3 a\sqrt{5}}{12}$
4	$\frac{a}{2}$	0	at	$\frac{(a)^3 t}{12}$	$\frac{(t)^3 a}{12}$
SUM	$\sum \bar{y}_i = \frac{3a}{2}$	$\sum \bar{z}_i = -\frac{2a}{2} - \frac{a\sqrt{5}}{2}$	$\sum A_i = 5at + at\sqrt{5}$		

Thus $A^{ASS} = \sum A_i = 5at + at\sqrt{5} = 7.236at$ **(7)**

$\bar{y}_G = 1.5a$ and $\bar{z}_G = -1.12a$ **8(a-b)**

b. MOMENT OF INERTIA, I^{ASS}

Since there is no axis of symmetry, the moment of inertia will be based on I_{YZ}.. Using thin – walled assumption,

$I_{YZ} = I_{YZ1} + I_{YZ2} + I_{YZ3} + I_{YZ4}$

Where $I_{YZ1} = I_{Yc1} * I_{Zc1} + A_1[-(\bar{z}_i - \bar{z}_G)][-\bar{Y}_G]]$

$$= \frac{(2a)^3 t}{12} * \frac{(t)^3 2a}{12} + 2at\,[-(1.12a - a)][-1.5a] = \frac{(2a)^3 t}{12} * \frac{(t)^3 2a}{12} + 0.36a^3 t \qquad \textbf{(9)}$$

By thin – walled assumption

$$\frac{(2a)^3 t}{12} * \frac{(t)^3 2a}{12} \approx 0$$

Thus $I_{YZ1} = 0.36a^3 t$ **(10)**

Similarly,

$I_{YZ2} = I_{Yc1} * I_{Zc1} + A_2[(\bar{Y}_2 - \bar{Y}_G)[\bar{Z}_G]] = 2at[1.12a][1.5a - a] = 1.12a^3 t$ **(11)**

Again

$I_{YZ3} = A_3[-(\bar{Z}_3 - \bar{Z}_G)][-\bar{Y}_G]] = at\sqrt{5}\,[-(\frac{a\sqrt{5}}{2} - 1.12a)][-1.5a] = 0$ **(12)**

And finally,

$I_{YZ4} = A_4[(\bar{Y}_4 - \bar{Y}_G)][\bar{Z}_G] = at\,[1.12a]\,[1.5a - 0.5a] = 1.12a^3 t$ **(13)**

Thus, $I^{ASS} = I_{YZ} = 2.60a^3 t$ **(14)**

3.2. DERIVATION OF TPEF FOR ASS DIFFERENT BOUNDARY CONDITION CASES

(a).CASE 1: PINNED-PINNED(S-S)- ASS TWBC.

The total potential energy functional (TPEF) for ASS-[S-S] thin-walled box column (TWBC), π_{ASS}^{S-S} can be formulated as follows:

$$\pi_{ASS}^{S-S} = k_1^{ASS}\varphi_1^{S-S} + k_2^{ASS}\varphi_2^{S-S} + k_3^{ASS}\varphi_3^{S-S} - k_4^{ASS}\varphi_4^{S-S} \qquad \textbf{(15)}$$

$$= k_1^{ASS}\,[\,24c_1^2 L^{10} - 60c_1^2L^{11} + \frac{390c_1^2L^{12}}{7} - 10c_1^2L^{13} + \frac{10c_1^2L^{14}}{3} - \frac{8c_1c_2\sqrt{6300}\,L^{10}}{5} + \frac{20c_1c_2\sqrt{6300}\,L^{11}}{3} - \frac{74c_1c_2\sqrt{6300}\,L^{12}}{7} + 8\,c_1c_2\sqrt{6300}\,L^{13} - \frac{26c_1c_2\sqrt{6300}\,L^{14}}{9} + \frac{2c_1c_2\sqrt{6300}\,L^{15}}{5} +$$

$$168c_2^2 L^{10} - 980c_2^2L^{11} + 2310c_2^2 L^{12} - \frac{5565c_2^2L^{13}}{2} + \frac{5390c_2^2L^{14}}{3} - 588c_2^2 L^{15} +$$

840c22L1611]

$$+ k_2^{ASS}[30c_1^2 - 60c_1^2 L + 40c_1^2L^2 - 2\,c_1c_2\sqrt{6300} + 8\,c_1c_2\sqrt{6300}\,L - 12c_1c_2\sqrt{6300}\,L^2 + 6\,c_1c_2\sqrt{6300}\,L^3$$

$$+ 210c_2^2 - 1260c_2^2L + 3360c_2^2L^2 - 3780c_2^2L^3 + 1512c_2^2L^4\,]$$

$$+ k_3^{ASS}\,[\frac{120c_1^2}{L^2} - \frac{24\,c_1c_2\sqrt{6300}}{L^2} + \frac{24\,c_1c_2\sqrt{6300}}{L} + \frac{7560c_2^2}{L^2} - \frac{15120c_2^2}{L} + 10080c_2^2]$$

$$-k_4{}^{ASS}\,[30c_1{}^2 - 60c_1{}^2 L + 40c_1{}^2L^2 - 2\,c_1c_2\sqrt{6300} + 8\,c_1c_2\,\sqrt{6300}\,L - 12c_1c_2\,\sqrt{6300}\,L^2 + 6\,c_1c_2\,\sqrt{6300}\,L^3 + 210c_2{}^2 - 1260c_2{}^2L + 3360c_2{}^2L^2 - 3780c_2{}^2L^3 + 1512c_2{}^2L^4\,] \quad \textbf{(16)}$$

