

# MHD unsteady flow and Heat transfer in $\text{SiO}_2\text{-H}_2\text{O}$ nanoparticles over a stretching cylinder.

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## Abstract

The goal of this study is to investigate  $\text{SiO}_2\text{-H}_2\text{O}$  Nano fluid's momentum and thermal unsteady flow on a stretching surface under the influence of a magnetic field. The physical structure of this research is modeled into PDE's which were representing basic laws of fluid flow and law of thermodynamics. BVP4C solver technique is used to solve the system of ODE's which was obtained by using similarity transformation on reduced system of PDS's. In this article impact of some pertinent parameter on velocity and heat transfer is investigated and results are shown graphically. Also heat transfer coefficient and skin friction coefficient calculated and represented through tables and bars chart. It is noticed that increase in volume fraction fluid accelerate more and highest velocity is noted for platelets shape of nanoparticle. Temperature distribution increased by the rise of Magnetic parameter and Eckert number while decreased for unsteadiness parameter.

**Keywords:** Unsteady flow; Nanomaterial; Stretching cylinder; skin friction coefficient; shape factors

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## 1. Introduction

Because of its importance and wide range of applications, nano technology has grown in popularity. Nanofluids are colloids comprised of a base liquid and nanoparticles ranging in size from 1 to 100 nanometers. When compared to the base liquid, they have higher thermal conductivities and heat transfer coefficients. Solid materials are utilised to improve the thermal characteristics of the basic liquids by including a particular amount of nanoparticles, which increases the thermal conductivity of the nanomaterials. The Flow and heat transfer through a copper–water nanofluid around circular cylinder has been numerically investigated by Mohammad Sadegh Valipour and Ariyan Zare Ghadi [1]. The impact of solid volume percentage and Reynolds number on flow pattern and heat transfer properties was examined in this study. The magnitude of the lowest velocity in the wake zone and the recirculation length rise as the solid volume percentage increases, while the separation angle decreases. It concluded that at a given Reynolds number, local and average Nusselt numbers were increased by addition of nanoparticles to base fluid. Sandip Sarkar et al. [2] in 2012 investigates about the buoyancy-driven mixed convective flow and heat transfer characteristics of a water-based nanofluid past a circular cylinder in cross flow using a SUPG-based finite element method, the effect of aiding and opposing buoyancy is obtained by taking two representative Richardson numbers of 1 & -1. Sheikholeslami [3] has investigated two-dimensional nanofluid flow due to a stretching permeable tube. The equations are solved numerically using the fourth-order Runge–Kutta method. The results demonstrate that introducing a nanoparticle into the problem's base fluid can alter the flow pattern. The Nusselt number is observed to increase as the nanoparticle volume fraction, suction parameter, and Reynolds number increase. A steady two-dimensional laminar forced convection boundary layer flow along a horizontal thin needle immersed in a nanofluid is considered by Siti Khuzaimah Soid et al. [4]. The impacts of the needle size and the strong volume part boundary on the stream and warmth move qualities just as on the speed and temperature profiles are researched. Mehdi Fakour et al. [5] in his study used the Least square method to solve the unsteady flow problem and related heat transfer of a nanofluid over a horizontal plate. The accuracy of the least square method is checked with the numerical model. In comparison to other forms of nanofluids, the water-alumina nanofluid displayed greater heat transfer improvement. Reseachers Maskeen et al. [6] addressed about the heat transfer and fluid flow parameters of alumina–copper/water ( $\text{Al}_2\text{O}_3\text{-Cu/H}_2\text{O}$ ) hybrid nanofluid flow via a stretched cylinder were improved. Work compared hybrid nanofluids to base fluids and single material nanofluids, and discovered that hybrid nanofluids are more efficient at heat transmission than traditional fluids or single nanoparticle-based nanofluids.

