

What four and eight Systems do for you

Gudrun Kalmbach H.E.

MINT, PF 1533, D-86818 Bad Woerishofen, Germany

Abstract

The author proposes color charges as an independent force, not only as a property of nucleons quarks. In this context analysis, objects like quarks are characterized by their properties. They are a color charge, a mass GR and an electrical charge EM. Mass relates to the gravitational force, EM to the electromagnetism as force. In this sense color charges act also as force.

Attributing to color charges a geometrical action, the proposal is that they present perspective projections, geometrically described by the cross ratios. It is defined by four points, variables or parameters and allows permutations of them.

It is investigated what kind of geometries can be induced from them for the other physical forces in the early development of the universe. The number four shows up for spacetime $xyzt$ -coordinates, for their presentation through Pauli matrices (including an identity id map). EM and the weak interaction use mainly these coordinates and symmetries. For the strong interaction the doubling for 8 gluons is presented by adding to Pauli 2×2 -matrix symmetries a third row and column of 0 entries. The nine GellMann matrices have for the third Pauli matrix only two extending matrices since the three are linearly dependent. Therefore only 8 gluons exist. For gravity as force physics has no geometrical suggestion concerning the number four. Higgs sets mass as barycenters of systems, the author suggests rgb -graviton field quantum whirls for gravity. In a coordinate combination of octonian coordinates, listed by indices $0,1,2,\dots,7$, rgb is 126 and mass 5, giving a 4-dimensional 1256 octonian subspace for GR. The energies mass frequency 5 m, 6 f replace spacetime coordinates 34, written as complex variable $z_1 = z + ict$. GR has only a real 2-dimensional space, complex written $z_2 = x+iy$. It is argued that $z_3 = (m.f)$ energies extension of spacetime is a complex cross product $z_3 = z_1 \times z_2$. Using again a complex cross product $z_4 = z_1 \times z_2 \times z_3$ generates octonians with $z_4 = (u,w)$, $u \in \{r,g,b,c(r),c(g),c(b)\}$ an octonian color charge parameter and w a linear octonian 7 coordinate. 7 is for the electromagnetic interactions symmetry $U(1)$ stereographic closed to a Kaluza-Klein circle. Attributing to the six color charges that they can be presented by cross ratios, the geometrical figure 1 shows that they project in a perspective view four collinear points, representing real coordinate lines, to another collinear quadruple. This makes eight from four for coordinates. More details to octonian presentations extending quaternionic spacetime vectors of physics are studied in relation to color charges cross ratios or for cross products.

Keywords: color charges, cross product, cross ratio, perspectivity, projections

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The $xyzt$ - and (m,f,u,w) -coordinates

The perspective projection of four collinear points onto four other collinear points in figure 1 are taken for $X = P,Q,R,S$ as x,y,z,t . The question is how X' associates with the the other octonian coordinates. Heisenberg suggests 15 (x,m) , 46 (t,f) , but his 23 uncertainty is not fitting. It is replaced by 03 (u,z) and 27 (y,w) .

Are they new kinds of uncertainties? The author suggested in another article that instead of the Heisenberg lower bound as scaled Planck number h , there is an upper bound c , speed of light, as uncertainty for 03. Systems in the universe have speeds smaller (or equal) to c . The Minkowski cone for spacelike vectors separates in physics them from timelike vectors inside the cone. The vectors use the coordinate 0. 3 can be taken as a radius variable r for the cone equation $r^2 - c^2t^2 = 0$, t time. As a kind of uncertainty, 03 ccan mean for EMI light that its cylindrical axis must be broken in a 3 spherical angle θ towards the cylinders z -axis when absorbing energy from a huge mass system it passes by (double lensing). The question is what 27 can present. In the list of Planck numbers there are the constants h,c,G,k , G gravitational constant, k for heat (Kelvin). Temperature as energy is associated with 2, the y -coordinate of space. Introducing polar complex coordinates (r,ϕ) the map $\phi \rightarrow \exp(i\phi)$ associates with an angle on 2 the exponential function \exp for wave descriptions of systems on 7. In a complex plane \exp has the usual equations for transferring linear xy to polar coordinates. For entropy in a volume an

uncertainty can mean that it cannot reach the absolute zero value for temperature. For temperature $T = 0$ no spacial volume is available. Since for energy holds $E = kT$, also energy $E = 0$, no frequency, no mass. Since time as interval is inverse to frequency, time is set at ∞ . It cannot be reached by the expanding universe or another version of the universes collapse can occur through the use of a projective map. As a kind of uncertainty for gravity can be mentioned redshift of light, the eletromagnetic interaction energy in its time expansion. Photons energy looses against gravity frequency f through $mc^2 = hf$ and get a larger wave length in $\lambda f = c$.

