

On various morphologies of propylene glycol-based SiO₂ nanofluid flow and heat transmission over a stretching cylinder

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Abstract

The Current study analyses the unsteady flow and heat transfer of SiO₂ nanoparticles based on propylene glycol over a stretching cylinder. The given partial differential equations (PDEs) are converted into a system of nonlinear ordinary differential equations and solved by using BVP4C in MATLAB. Graphical simulation is used to examine how different shaped SiO₂ nanoparticles affect flow and heat transfer rate along with other different parameters like Prandtl, Eckert, and biot numbers. Additionally, the Nusselt number and the skin friction coefficient are computed and numerically calculated. The research shows that different SiO₂ nanoparticle morphologies have faster flow and heat transmission rates.

Keywords: Axial stretching, Shape factor, Propylene based SiO₂-nanofluid; Heat transmission.

Date of Submission: 22-06-2022

Date of Acceptance: 04-07-2022

1. Introduction

The creation of materials and technologies with novel or radically altered features using nanotechnology involves the manipulation or self-assembly of individual atoms, molecules, or molecular clusters into structures. Sulfide nano crystals were employed in stained glass windows for hair dyeing 2000 years ago by the Greeks and Romans. Different-sized gold nanoparticles are used in stained glass windows to create a variety of colors.

The class of fluids known as nanofluids, which includes particles with sizes ranging from 1 to 100 nm suspended in a base fluid, is thought to be a relatively new one. By adding various base fluids, such as water or propylene glycol, the thermal conductivity of nanofluids is increased.[1]





Metals or metal oxides, such as silver, copper, copper oxide, silica, and aluminium oxide, are colloidal nanomaterials with outstanding electrical, thermal, and structural properties. For the manufacture of nano fluids, various base fluids like water, ethylene glycol, and glycerol are employed. [2]

Nanofluid is used to improve the performance of engineering devices such automobiles, micro and minichannels, nuclear reactors, and thermal engineering systems[3-5]. Small amounts of nanoparticles have been shown to significantly boost the thermal conductivity of nanofluids [6]. Using ethylene glycol as a nano fluid and copper nanoparticles with a volume fraction of 0.3 nm results in an increase of 0.4 in effective thermal conductivity[7]. Additionally, alumina/water nanofluid with 1-4 percent alumina volume increased effective thermal conductivity by 0.1–0.3 [8]. The increase in energy absorbing capacity of nano fluid has also been calculated using the single phase coefficient of heat transfer. Further, if the flow is turbulent, the Nusselt Number rises by 0.3 compared to the assumption by taking into account the influence of heat conductivity and the increase in viscosity in circular tubes [9-13].

Table 1: Thermophysical properties of Propylene glycol base fluid and SiO₂ nanoparticle [14]

Nanoparticle / Base fluid	Density (kg/m ³)	Thermal conductivity (W/m K)	Specific heat (J/kg K)	Electric conductivity (S/m)
SiO ₂	2200	1.2	703	5.5 × 10 ⁻⁶
C ₃ H ₈ O ₂	938.5	0.684	4338	0.10 × 10 ⁻⁴

Table 2: Numerical values of viscosity and shape factor of nanoparticles [15]

Nanoparticles	Cylinder	Sphere	Blades	Platlets
Parameters				
A1	13.5	2.5	14.6	37.1
A2	904.4	6.5	123.3	612.6
M	4.82	3.0	8.26	5.72

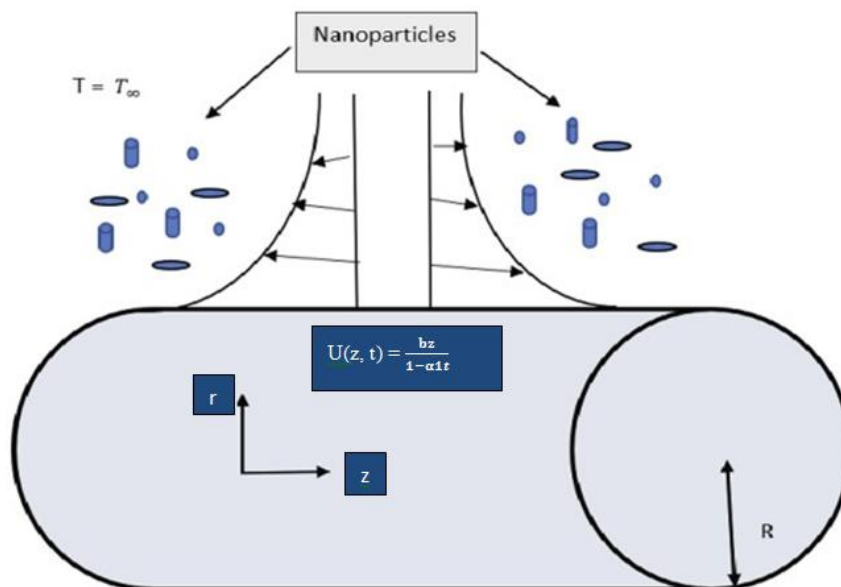
Applications of heat transfer of nano fluid taking into account its flow across a stretching cylinder are now influencing many industries, such as insulating materials, roofing shingles, paper manufacture, condensation process, etc [16-19].