Nadeem Abbas et al. [7] discussed the computational analysis of hybrid Nano fluid flow with inclined magnetic hydrodynamics over a movable stretching cylinder. Thermal and velocity slip effects are highlighted in this study. The magneto-hydrodynamic stagnation point Powell–Eyringnanofluid flow past an inclined cylinder is investigated by S. R. R. Reddy et al. [8]. Buongiorno nanofluid model utilized to express the flow equations. Proper converts drive to strongly nonlinear differential systems, which are solved by shooting procedure with the fourth-order R–K (Runge–Kutta) method. Some of his findings are; velocity and magnetic field are in inverse relation, in velocity profile Powell–Eyring fluid parameters have an opposite trend and Drag force decreases for large estimations of the curvature variable.

Many studies of the heat transfer rate in nano fluid have been conducted in recent years [9-12]. The thermal conductivity of nanofluid is influenced by concentration, temperature, nanoparticle diameter, and particle shape. In this research work we studied the heat transfer and fluid flow in water based silica nano fluid over stretching cylinder. Sphere, cylinder, platelet, and blade nanoparticles were used to calculate the Nusselt number and skin friction.

**Nomenclature:**

$U$	Surface velocity	$h_f$	Convective heat transfer constant
$\mathbf{V}$	Velocity field	$u, w$	Velocity components along $r, z$ directions
$B$	Magnetic field	$k_f$	Thermal conductivity of water
$\alpha, \alpha_1$ and $b$	Dimensional constants	$k_{nf}$	Thermal conductivity of nanofluid
$T$	Temperature of the nanofluid	$C_p$	Specific heat of water
$\nu_f$	Kinematic viscosity of water	$\alpha_f$	Thermal diffusion of base fluid
$\nu_{nf}$	Kinematic viscosity of nanofluid	$\alpha_{nf}$	Thermal diffusion of nanofluid
$(\rho C_p)_{nf}$	Heat capacity of nanofluid	$Nu$	Nusselt number
$\sigma_{nf}$	Electrical conductivity	$Re$	Reynolds number
$\rho_{nf}$	Density of nanofluid	$Pr$	Prandtl number
$\rho_f$	Density of fluid	$\eta$	Similarity variable
$\mu_{nf}$	Dynamic viscosity of nanofluid	$\phi$	Volume fraction of nanoparticles

**2. Problem Statement and Method of solution:**

In two dimensions, a Nanofluid is examined over a stretched cylinder. The flow coordinates are chosen so that the  $z$ -axis corresponds to a cylinder's axis. It is assumed that the fluid motion is unsteady and in the  $z$ -direction. Furthermore, the magnetic field is perpendicular to the flow in the  $r$ -direction and defined as  $B(t) = \frac{B_0}{(1-\alpha_1 t)^2}$ .

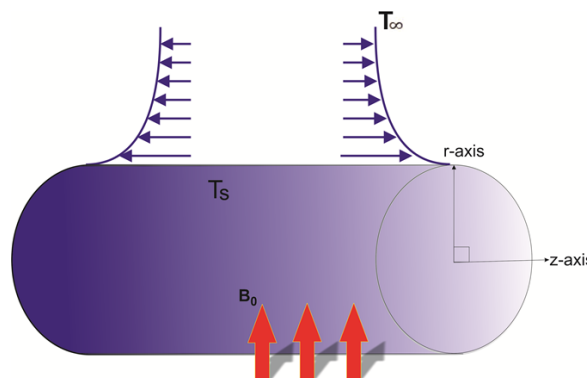


Figure. 2 Physical description of fluid flow model.

Surface velocity  $U(z, t) = \frac{bz}{1-\alpha_1 t}$  and  $T_s(z, t) = T_0 - T_r \frac{bz^2}{2v_f} (1 - \alpha_1 t)^{-\frac{3}{2}}$  is the temperature field.