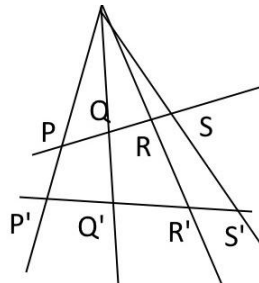


Figure 1 cross ratio as perspective projection, mapping the four collinear points PQRS on the upper line to the lower line with PQRS and P'Q'R'S' having the same cross ratio value $(P,Q;R,S) = (P'R';Q'S') / (Q'R';P'S') = (P',Q';R',S')$

In another projective construction, four collinear points occur for the construction of a harmonic middle Q between P,R when S is considered as a projective point at infinity.

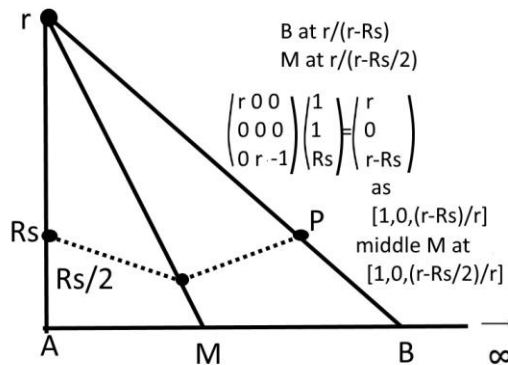


Figure 2 construction of a harmonic middle M for the Schwarzschild radius $R_s = 2Gm/c^2$

In another article of the author, the third Einstein general relativistic effect beside double lensing and redshift is spacetime curvature, computed by his energy-momentum tensor. The rescaling of Minkoskwi metric by $\sin^2\beta = R_s/r$ provides another (uncertainty) angular effect for the polar angle ϕ . As a speed acceleration, due to gravity, a periodic angle ϕ_0 is added after one revolution fro the rosette motion of a planet rotating about a central sun. Its Kepler orbit ellipse gets a shift by this angle for its main ellipse diagonal. All three spherical space coordinates are then renormed for gravity by 03 for θ and c (double lensing), radial interpreted wave length (redshift, G) and polar angle, R_s for rotations (rosette orbits of planets). Recall: relativistic mass as frequency 6 of light allows with θ double lensing, mass acts for wave length 1 as redshift frequency and a GR accelerated speed of planets changes for 2 Kepler ellipses orbits to rosettes for planets. Gravitons are presented as 126 *rgb* whirls in octonians. 2 is added as 2356 to the SI rotor measuring triple 356, 6, 6 is added as 1456 to the EM measuring triple 145 and 1 (taken) as real z-coordinate and cross product is added to 23 space xy-coordinates in 1234, where 4 is for time. In figure 1, the perspective projection can map PQRS as 2356 (56) or 1456 (56) to 1234 where SI or EM/WI are observed.

In the definition of Planck numbers, the gravitational constant G in the Schwarzschild radius scales mass. In figure 2 the point marked on the interval rM is D. (D,P,r,R_s) are the points of an affine geometrical configuration which allows the computation of a harmonic middle M between AB.

Instead of redshift involving G, this constant is seen for providing to 3 collinear points like 0,1, ∞ a harmonic middle, in this case the number $\frac{1}{2}$. If the three values are permuted in a cross ratio $(z,0;1,\infty)$, the six Moebius

transformations for color charges, written as complex fractions, are $z, 1/z, (1-z), 1/(1-z), z/(z-1), (z-1)/z$. They are invariants of the complex Riemannian sphere S^2 having as symmetry the Moebius transformations. A degenerate numerical orbit of them are the scaled three basic spin values $z = -1, 2, 1/2$ for bosons (as 1), graviton and fermions. In figure 2 for the second cosmic speed v_2 which uses R_s , the harmonic middle is computed as $R_s/2$ used as gravitational potential for the first cosmic speed v_1 of a mass system W . For an orbiting mass system U about a central W with Schwarzschild radius R_s , the kinetic energy with speed v of U is compared with v_j as usual. The bounds are in this case due to a catastrophe cusp which allows in its control space potential jumps between free fall of U , rotation or escape of U from W when the gravitational interaction ends.