To fully comprehend the rate of heat transfer in both analytical and numerical investigations, slip condition plays a crucial role during the motion of nanofluids [20-25].

Thermal conductivity and heat transfer of nano fluid is explained by plucking the valuable contribution of Hamid and khan [26], Sheremet and Ramesh et al [27-31], Tiwari and Das, Maiga, Alawi, Dinarvand [32-35].

2. Mathematical Formulation:

The proposal given in this article is to investigate viscous fluid over a stretching cylinder of radius R and heat transfer of SiO₂ propylene glycol based nanofluid over a solid cylinder. Stretching of the cylinder is responsible for motion of the fluid along the axial direction z of the cylindrical coordinate system. $U(z, t) = \frac{bz}{1-\alpha_1 t}$ is the surface velocity, where b and α_1 both are positive constants with dimension t^{-1} , $T = T_s = T_o - T_r \left[\frac{bz^2}{2v} \right] [1 - \frac{\alpha_1 t - 32}{1 - \alpha_1 t}]$ is the temperature of the stretching cylinder. T_r is the constant reference temperature. Stretching rate $\frac{bz}{1-\alpha_1 t}$ increase with time. $U(z,t)$, $T_s(z,t)$, $B(r,t)$ are taken in such a way such that it helps in formulating new similarity transformations. Law of Conservation of mass and momentum equation of PDEs into system of ODEs.



Velocity profile

$$\vec{V} = [u(r, z, t), 0, w(r, z, t)]$$

Law of Conservation of mass:

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0$$

Momentum Equation

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \frac{v}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) - \frac{1}{\rho} \nabla \sigma \frac{B_0^2}{(1-\alpha_1 t)}$$

Energy equation

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{v_{nf}}{C_p} \left(\frac{\partial w}{\partial r} \right)^2 + \alpha_{nf} \left[\frac{r^2 T}{\partial r^2} + \frac{1}{r} \left(\frac{\partial T}{\partial r} \right) \right]$$

Subject to the boundary condition

$$\begin{aligned} W=U_w, u=0 \text{ and } T=T_s & \text{ at } r=R \\ w=0, T \rightarrow T_0 & \text{ as } r \rightarrow \infty \end{aligned}$$

Similarity transformation:

$$\begin{aligned} \Psi &= (Uvz)^{\frac{1}{2}} Rf(\eta), \eta = \frac{r^2 - R^2}{zR} \left(\frac{U}{vz} \right)^{\frac{1}{2}} \\ \theta(\eta) &= \frac{T - T_0}{-T_{ref} \left[\frac{bz^2}{2v} \right]^{1-\alpha_1 t} } \end{aligned}$$

$$T = T_0 - T_r \left[\frac{bz^2}{2v} \right]^{1-\alpha_1 t} \theta(\eta)$$

Now above system of equations together with boundary conditions will become

$$\begin{aligned} [1 + 2C\eta] \epsilon_1 f''' + 2C\epsilon_1 f'' - [\epsilon_3 M + S] f' - \frac{S\eta}{2} \cdot f'' + f \cdot f'' - (f')^2 &= 0 \\ [1 + 2C\eta] \theta'' + 2C\theta' + \frac{Pr}{\epsilon_2} f \theta' - 2 \frac{Pr}{\epsilon_2} f' \theta - \frac{Pr}{2\epsilon_2} S [3\theta + \eta \theta'] + [1 + 2C\eta] Pr \epsilon_2 \left[\frac{\epsilon_1}{\epsilon_2} \right] (f'')^2 &= 0 \end{aligned}$$