We considered  $\vec{V} = [u(r, z, t), 0, w(r, z, t)]$  be the velocity field and  $T = T(r, z, t)$  is the temperature of the nanofluid. The conservation of mass, momentum and thermodynamic equations can be expressed as

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0 \tag{1}$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} = \frac{v_{nf}}{r} \frac{\partial}{\partial r} \left( r \frac{\partial w}{\partial r} \right) - w \frac{\sigma_{nf}}{\rho_{nf}} \cdot \frac{B^2_0}{(1 - \alpha_1 t)} \tag{2}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{\mu_{nf}}{\rho_{nf} C_p} \left( \frac{\partial w}{\partial r} \right)^2 + \alpha_{nf} \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \left( \frac{\partial T}{\partial r} \right) \right] \tag{3}$$

Subject to the boundaries conditions

$$w = U_w, u = 0, T_s = T \text{ at } r = R, w \rightarrow 0, T \rightarrow 0 \text{ as } r \rightarrow \infty. \tag{4}$$

In the above equations thermo-physical properties are defined as

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, (\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s, \frac{k_{nf}}{k_f} = \left[ \frac{k_s + (m-1)k_f + (m-1)(k_s - k_f)\phi}{k_s + (m-1)k_f - (k_s - k_f)\phi} \right]$$

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s, \mu_{nf} = \mu_f(1 + A_1\phi + A_2\phi^2) \text{ and } \sigma_{nf} = \sigma_f(1 - \phi) + \phi\sigma_s$$

Whereas  $h_f$  is convective heat transfer constant,  $A$  is proportionality constant,  $A_1$  and  $A_2$  are the viscosity enhancement heat capacitance coefficients, and  $\phi$  is volume fraction of silica.

The equations (1 – 3) subject to the boundaries conditions (4) are transformed by using the following Similarity transformation:

$$\Psi = (Uvz)^{\frac{1}{2}} R f(\eta), \eta = \frac{r^2 - R^2}{2R} \left( \frac{U}{vz} \right)^{\frac{1}{2}}, \theta(\eta) = \frac{T - T_0}{-T_r \left[ \frac{bz^2}{2v} \right]^{1-\alpha_1 t} \frac{1}{2}} \tag{5}$$

Here  $\Psi$  is stream function while  $u = \frac{-1}{r} \frac{\partial \Psi}{\partial z}, w = \frac{1}{r} \frac{\partial \Psi}{\partial r}$ , for which continuity equation remains true. And hence components of velocity can be written as

$$u = \frac{-R}{r} \left( \frac{bv}{1-\alpha_1 t} \right)^{\frac{1}{2}} f'(\eta) \text{ and } w = \frac{bz}{(1-\alpha_1 t)} f'(\eta)$$

So by using (5) after transforming equations (1 – 4) we obtained

$$[1 + 2C\eta]\epsilon_1 f''' + 2C\epsilon_1 f'' - [\epsilon_3 M + S]f' - \frac{S\eta}{2} f'' + f \cdot f'' - (f')^2 = 0 \tag{6}$$

$$[1 + 2C\eta]\theta'' + 2C\theta' + \frac{Pr}{\epsilon_2} [f \cdot \theta' - 2f' \theta - \frac{S}{2}(3\theta + \eta\theta')] + E_c \epsilon_1 (1 + 2C\eta)(f'')^2 = 0 \tag{7}$$

And boundary conditions are

$$f'(\infty) = 0, f'(0) = 1, \theta(0) = 1 \text{ and } \theta(\infty) = 0 \tag{8}$$