Where

$$k_1{}^{ASS} = \frac{A^{ASS}p^2}{8EI^{2(ASS)}}, \quad k_2{}^{ASS} = \frac{A^{ASS}\mathrm{G}}{2}, \; k_3{}^{ASS} = \frac{E\,I^{ASS}}{2} \;\&\; k_4{}^{ASS} = \frac{P}{2} \quad \textbf{17(a-d)}$$

A^{ASS} and I^{ASS} are defined in Eqns.(7) and (13) respectively.

(b).CASE 2: FIXED-FIXED[C-C]- ASS TWBC.

The total potential energy functional for ASS-[C-C] thin-walled box column can be obtained as follows:

$$\pi_{ASS}^{C-C} = k_1{}^{ASS}\,\varphi_1{}^{C-C} + k_2{}^{ASS}\,\varphi_2{}^{C-C} + k_3{}^{ASS}\varphi_3{}^{C-C} - k_4{}^{ASS}\varphi_4{}^{C-C} \quad \textbf{(18)}$$

$$= k_1{}^{ASS}\,[360c_1{}^2\,L^{12} - 1575c_1{}^2L^{13} + 2870c_1{}^2L^{14} - 2772c_1{}^2L^{15} + \frac{16380c_1{}^2L^{16}}{11} - 420c_1{}^2L^{17} + \frac{630c_1{}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} + \frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} + \frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960c_2{}^2\,L^{12} - 31185c_2{}^2L^{13} + 105490c_2{}^2\,L^{14} - 1995840c_2{}^2L^{15} + 230580c_2{}^2\,L^{16} - 166320c_2{}^2L^{17} + \frac{949410c_1{}^2L^{18}}{13} - 17820c_2{}^2L^{19} + 1848c_2{}^2L^{20}$$

$$+k_2{}^{ASS}\,[840c_1{}^2L^2 - 3780c_1{}^2L^3 + 6552c_1{}^2L^4 - 5040c_1{}^2L^5 + 1440c_1{}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240c_2{}^2L^2 - 83160c_2{}^2L^3 + 310464c_2{}^2L^4 - 600600c_2{}^2L^5 + 633600c_2{}^2L^6 - 346500c_2{}^2L^7 + 77000c_2{}^2L^8]$$

$$+ k_3{}^{ASS}\,[\frac{2520c_1{}^2}{L^2} - \frac{8820c_1{}^2}{L} + 40320c_1{}^2 - 45360c_1{}^2L + 18144c_1{}^2L^2 - \frac{72c_1c_2\,\sqrt{53900}}{L^2} + \frac{648c_1c_2\,\sqrt{53900}}{L} - 2592c_1c_2\,\sqrt{53900} + 4896c_1c_2\,\sqrt{53900}\,\mathrm{L} - 4320c_1c_2\,\sqrt{53900}L^2 + 1440c_1c_2\,\sqrt{53900}L^3 + \frac{27720c_2{}^2}{L^2} - \frac{332640c_2{}^2}{L} + 1884960c_2{}^2 - 5266800c_2{}^2L + 7650720c_2{}^2L^2 - 5544000c_2{}^2L^3 + 1584000c_2{}^2L^4\,]$$

$$-k_4{}^{ASS}\,[840c_1{}^2L^2 - 3780c_1{}^2L^3 + 6552c_1{}^2L^4 - 5040c_1{}^2L^5 + 1440c_1{}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240c_2{}^2L^2 - 83160c_2{}^2L^3 + 310464c_2{}^2L^4 - 600600c_2{}^2L^5 + 633600c_2{}^2L^6 - 346500c_2{}^2L^7 + 77000c_2{}^2L^8\,] \quad \textbf{(19)}$$

Where $k_1{}^{ASS}$, $k_2{}^{ASS}$, $k_3{}^{ASS}$ and $k_4{}^{ASS}$ are defined in Eqns. 16(a-d) respectively.

©.CASE 3: FIXED-PINNED[C-S]- ASS TWBC

The total potential energy functional for ASS-[C-S] thin-walled box column can be obtained as follows:

$$\pi_{ASS}^{C-S} = k_1{}^{ASS}\,\varphi_1{}^{C-S} + k_2{}^{ASS}\,\varphi_2{}^{C-S} + k_3{}^{ASS}\varphi_3{}^{C-S} - k_4{}^{ASS}\varphi_4{}^{C-S} \quad \textbf{(20)}$$