Here

$$C = \sqrt{\frac{(1-\alpha_1 t)v_f}{bR^2}}, \epsilon_1 = \frac{1+A_1\phi+A_2\phi^2}{1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)}, \epsilon_2 = \frac{\frac{k_{nf}}{k_f}}{1-\phi+\frac{\phi(\rho C_p)_3}{(\rho C_p)_f}}, \epsilon_3 = \frac{1-\phi+\phi\left(\frac{\sigma_s}{\sigma_f}\right)}{1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)}$$

$$Pr = \frac{v_f(\rho C_p)_f}{K_f}, E_c = \frac{bz^2}{C_p \Delta T (1-\alpha_1 t)^2} = \frac{U^2}{C_p \Delta T}, S = \frac{\alpha_1}{b}$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, v_{nf} = v_f \epsilon_1, (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_o)_p$$

$$\frac{k_{nf}}{k_f} = \left[\frac{k_{s+(m-1)k_f+(m-1)(k_s-k_f)\phi}}{k_{s+(m-1)k_f-(k_s-k_f)\phi}} \right]$$

where ϕ is the volume-fraction of the nanofluid, A_1, A_2 are the coefficients of viscosity enhancement heat capacitance, K is the slip parameter, γ is the biot number, Ec is the Eckert number, M is the magnetic parameter, Pr is the Prandtl number and S is the unsteadiness parameter.

So we have

$$\begin{aligned} [1 + 2C\eta] \epsilon_1 f''' + 2C\epsilon_1 f'' - [\epsilon_3 M + S] f' - \frac{S\eta}{2} f'' + f \cdot f'' - (f')^2 &= 0 \\ [1 + 2C\eta] \theta'' + 2C\theta' + \frac{Pr}{\epsilon_2} [f \cdot \theta' - 2f' \theta - \frac{S}{2} (3\theta + \eta \theta')] + E_c \epsilon_1 (1 + 2Cv) (f'')^2 &= 0 \end{aligned}$$

Related with boundary conditions

$$\begin{aligned} f(0) = 0, f'(0) = 1, \theta(0) = 1 \\ f'(\infty) = 0, \theta(\infty) = 0 \end{aligned}$$

3. Method of Solution:

For solving above system in mathematical form, the majority of scholars used various techniques as shooting, kellor boxes, Runge- Kutta, RK-2,4 Euler, and finite difference method. The BVP4C approach is suggested in this paper as a potential solution to the abovementioned systems. In publications, errors and convergence analysis in BVP4C are briefly explored. Analysis of [36] leads to the conclusion that BVP4C is an effective method with fewer errors that can be applied to both multi- and two-point BVPs. These features of BVP4C made it a revolutionary star in the disciplines of research. To solve the above system of

equations numerically , we consider

$$\begin{aligned} y_1 &= f \\ y_1' &= y_2 \\ y_2' &= y_3 \\ \theta &= y_4 \\ \theta' &= y_5 \end{aligned}$$

4. Results and Discussion:

This section of research work is reserved for discussion of numerically computed results of the given equations using a reliable technique BVP4C in MATLAB. For a comprehensive study the multi-shaped nanoparticles of SiO_2 are dispersed in the propylene glycol base fluid are considered. The numerical results computed for velocity and temperature are illustrated and analyzed through graphs.

Impact of Significant physical Parameters on Velocity Profile

Impact of volumetric fraction ϕ , unsteadiness parameter S , magnetic parameter M on velocity profile. Influence of volumetric fraction ϕ on velocity profile for different shaped nanoparticles of SiO_2 . It is noted that nanofluid velocity increases for rising values ϕ for all shaped nanoparticles of SiO_2 . It means that all type of nanoparticles of SiO_2 play a significant role to decrease the viscosity of regular Propylene glycol base fluid as a result fluid flow accelerates. The impact of magnetic parameter M on velocity profile. It is observed that fluid flow decelerate for increasing values of M all shaped nanoparticles of SiO_2 . The main reason behind this deceleration is that magnetic field acts like a drag force. Furthermore, solutions of velocity and temperature fields are discussed in table (3, 4) and figure (1-8).