Where in the above dimensionless equations (6 – 8)  $C = \sqrt{\frac{(1-\alpha_1 t)v_f}{bR^2}}$  is curvature parameter,  $S = \frac{\alpha_1}{b}$  is

unsteadiness parameter,  $M = \frac{\sigma_f B^2_0}{\rho_f b}$  is magnetic parameter,  $Pr = \frac{v_f(\rho C_p)_f}{K_f}$  is Prandtl number

$$Re^{\frac{1}{2}} C_f = (1 + \phi A_1 + \phi^2 A_2) f''(0) \tag{9}$$

Also  $N_u = \frac{r q_w}{K_f (T_s - T_0)}$  with  $q_w = -K_{nf} \left[ \frac{\partial T}{\partial r} \right]_{r=R}$  and  $\frac{\partial T}{\partial r} = -\frac{rbz^2}{2vR} T_r \sqrt{\frac{b}{r}} \left[ \frac{1}{1-\alpha_1 t} \right]^2 \theta'$  and to get

$$Re^{\frac{1}{2}} N_u = -\frac{k_{nf}}{k_f} \theta'(0). \tag{10}$$

Here Reynold number  $Re$  defined as  $(=\frac{Uz}{\nu})$ .

### 3. Results and discussions:

This section includes discussion of numerical results that shows how significant physical parameters affect velocity and temperature profiles.

Figure 3 to 6 shows the impact of Volumetric fraction on velocity profile of nanofluid. The velocity of each multi-shape nanoparticle is increased for increasing value of nanoparticles volume fraction

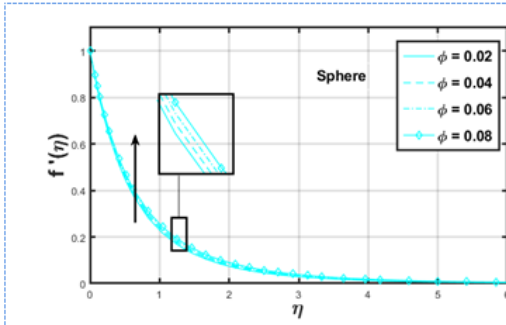


Figure 3.

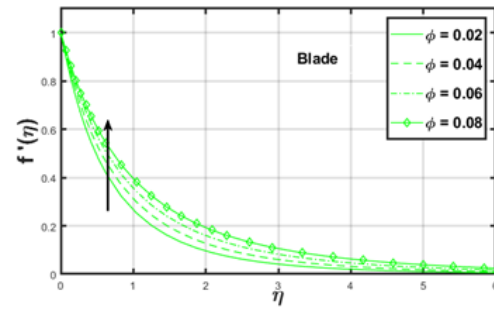


Figure 4.

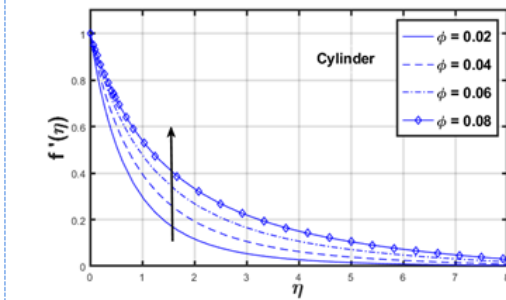


Figure 5.

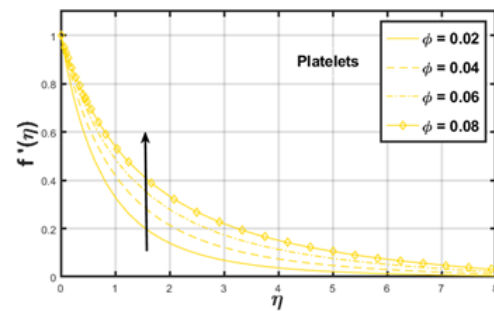


Figure 6.

Figure (3-6). Impression of  $\phi$  on  $f'(\eta)$  for sphere, blade, cylinder and platelets shaped nanoparticles, keeping  $M = 1.0, Ec = 1.0, S = 0.3, K = 0.5$  are fixed.

Impact of shape factors on velocity profile is plotted in figure 7. The highest value is observed for platelet shape. While changing shape factor other parameters are kept fixed.