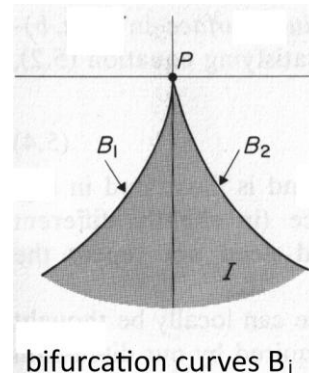
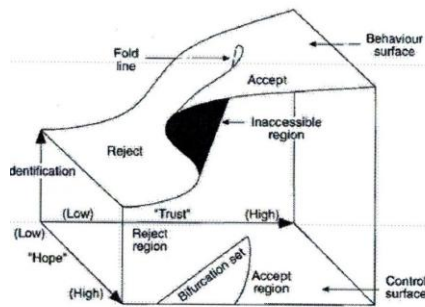


Figure 3 with a catastrophe potential $V = x^4/4 + ax^2/2 + bx$, x a variable, a, b parameters for the control space (at right) with the equation $4a^3 + 27b^3 = 0$. The catastrophe manifold has the equation $x^3 + ax + b = 0$.

Harmonic middle and projective quadruple

Back to the projective figure 2: the basic harmonic configuration in figure 4 has four (affine) points A, B, C, D , no three are collinear. They are pairwise connected by lines and projective extended in this configuration by intersection points of lines P, Q, R and the line PQ which gives to more intersection points M, N in the order first P then M then Q then N . The last point is often taken at projective infinity and M is the projective middle between P, Q .

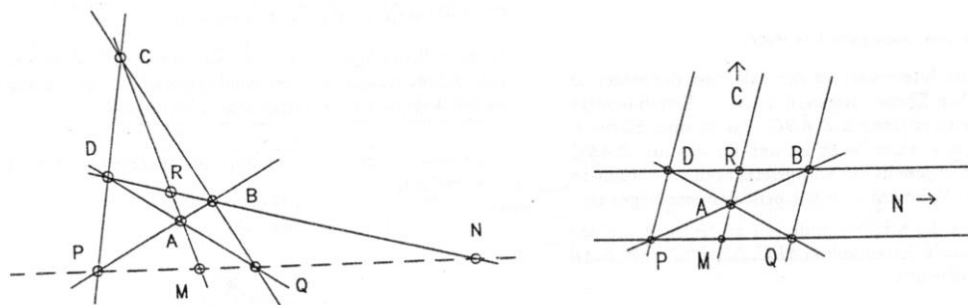


Figure 4 harmonic projective configuration, M as harmonic middle between P, Q

In the projective configuration at left in figure 4, the 4 points A, B, C, D are generating through their connecting lines first the three new points P, Q, R ; they are joined by lines PQ or contain the intersecting point of AC with DB as R . The intersecting points of BD, PQ as N and of AC with PQ as M complete the harmonic configuration, having 7 lines and 9 points. As perspective projection from C , the quadruple D, R, B, N is mapped to P, M, Q, N . Four more points can be added as in figure 5 and 6 more lines. This configuration has 13 points and 13 lines. Used is the Desargues theorem. In this configuration the points C or N can be shifted onto a projective line at infinity in the real projective plane P^2 . It arises for instance by mapping rays in space R^3 onto their intersection with the unit sphere S^2 and then identifying diametrical opposite points $A, -A$ to one point $|A|$. Homogeneous coordinates for P^2 can be $[x, y, w]$. Some interpretations of the new points and lines or coordinates in the harmonic configuration are for a research project, one is for M as middle of PQ , N as a point at infinity $[1, 0, 0]$, C can be another such point at infinity $[0, 1, 0]$, the line g at infinity is $[x, 1, 0]$ and renamed P^2 coordinates are listed as $[x, y, 1]$.

For metrical quadrics figure 5 shows the well used intersections of a circle with g for Kepler orbits as rotational ellipse orbits, or parabola, hyperbola as escape orbits. In metrical form, the ellipse quadric extends 3-dimensional to the Euclidean metric for space. The hyperbola orbit has for the hyperbola axes two intersecting lines which in rotation generate a Minkowski cone, used for the 4-dimensional Minkowski metric. The parabola can serve as a catastrophe fold where the potential function $V = x^3/3 + ax$, (x as variable, a a constant or parameter) has two energy states maximum, minimum and one bifurcation point for sudden changes, jumps between max-min. The catastrophe manifold is $x^2 + a = 0$, a parabola ($x, -x^2$), setting $a = -x^2$.