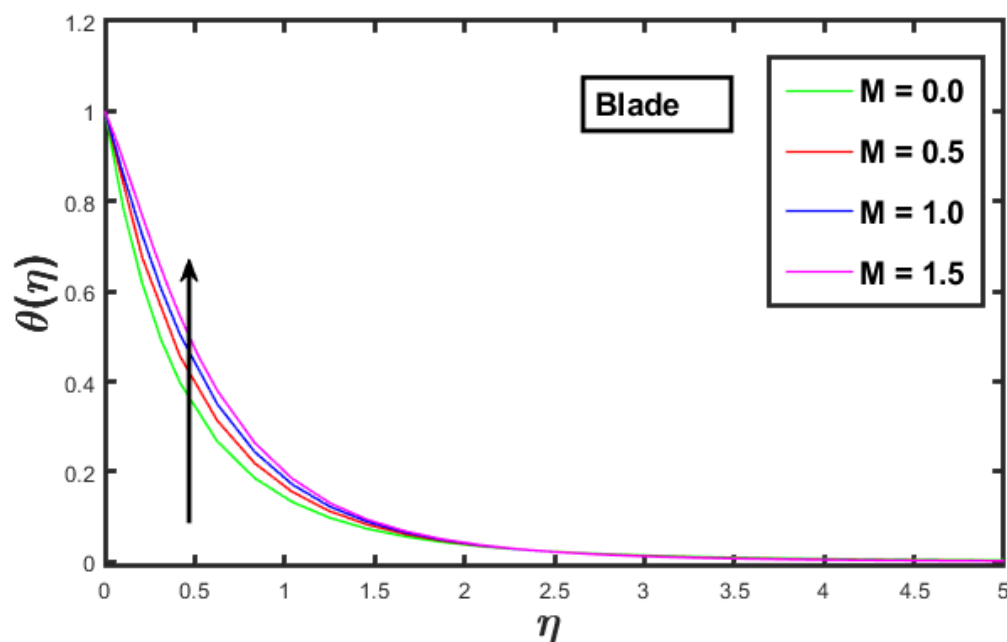


Figure: 1 represents the impact of M on temperature field for Blade shaped nano particles by taking $M = 0.0, 0.5, 1.0$ and 1.5 respectively while other parameters are fixed i.e $K = 0.5, S = 0.3, \phi = 0.08$ and $Ec = 1.0$. The graph shows clear increase of temperature field by increasing value of M using Blade shaped nano particles.

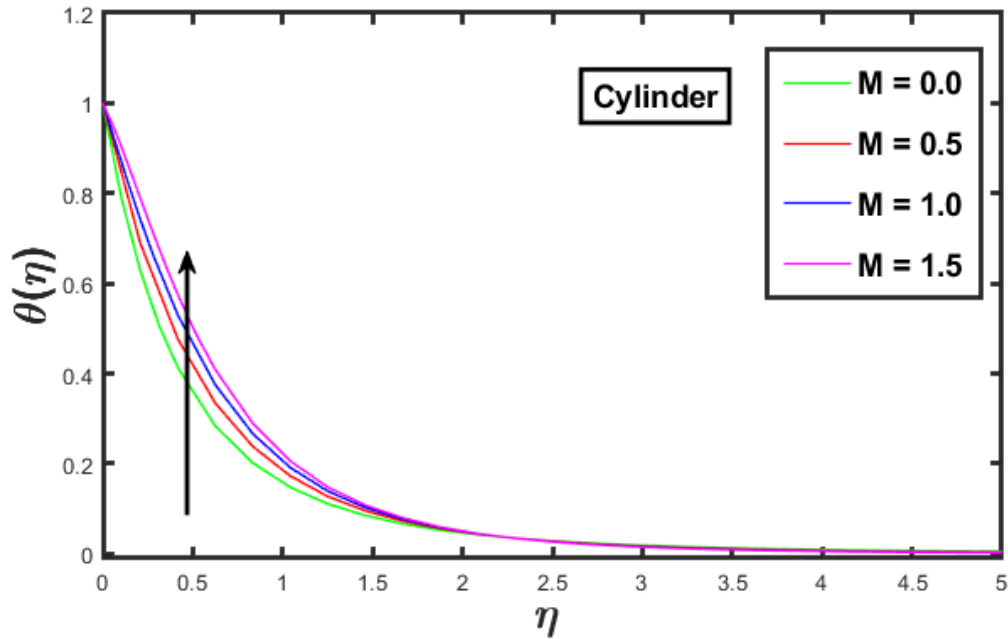


Figure: 2 represents the impact of M on temperature field for Cylinder shaped nano particles by taking $M = 0.0, 0.5, 1.0$ and 1.5 respectively while other parameters are fixed i.e $K = 0.5, S = 0.3, \phi = 0.08$ and $Ec = 1.0$. The graph shows clear increase of temperature field by increasing value of M using Cylinder shaped nano particles.