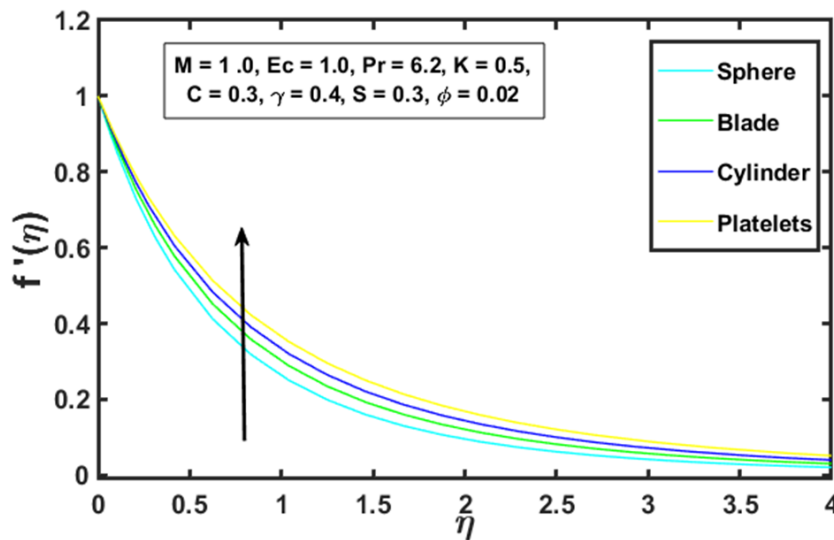


Figure 7. Demeanor of  $\phi$  on  $\theta(\eta)$

Figure 8 indicates the effect of magnetic parameter on temperature profile. It is noticed that the temperature distribution is increased by increasing values of the magnetic parameter  $M$ . As rise in the magnetic parameter indicates an increase in the magnetic field, which opposes fluid movement and, as a frictional force, leads to an increase in nanofluid temperature.

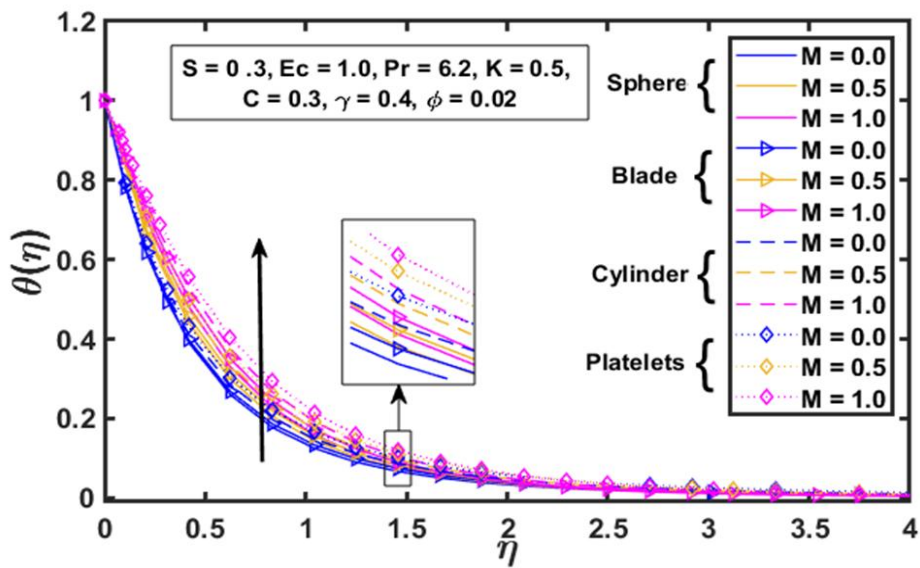


Figure 8. Effect of  $M$  on  $\theta(\eta)$

Figure 9 shows the relationship between Eckert number and temperature. As  $Ec$  is raised, the temperature rises because the increase in kinetic energy causes a rise in the temperature distribution. Figure 10 demonstrates that a decrease in the temperature profile results from increasing the value of the unsteadiness parameter  $S$ . This is a real physical phenomenon because cylinder stretching increases heat loss with increasing unsteadiness, which lowers the temperature profile.