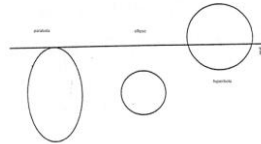


Figure 5 line at infinity, ellipse has no point in common, parabola one, hyperbola two

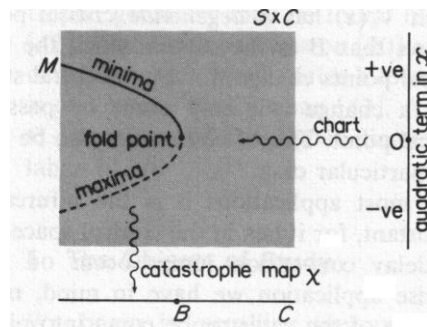


Figure 6 fold with two max -ve, min +ve states for its potential

For a Feigenbaum subspace bifurcation of energies (in figure 8 as Pascal configuration), the authors book on orthonormal lattices is quoted. In a real or complex 4-dimensional lattice diagram, the block structure of commuting projection maps listed as subspaces require for a 4-cycle a central astroid. The (curved) intervals are for Boolean sublattices having four points on a connecting interval their intersections consist of 1 or 2 points. The Pascal bifurcation is listed as 1456 interval, 0237 interval and other combinations in figure 7. At right in figure 7 is the required Boolean 4D-extension for a 3-cycle of commuting subspaces (projections). The octonians are used for the astroid coordinates. 1234 for the 4-cycle.

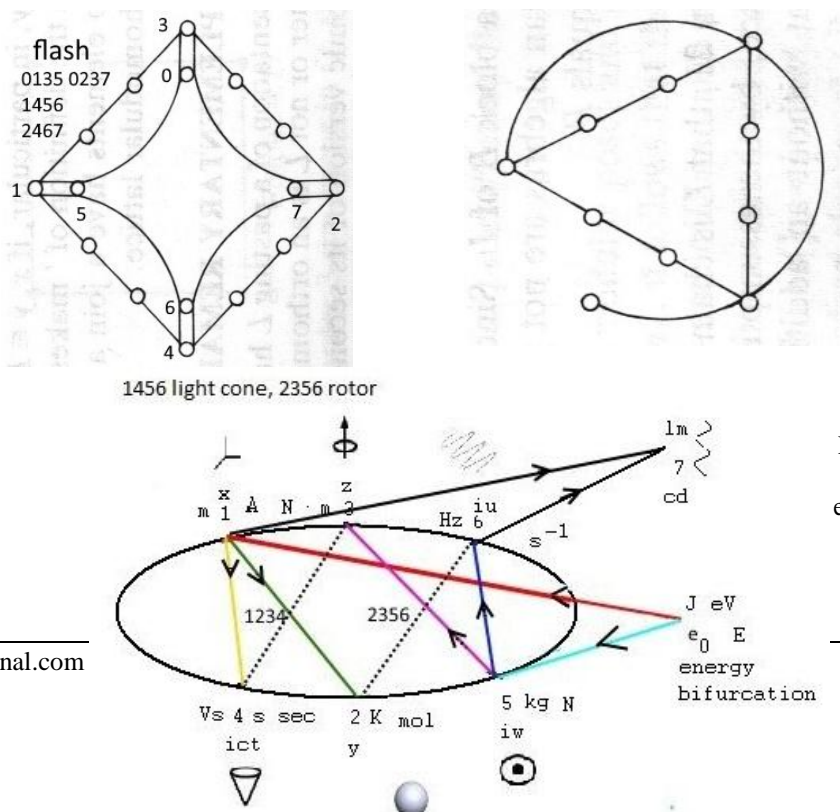


Figure 7 astroid and Boolean 4D-extension (circle) for a 3-cycle at right

Figure 8 Feigenbaum bifurcation of energies

The orthomodular lattice figures in figure 7 are not available in 3 dimensions where 3- and 4-cycles cannot exist. The real or complex Hilbert space for them can be taken having a Euclidean or Hermitian metric. The Minkowski metric has for the real case a projective operator T_u for its coordinates u which has a Morse function metric, in diagonal form $\text{diag}[1,1,1,-1]$ for $u = (x,y,z,ict)$ coordinates of spacetime.