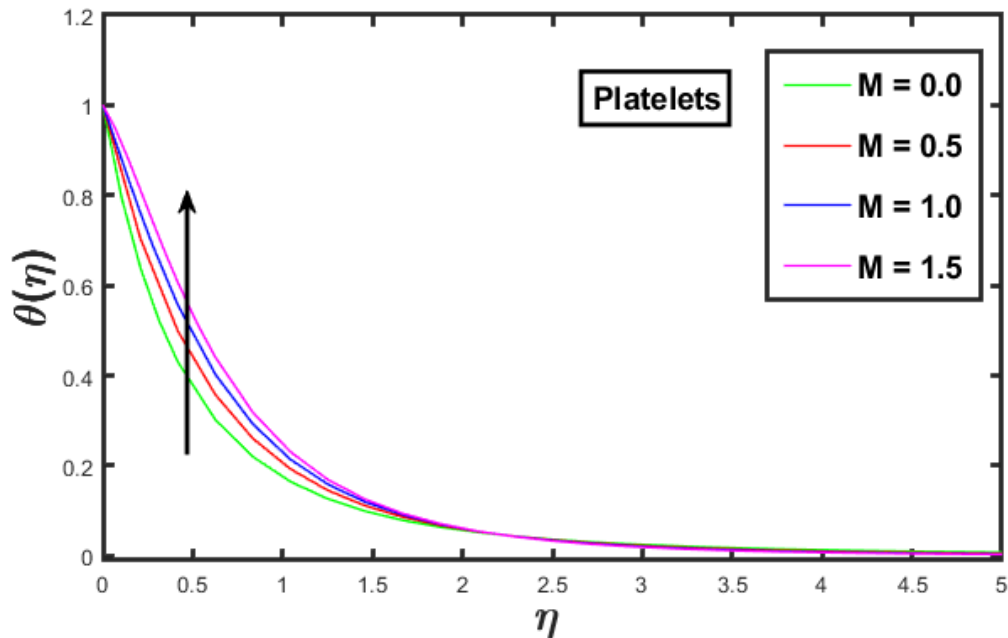


Figure: 3 represents the impact of M on temperature field for Platelet shaped nano particles by taking $M = 0.0, 0.5, 1.0$ and 1.5 respectively while other parameters are fixed i.e $K = 0.5, S = 0.3, \phi = 0.08$ and $Ec = 1.0$. The graph shows clear increase of temperature field by increasing value of M using Platelet shaped nano particles.