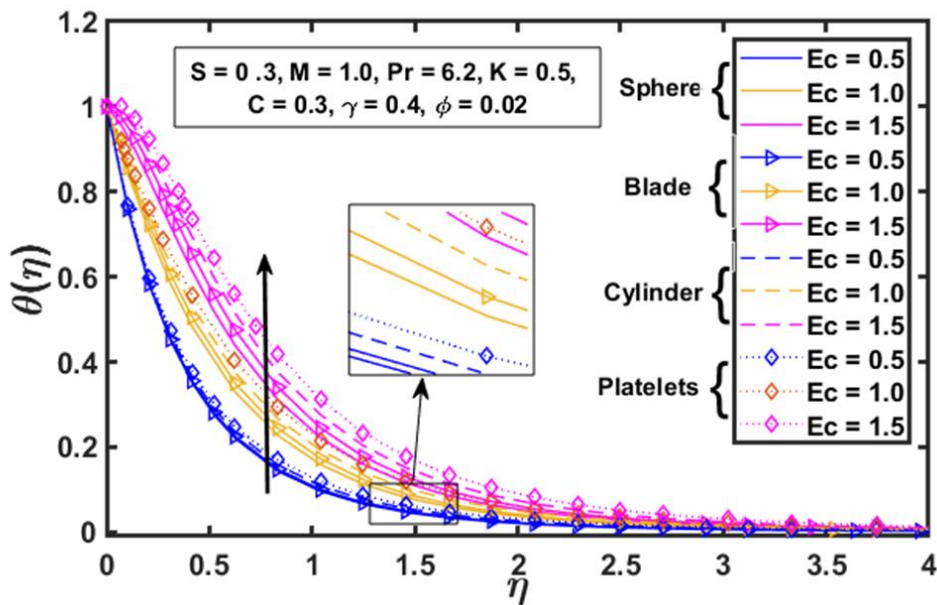


Figure 9. Effect of  $Ec$  on  $\theta(\eta)$

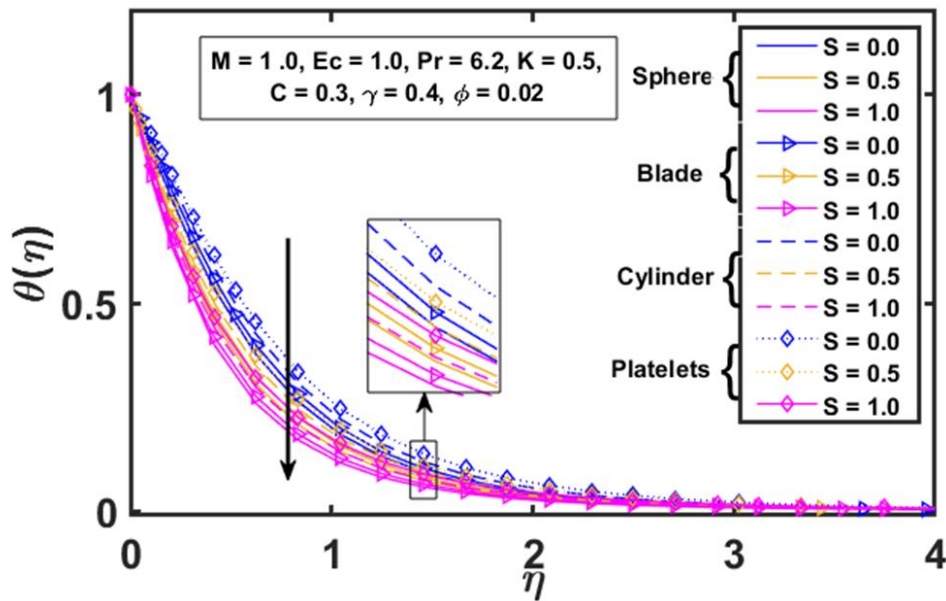


Figure 10. Effect of  $S$  on  $\theta(\eta)$

Nanoparticle volumetric fractions played a significant role in rising skin friction of  $SiO_2$  nanofluid. The value of skin friction increases for volumetric fraction and unsteadiness parameters. Figure 11 displays values for skin friction in the form of a column chart. Sphere shaped nanoparticles showed lowest values of skin friction while highest value is noted for platelets shape. Figure 12 shows the Nusselt number values