Tetrahedron configuration and cross ratios

The author uses a tetrahedron configuration for nucleons in the octonian subspace 2356. Its four vertices have no three collinear points. In case the base triangle has vertices $0,1,\infty$, the circumference of the triangle can be taken as a projective line, containing them. The tip of the cone is taken as origin for a *rgb*-graviton base triple, adding to the triangle base vertices these color charges *r,g,b* on quarks vertex barycenters in a nucleon. The spin-like *rgb*-graviton 126 acts as a rotation axis of the tetrahedron of order 3 for the reference triples $0,1,\infty$ of three triangle reflections $1/z, z/(z-1), (1-z)$ of the D_3 cross ratios symmetry, generating barycentric coordinates for the triangle. Higgs sets nucleons mass at their intersection as barycenter. For the nucleons dynamical SI rotor, the gluon exchanges use two plane reflections of the tetrahedron of order 2 which interchange alternatively *gb* and *rb* vertices in a 6 cycle for $rgb \rightarrow rbg \rightarrow brg \rightarrow grb \rightarrow gbr \rightarrow bgr \rightarrow rgb \dots$. The nucleon dynamics is described in other articles of the author. Here is added that the S_4 symmetry of the tetrahedron of order 24 projects by the normal Klein group $Z_2 \times Z_2$ to D_3 , the symmetry of permuting the 3 triangle vertices. D_3 can be taken as symmetry for six color charges, adding to $w = r,g,b$ their conjugates $c(w)$. The complex Riemannian sphere S^2 mentioned above has been used for presenting color charges as cross ratios. As a new force their geometry can be a G-compass with the six segments for the sixth roots of unity. In rotation, clockwise *cw* or counterclockwise *mpo*, the vectors cover the next segment with the charge. Rolling up the cutted out segments to cones, the color charges are conic whirls like magnetic field quantum. The G-compass has as 2x2-matrix the first row $(1 \ -1)$, as second row $(0 \ 1)$ which suitable scaled presents the quotient $(r - Rs)/r$ for the Schwarzschild scaling factor of Minkowski metric. The rotations cross ratio is $(z-1)/z$.

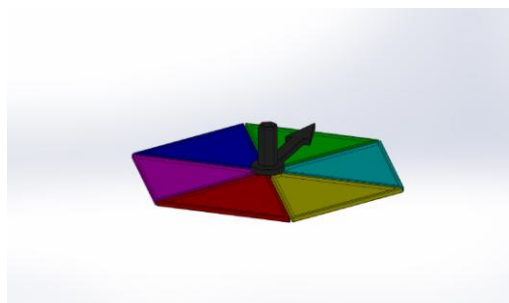


Figure 9 G-compass

The factor classes of S_4 contain a color charge, an octonian coordinate 1,2,...,6, an associated D_3 symmetry whose eigenvector measures as unit an energy as potential EM or GR energy 1,5, speeds as kinetic or rotational energy 6,2, mass and magnetic energy 5,4.

Concerning *rgb*-graviton whirls, their wave presentation ia compared with the electrical case. As electromagnetic waves EMI have been generated, from the Hopf fiber bundle the Hopf sphere $S^2 = h(S^3)$ contributes with main quantum numbers as latitude circles for the rotating electrical charge the frequency when the electron in an atoms shell jumps between radii and sends out or absorbes energy in form of photons, spectral series. The EMI symmetry is a circle $U(1)$, arising as a Lissajous figure from two orthogonal hitting frequencies in proportion 1:1. It uses the universal cover of the circle for mapping in time the revolution frequency $\omega = 2\pi f$ about $U(1)$ onto a helix winding as photon to $f = 1/\Delta t \equiv (1/n - 1/n')$, generated in a time interval Δt and having a wave length λ with speed $\lambda f = c$.