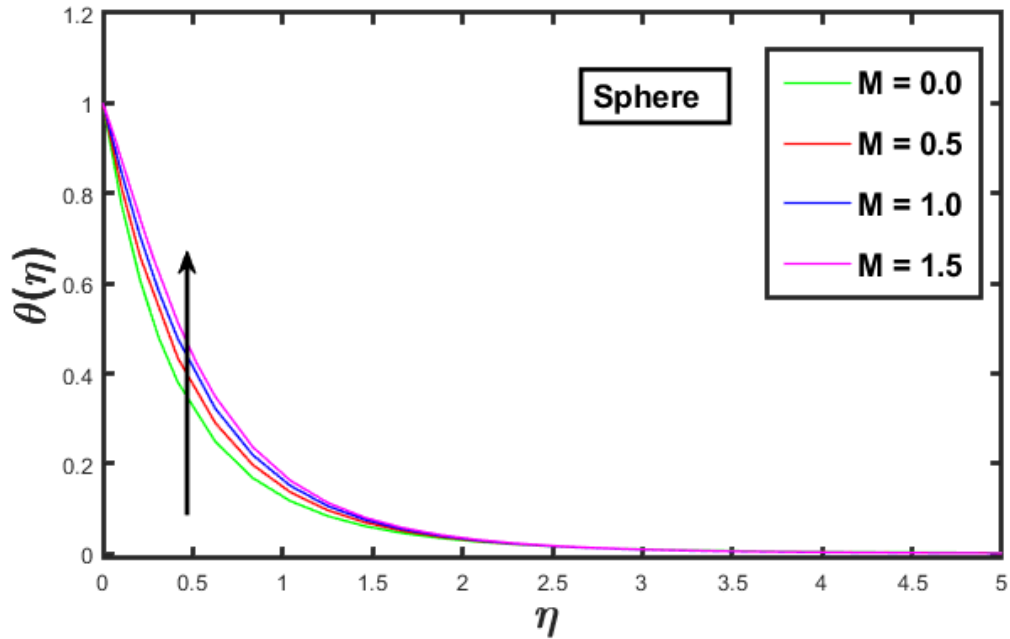


Figure: 4 represents the impact of M on temperature field for Sphere shaped nano particles by taking $M = 0.0, 0.5, 1.0$ and 1.5 respectively while other parameters are fixed i.e $K = 0.5$, $S = 0.3$, $\phi = 0.08$ and $Ec = 1.0$. using Sphere shaped nano particles.

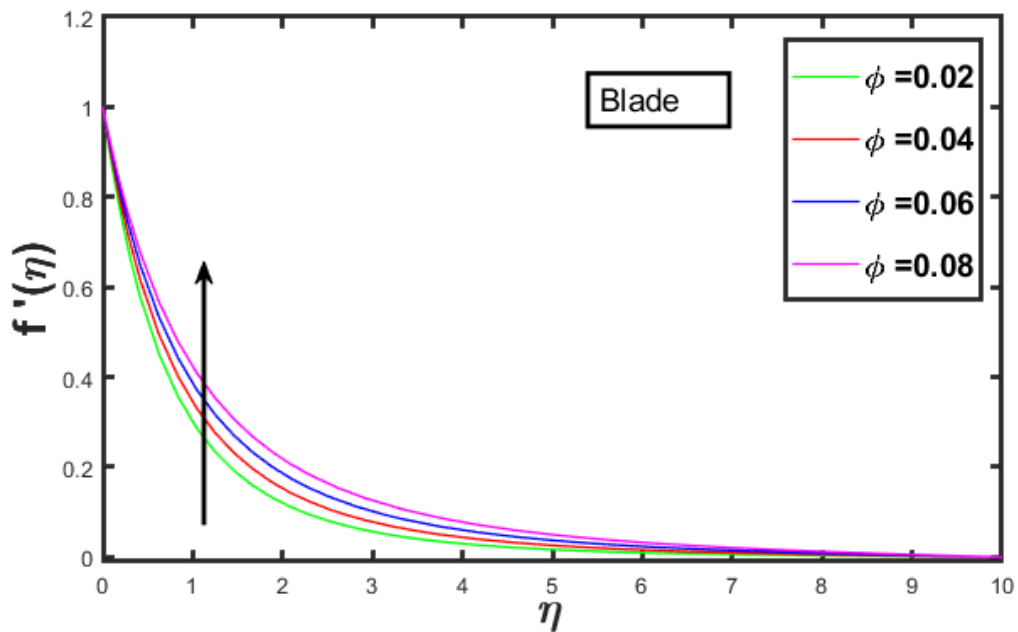


Figure: 5 represents the impact of M on Velocity field for Blade shaped nano particles by taking $\phi = 0.02, 0.04, 0.06$ and 0.08 respectively while other parameters are fixed i.e $K = 0.5$, $S = 0.3$, $M = 1.0$ and $Ec = 1.0$. The graph shows clear increase of temperature field by increasing value of ϕ using Blade shaped nano particles.