for the variations in the Eckert number, volume %, unsteadiness parameter, and magnetic parameter. The magnitude of Nusselt number decreases as  $M$  and  $E_c$  increases, whereas by increasing in volumetric percentage and unsteadiness parameter. Nusselt number also increased. Additionally, the platelet-shaped  $SiO_2$  nanoparticles have the greatest Nusselt number whereas sphere-shaped  $SiO_2$  nanoparticles have the lowest value.

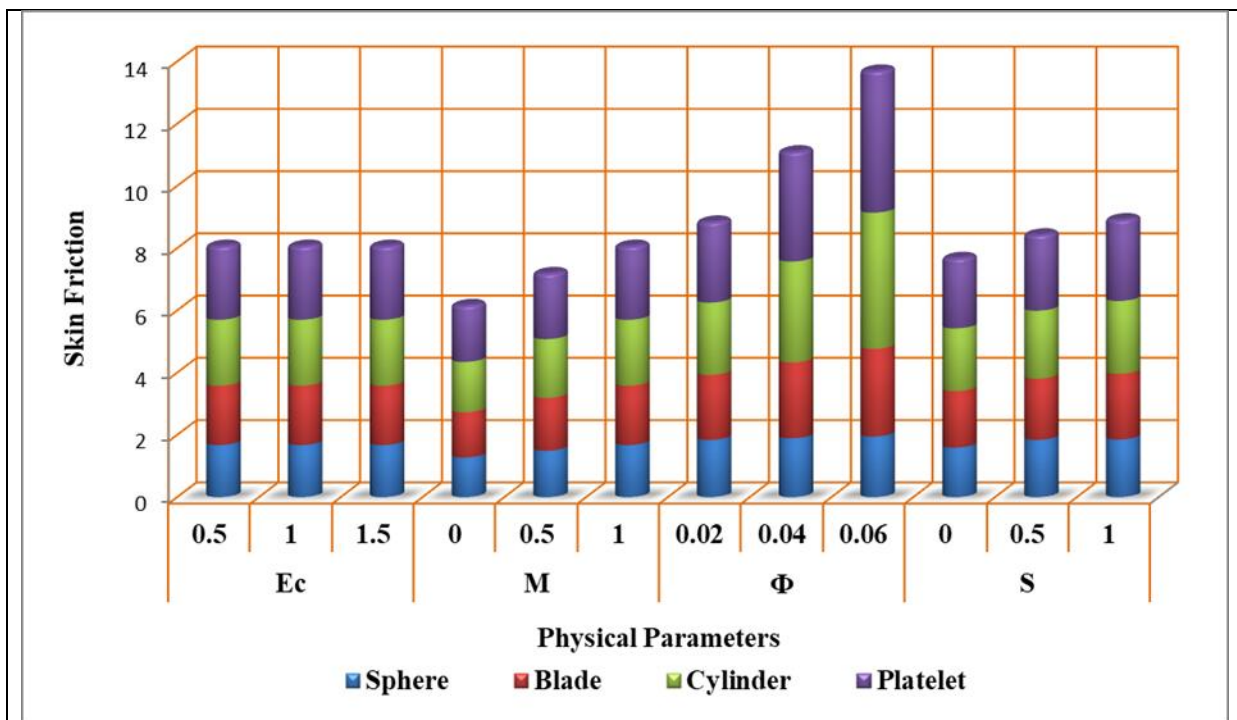


Figure 11. Skin Friction results for the Impact of  $Ec$ ,  $M$ ,  $\phi$ ,  $S$  on sphere, Blade, Cylinder and Platelet shape