The *rgb*-graviton case uses Δt and $f = 1/\Delta t$ for the frequency of a graviton wave where Δt is the discrete time interval in which the SI rotor changes the states of a nucleon by a gluon exchange between paired quarks. The tetrahedron configuration shows the *rgb*-graviton as tip with base the quark triangle (figure 10). The Hopf S^2 and an electron is replaced by a presentation of the traingles

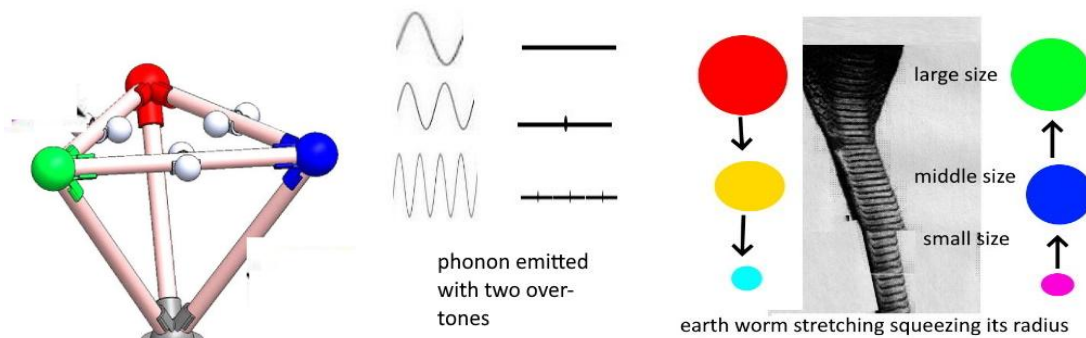


Figure 10 nucleon tetrahedron with r,g,b colored quarks and gluons on the triangle sides; earthworm

D_3 symmetry as SI rotor. Two Bohr radii are replaced by three radial sizes of the nucleon triangle as stretching squeezing of the *rgb*-graviton. In frequencies proportion 2 for $1/2:1$ and $1:2$ an $\exp(i\beta)$ angle replaces the EMI $\exp(i\varphi)$ angle as $\sin^2\beta = Rs/r$, related to the second cosmic speed of the nucleon, while EMI uses $\sin\varphi = v/c$, the Minowski relative speed. The three normed basic spin values $1/2:1:2$ are used for the SI rotor as Bohr radii of the nucleons quark triangle. The $U(1)$ circle is replaced by the three radius scaled boundary of the SI rotors conic rotations. They arise in a 6-cycle for time Δt . There is a catastrophe computation for discrete water drops one above the other. For *rgb*-gravitons this is modified for six circles one above another, alternating in size. The small and large size are repeated in the sequence. The cylinder for the helix line of an EMI wave is radius stretched and squeezed for the graviton wave, curved like an earthworm (figure 10). Instead of one helix winding for a photon on the cylinder, there are two helix lines in diametrical distance on the circle. One winding is done between the large and small circle. For the middle size circle it is half a winding $\beta = \pi, 2\pi$. The sinus is used in this case for $\sin^2\beta = Rs/r$, the Schwarzschild metrical rescaling factor instead of the Minkowski speed. In both gravitational cases the change of size in proportion $1/2:1:2$ has a phonon exchange with the environment, observed as moise in the universe or as send out ba a pulsar (star). For the earthworm the length of the modified helix is vibrating as tone c . For the middle size circle is added the vibration as overtone c' , for the small size circle the overtone c'' is generated and a phonon can be emitted. For phonon absorbtion this is reversed. The SI rotor is descibed in the quoted literature. The gluon exchange of the SI rotor is replaced by a frequency color charge $uc(u)$ interaction between the two modified helix lines of the earthworm. The earthworm generates the rescaling of Minkowskl to Schwarzschild metric. One more comment is for the EMI redshift. Also for EMI waves the wave equation can have a function depending on the length the wave has travelled in time. Periodically or stochastically the wave emits a phonons energy, its frequency decreases, its wave length is stretched.

Doubling of coordinates and cross product space extensions

The change of space dimensions is also visible in matrix form. The first extension from real linear coordinates to complex 2×2 -matrices is due to base vectors of xy -space $(1,0)$ for x and $(0,1)$ or $(0,-1)$ for y . If taken as rows of a matrix, the identity and the Pauli σ_2 2×2 -matrices are obtained. The second matrix multiplies like the imaginary unit i . In this added matrix form, complex numbers are written as (x, y) as first matrix row and $(-y, x)$ as second row. The determinant is the radial metrical quadric $r^2 = x^2 + y^2$. The fourth roots of unity are for sigend $1, i$ numbers. For the magnetic symmetry, the quadrangle has eight segments (figure 10). The doubling of complex coordinates to quaternions uses Euler angle rotations, demonstrated for instance by the wheel where cw or mpo

rotations about a x-, y- or z-axis generate from angles the quaternions, written in 2x2-matrix form with complex entries $(z_1 z_2)$ as first row and $(-z_2 z_1)$ as second row. This kind of doubling is repeated for octonians as 2x2-matrices with first row quaternions q_j in $(q_1 q_2)$, second row $(-q_2 q_1)$.