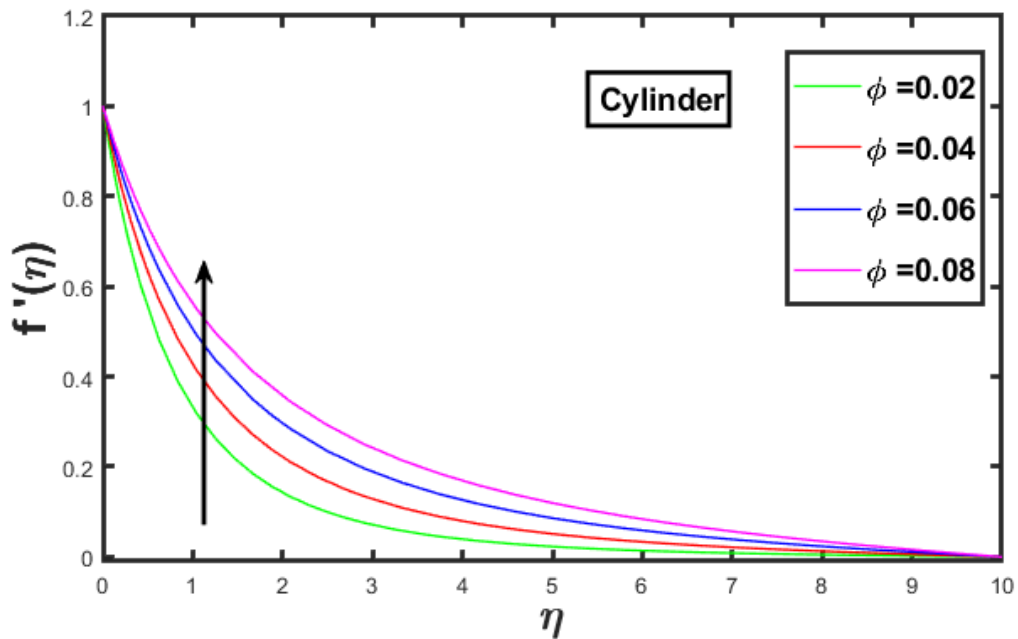


Figure: 6 represents the impact of M on Velocity field for Cylinder shaped nano particles by taking $\phi = 0.02, 0.04, 0.06$ and 0.08 respectively while other parameters are fixed i.e $K = 0.5, S = 0.3, M = 1.0$ and $Ec = 1.0$. The graph shows clear increase of temperature field by increasing value of ϕ using Cylinder shaped nano particles.

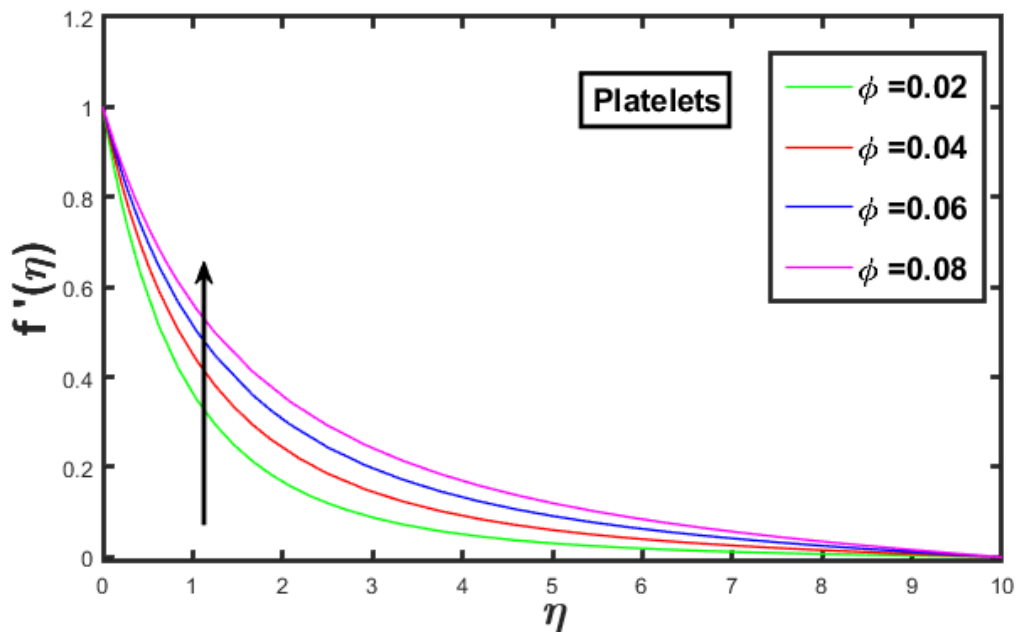


Figure: 7 represents the impact of M on Velocity field for Platelet shaped nano particles by taking $\phi = 0.02, 0.04, 0.06$ and 0.08 respectively while other parameters are fixed i.e $K = 0.5, S = 0.3, M = 1.0$ and $Ec = 1.0$.

The graph shows clear increase of temperature field by increasing value of ϕ using Platelet shaped nano particles.