$$= k_1{}^{ASS}\,[\frac{22680c_1{}^2\,L^{12}}{133} - \frac{98280c_1{}^2\,L^{13}}{152} + \frac{174510c_1{}^2\,L^{14}}{171} - \frac{162540c_1{}^2\,L^{15}}{190} + \frac{83790c_1{}^2\,L^{16}}{209} - \frac{22680c_1{}^2\,L^{17}}{228} + \frac{2520c_1{}^2\,L^{18}}{247} - \frac{216}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{12} + \frac{10728}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{13} - \frac{39078}{9}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{14} + \frac{58500}{10}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{15} - \frac{44640}{11}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{16} + \frac{18000}{12}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{17} - \frac{3546}{13}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{18} + \frac{252}{14}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^{19} + \frac{27720c_2{}^2\,L^{12}}{1729} - \frac{2633400c_2{}^2\,L^{13}}{1976} + \frac{66784410c_2{}^2\,L^{14}}{2223} - \frac{203312340c_2{}^2\,L^{15}}{2470} + \frac{250637310c_2{}^2\,L^{16}}{2717} - \frac{151295760c_2{}^2\,L^{17}}{2964} + \frac{45952830c_2{}^2\,L^{18}}{3211} - \frac{6500340c_2{}^2\,L^{19}}{3458} + \frac{339570c_2{}^2\,L^{20}}{3705}\,]$$

$$+k_2{}^{ASS}\,[\frac{22680c_1{}^2\,L^2}{57} - \frac{113400c_1{}^2\,L^3}{76)} + \frac{202230c_1{}^2\,L^4}{95} - \frac{151200c_1{}^2\,L^5}{114} + \frac{40320c_1{}^2\,L^6}{133} - \frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^2 + \frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^3 - \frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^4 + \frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^5 - \frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^6 + \frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}\,L^7$$

$$+\frac{27720c_2{}^2L^2}{741}-\frac{3908520c_2{}^2L^3}{988}+\frac{143497970c_2{}^2L^4}{1235}-\frac{415273320c_2{}^2L^5}{1482}$$
$$+\frac{379861020c_2{}^2L^6}{1729}-\frac{102841200c_2{}^2L^7}{1976}+\frac{8489250c_2{}^2L^8}{2223}]$$
$$+k_3{}^{ASS}\,[\frac{22680c_1{}^2}{19L^2}-\frac{226800c_1{}^2}{38L}+\frac{748440c_1{}^2}{57}-\frac{907200c_1{}^2L}{76}+\frac{362880c_1{}^2L^2}{95}$$
$$-\frac{216}{L^2}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}+\frac{31536}{2L}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}-\frac{221832}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}+\frac{480384}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L-$$
$$\frac{350352}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2+\frac{60480}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3$$
$$+\frac{24640c_2{}^2}{247L^2}-\frac{7817040c_2{}^2}{494L}+\frac{568731240c_2{}^2}{741}-\frac{2489699520c_2{}^2L}{988}$$
$$+\frac{3350350080c_2{}^2L^2}{1235}-\frac{123094400c_2{}^2L^3}{1482}+\frac{135828000c_2{}^2L^4}{1729}]$$
$$-k_4{}^{ASS}\,[\frac{22680c_1{}^2L^2}{57}-\frac{113400c_1{}^2L^3}{76)}+\frac{202230c_1{}^2L^4}{95}-\frac{151200c_1{}^2L^5}{114}+\frac{40320c_1{}^2L^6}{133}-$$
$$\frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2+\frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3-\frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^4+$$
$$\frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^5-\frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^6+\frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^7$$
$$+\frac{27720c_2{}^2L^2}{741}-\frac{3908520c_2{}^2L^3}{988}+\frac{143497970c_2{}^2L^4}{1235}-\frac{415273320c_2{}^2L^5}{1482}$$
$$+\frac{379861020c_2{}^2L^6}{1729}-\frac{102841200c_2{}^2L^7}{1976}+\frac{8489250c_2{}^2L^8}{2223}] \qquad \textbf{(21)}$$

Where $k_1{}^{ASS}$, $k_2{}^{ASS}$, $k_3{}^{ASS}$ and $k_4{}^{ASS}$ are defined in Eqns.16 (a-d) respectively.

4. CONCLUSIONS

In this study, Raleigh –Ritz Method (RRM) of structural stability TPEF formulation was presented. The study was able to evaluate cross sectional properties of ASS cross sections, which are moment of Inertia and cross sectional area, given in Eqns.(7) and (14). The study also formulated peculiar TPEF for ASS cross-section. The formulated Raleigh- Ritz based ASS TPEF given in Eqns.(16), (19) and (21) are found handy and convenient to be used in the bulking/stability analysis of ASS TWBC cross- section. . The Raleigh- Ritz based formulated TPEF equations are also found suitable, handy and simple to be used in the Flexural(F) , Flexural-Torsional(FT) and Flexural- Torsional- Distortional(FTD) buckling/stability analysis of ASS cell TWBC cross-section where data obtained (critical bulking loads) will be compared with the works of other authors in subsequent papers. The Raleigh- Ritz based formulated TPEF equations are found suitable, handy and simple to be used in the Flexural(F) , Flexural- Torsional(FT) and Flexural- Torsional- Distortional(FTD) buckling/stability analysis of MSS cell TWBC cross-section where data obtained (critical bulking loads) will be compared with the works of other authors in subsequent papers

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