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On Finding Integer Solutions to Non- homogeneous Ternary Bi-quadratic Equation

$$5(x^2+y^2)-2xy=140z^4$$

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Abstract:
This paper concerns with the problem of obtaining non-zero distinct integer solutions to the non-homogeneous ternary bi-quadratic equation $5(x^2+y^2)-2xy=140z^4$. Different sets of integer solutions are illustrated.

Keywords: non-homogeneous bi-quadratic ,ternary bi-quadratic integer solutions

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I.INTRODUCTION:

The Diophantine equations are rich in variety and offer an unlimited field for research [1-4]. In particular refer [5-28] for a few problems on bi-quadratic equation with 3 unknowns. This paper concerns with yet another interesting bi-quadratic diophantine equation with three variables given by $5(x^2+y^2)-2xy=140z^4$ for determining its infinitely many non-zero distinct integral solutions

II. METHOD OF ANALYSIS:

The non-homogeneous ternary bi-quadratic equation under consideration is

$$5(x^2+y^2)-2xy=140z^4 \qquad (1)$$

Introduction of the linear transformations

$$x=(3u+11v),\ y=5(u-v),\ u\neq v\neq 0 \qquad (2)$$

in (1) leads to

$$u^2+6v^2=z^4 \qquad (3)$$

Solving (3) for u,v,z through different ways as illustrated below and using (2), one obtains the corresponding integer solutions to (1).

Way 1:

Let

$$z=a^2+6b^2 \qquad (4)$$

Substituting (4) in (3) and employing the method of factorization, consider

$$u+i\sqrt{6}v=(a+i\sqrt{6}b)^4 \qquad (5)$$

On equating the real and imaginary parts in (5), the values of u,v are obtained.

In view of (2), the corresponding integer solutions to (1) are obtained as

$$\left.\begin{array}{l} x=3(a^4-36a^2b^2+36b^4)+11(4a^3b-24ab^3),\\ y=5(a^4-36a^2b^2+36b^4-4a^3b+24ab^3)\end{array}\right\}$$

along with (4).

Way 2:
Rewrite (3) as

$$u^2 + 6v^2 = z^4 * 1 \qquad (6)$$

Consider 1 on the R.H.S. of (6) as

$$1 = \frac{(1+i2\sqrt{6})(1-i2\sqrt{6})}{25} \qquad (7)$$

Following the procedure as in Way1, the corresponding integer solutions to (1) are found to be

$$x = 5(a^4 - 36a^2b^2 + 36b^4 - 4a^3b + 24ab^3),$$
$$y = -(a^4 - 36a^2b^2 + 36b^4) - 13(4a^3b - 24ab^3),$$
$$z = (a^2 + 6b^2)$$

Note 1:
Apart from (7),the integer 1 on the R.H.S. of (6) is also expressed as below:

$$1 = \frac{(5+i4\sqrt{6})(5-i4\sqrt{6})}{11^2},$$
$$1 = \frac{(6r^2 - s^2 + i2\sqrt{6}rs)(6r^2 - s^2 - i2\sqrt{6}rs)}{(6r^2+s^2)^2}$$

Repeating the above process, two different sets of solutions to (1) are obtained.

Way 3:
Observe that (3) is satisfied by

$$v = 2rs, u = 6r^2 - s^2, z^2 = 6r^2 + s^2 \qquad (8)$$

In (8),solving for z , one obtains

$$r = 2pq, s = 6p^2 - q^2 \qquad (9)$$

and

$$z = 6p^2 + q^2 \qquad (10)$$

Using (9) in (10) and employing (2), one has

$$\left.\begin{aligned} x &= -108p^4 - 3q^4 + 108p^2q^2 + 264p^3q - 44pq^3, \\ y &= -180p^4 - 5q^4 + 180p^2q^2 - 120p^3q + 20pq^3 \end{aligned}\right\} \qquad (11)$$

Thus,(10) and (11) represent the integer solutions to (1).

Way 4:
In (8),the third equation is written as the system of double equations as below in Table:1

Table:1-system of double equations

System	I	II	III	IV
$z+s$	r^2	$3r^2$	$6r$	$3r$
$z-s$	6	2	r	$2r$

Solving each of the above system of equations, the corresponding values of z, r, s are obtained.
From (8),the values of u, v are found .In view of (2), the corresponding integer solutions to (1) are determined.
For brevity ,the integer solutions obtained from each of the above four systems are exhibited as follows:
Solutions from system I:

$$x = -12k^4 + 88k^3 + 108k^2 - 132k - 27,$$
$$y = -20k^4 - 40k^3 + 180k^2 + 60k - 45,$$
$$z = 2k^2 + 3$$

Solutions from system II:

$$x = -108k^4 + 264k^3 + 108k^2 - 44k - 3,$$
$$y = -180k^4 - 120k^3 + 180k^2 + 20k - 5,$$
$$z = 6k^2 + 1$$

Solutions from system III:

$$x = 217k^2, y = -105k^2, z = 7k$$

Solutions from system IV:

$$x = 113k^2, y = 95k^2, z = 5k$$

Way 5:

In (8),the third equation is written as

$$z^2 - 6r^2 = s^2 * 1 \qquad (12)$$

Let

$$s = c^2 - 6d^2 \qquad (13)$$

Write 1 0n the R.H.S. of (12) as

$$1 = (5 + 2\sqrt{6})(5 - 2\sqrt{6}) \qquad (14)$$

Substituting (13),(14) in(12) and employing the method of factorization, consider

$$z + \sqrt{6}r = (5 + 2\sqrt{6})(c + \sqrt{6}d)^2$$

Equating the rational and irrational parts,the values of z,r are obtained.Substituting the values of r,s in (8) and using (2),the corresponding integer solutions to (1) are found to be

$$x = 113c^4 + 900d^4 + 2700c^2d^2 + 940c^3d + 3000cd^3,$$
$$y = 95c^4 + 4860d^4 + 4500c^2d^2 + 110c^3d + 7800cd^3,$$
$$z = 5c^2 + 30d^2 + 24cd$$

Note 2:

Apart from (14),the integer 1 on the R.H.S. of (12) is also expressed as below

$$1 = \frac{(11 + 4\sqrt{6})(11 - 4\sqrt{6})}{5^2}$$

from which a different set of solutions to (1) is obtained.

III.CONCLUSION:

An attempt has been made to obtain non-zero distinct integer solutions to the non-homogeneous bi-quadratic diophantine equation with three unknowns given by $5(x^2 + y^2) - 2xy = 140z^4$.One may search for other sets of integer solutions to the considered equation as well as other choices of the fourth degree diophantine equations with multi-variables

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