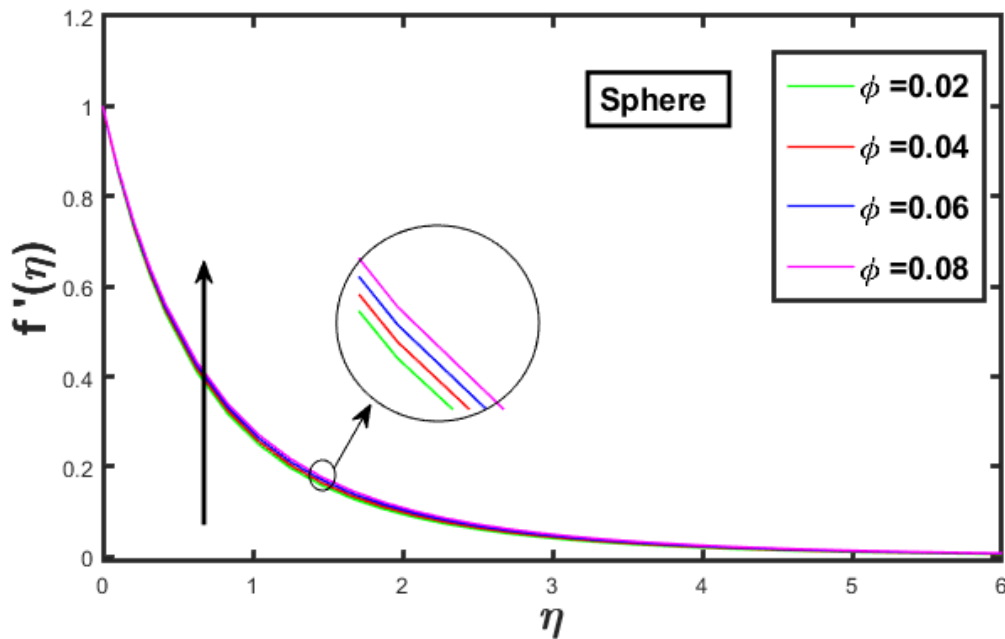


Figure: 8 represents the impact of M on Velocity field for Sphere shaped nano particles by taking $\phi = 0.02, 0.04, 0.06$ and 0.08 respectively while other parameters are fixed i.e $K = 0.5, S = 0.3, M = 1.0$ and $Ec = 1.0$. The graph shows clear increase of temperature field by increasing value of ϕ using Sphere shaped nano particles.

Table 3: Numerical values of skin-friction coefficient of multi-shape nanoparticles

K	Phi	Ec	S	M	sphere	Cylinder	blade	Platelet
0.5	0.02	1.0	0.3	1.0	-1.6757832	-2.1272397	-1.9111774	-2.3709318
-	0.04	-	-	-	-1.7330343	-2.9896493	-2.2424432	-3.2510739
-	0.06	-	-	-	-1.7938827	-4.0627124	-2.6068329	-4.2316768
-	0.08	-	-	-	-1.8581577	-5.3004602	-2.9987578	-5.3088485
-	-	-	-	0.0	-1.2719793	-1.6229155	-1.4543973	-1.8143759
-	-	-	-	0.5	-1.490298	-1.8952504	-1.7012532	-2.1145454
-	-	-	-	1.0	-1.6757832	-2.1272397	-1.9111774	-2.3709318
-	-	-	-	1.5	-1.8402061	-2.3328059	-2.0972002	-2.5981987

Table 4: Numerical values of Nusselt number of multi-shape nanoparticles

K	phi	Ec	S	M	Sphere	Cylinder	Blade	Platelet
0.5	0.02	1.0	0.3	1.0	1.5385163	1.2721682	1.3978275	1.1381833
-	0.04	-	-	-	1.5648209	0.90233308	1.2806772	0.78017119
-	0.06	-	-	-	1.5883489	0.50590055	1.1644155	0.43511613
-	0.08	-	-	-	1.6094517	0.088905918	1.0514986	0.086115226
-	-	-	-	0.0	2.5414549	2.3851236	2.461304	2.301599
-	-	-	-	0.5	2.0113645	1.8008983	1.9012195	1.6930965
-	-	-	-	1.0	1.5385163	1.2721682	1.3978275	1.1381833
-	-	-	-	1.5	1.1048287	0.78363093	0.93431242	0.62347619

5. Conclusion

Exploring the dependence of physical parameters on velocity and temperature profiles through graphs, in depth. Following are the conclusions of the current study:

- volume-fraction ϕ is directly related with both velocity and temperature field for all shapes of nano particle
- velocity of fluid flow and heat transfer rate increase with magnetic parameter M .
- impact of volume-fraction ϕ have constant behavior for blade,cylinder,sphere and platelets.
- Influence of magnetic parameter M have also the constant increasing behavior for all considered shapes of nano particles

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