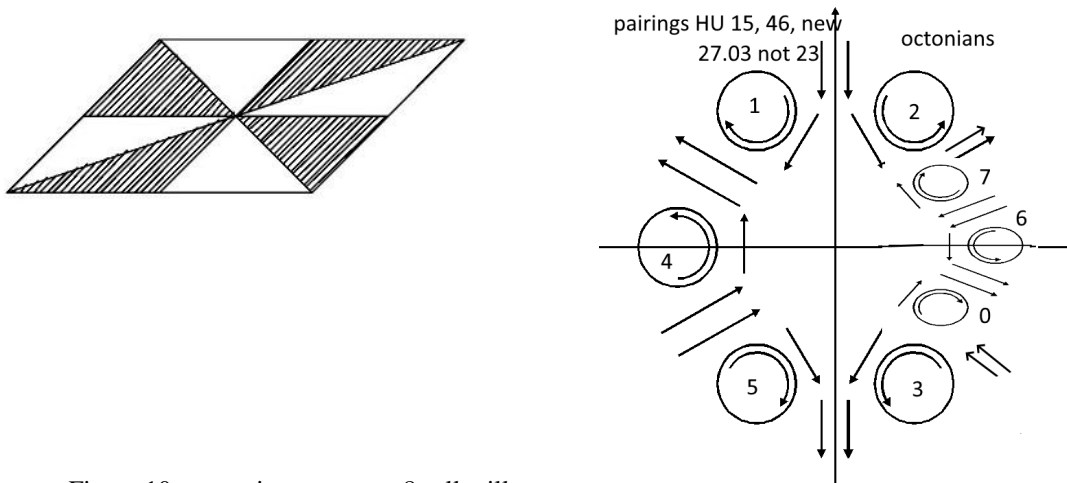


Figure 10 magnetic symmetry; 8 roll mill

The cross product expansions add first to xy-space coordinates the third z-coordinate, then to xyz the time t-coordinate. The 56 mass-frequency coordinates are added as complex z_3 cross product of space (z_1, z_2) coordinates and a complex written z_4 coordinate as cross product to the (z_1, z_2, z_3) coordinates for the octonians 07. There are three 8-dimensional coordinate descriptions, as linear octonians or complex 4D-space, and for the strong interaction SI as eight GellMann 3x3-matrices. Using Pauli matrices multiplied by complex numbers, a possibility for added GellMann matrices has in the first row $(z_1 z_2 z_3)$, second row $(-c(z_2) c(z_1) z_4)$, third row $(-c(z_3) -c(z_4) w)$ where w is determined by the scaled addition of extended third Pauli matrices. They are linearly dependent. The use of 3-dimensional presentations is not GellMann, but lets the extended third Pauli matrices for use in complex 4-dimensional spaces $(z_1, z_2, 0)$, $(0, z_2, z_3)$ and $(z_1, 0, z_3)$. The homogeneous shift can use 1 instead of 0 for a projective CP^2 space. One is used for the strong interactions fiber bundle, projecting its geometry factor S^5 onto CP^2 and having S^1 as fiber. The $(0, z_2, z_3)$ is the authors nucleon space 2356 in octonian notation. $(z_1, z_2, 0)$ is spacetime 1234 and $(z_1, 0, z_3)$ can be used for the EM/WI space 1456. There are no operators transforming these coordinates into SI matrices which can make a research project.

Conclusions

Many items in this article can be seen as suggestions. If researchers look at it they may have better ideas. The coordinate study has not a fixed transformation scheme as known between Euclidean and spherical space coordinates. Many items are explanations how, for instance, gravity can be fitted in. The author suggests a research project for this proposal.

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MINT-Wigris tool box in the Emmy Noether Memorial museum, lower left the SI rotor, 2nd box the leptons and weak bosons, 3rd box sterteching squeezing, fusion, handcrafts by a template, 4th box 6 roll mill, gluon exchange, barycentral coordinates, 5th box g-compass for color charges, dark matter, dark energy, hedgehog