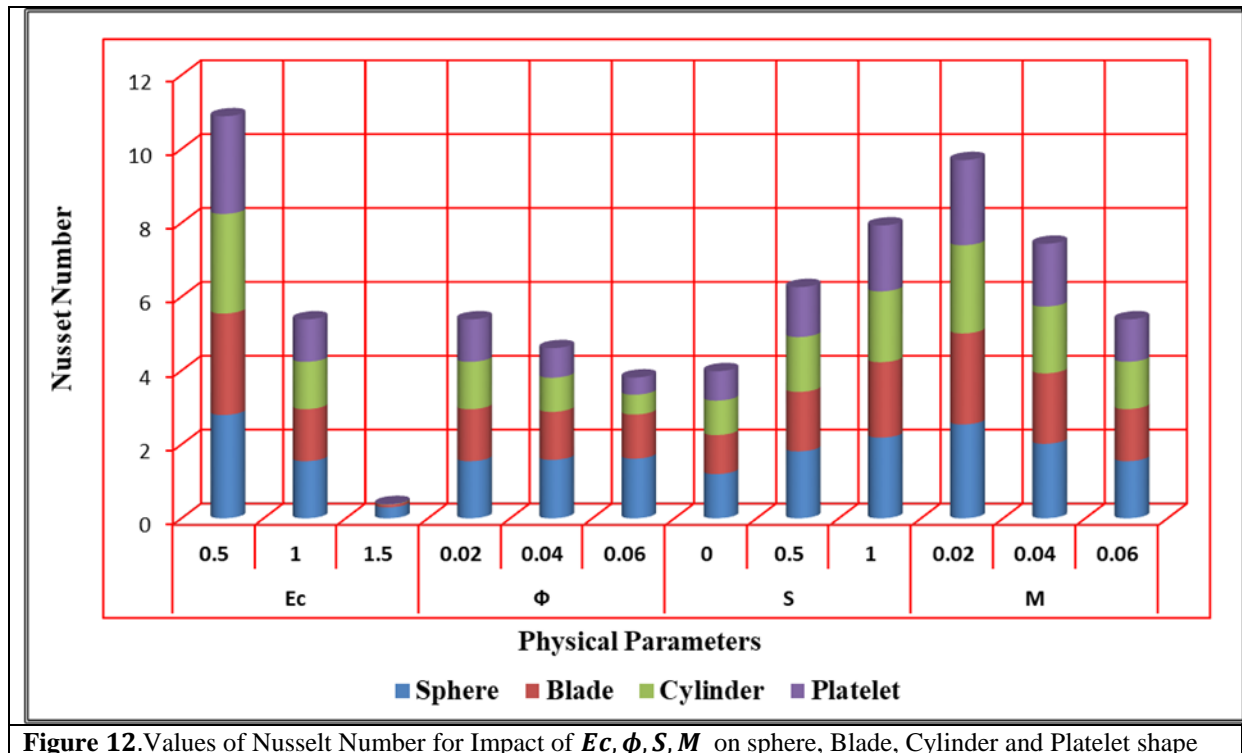


Figure 12. Values of Nusselt Number for Impact of  $E_c, \phi, S, M$  on sphere, Blade, Cylinder and Platelet shape

## 5. Conclusion:

The current problem's final findings are listed below:

- The increase in Volume fraction  $\phi$  upsurges the velocity profile.
- When the Eckert number  $E_c$  and the magnetic field parameter  $M$  are increased, the temperature field also surges, while decreasing trend is noted for the rising value of unsteadiness parameter  $S$ .
- In contrast to Eckert number, magnetic parameter, and volume fraction, Nusselt number has shown growing behavior for the increasing values  $S$ .
- By elevating Magnetic Parameter, Volume Fraction, and Unsteadiness parameter values, skin friction value rose.

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