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The use of Maxwell's equations to control absorption coefficient through conductivity, refractive index and wave guide dimensions

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ABSTRACT:

Absorption of light and electromagnetic waves plays an important role in the performance of the solar cells and in selecting construction materials for appropriate temperature. This requires using Maxwell's equations to see how to control absorption. Treating the wave number and conductivity as complex quantities a useful expression for the conductivity has been found. It was found that for continuous media the absorption coefficient increases with the refractive index and conductivity, however when the medium is assembled in the form of small isolated rectangular wave guides. The absorption can be increased by decreasing the dimensions of the wave guides.

Keywords: Maxwell's equations, absorption coefficient, conductivity, refractive index, wave guide.

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I. INTRODUCTION

Electromagnetic fields are widely used in civilization. It is used in telecommunication like internet, mobile phone, radio and television. The characteristic electromagnetic waves emitted from elements is widely used as a finger print for identification of the elements present in a soils, rocks, plants and air beside water. Different spectral techniques were utilized for this identification and purposes. Through the wide range of electromagnetic spectrum it is well known that light is one of the most well-known types or ranges of electromagnetic filed types. It stimulate the vision sense and can also be used to generate electricity using solar cells [1,2]. Light energy consists also of infrared waves which can affect weather and climate [3]. The absorption degree of cells to light affect its performance, whereas the absorption rate of earth crust to light effect the air and weather temperature [4,5]. The buildings material are carefully selected by civil engineers so as to prevent heat flow inside or outside the building. Therefore it is very important to promote the physics of absorption by trying to develop the solutions of Maxwell's equation's to see how one can easily control the light absorption using new concepts by designing the materials in the form of wave guides.
Different attempts were made to see empirically the factors affecting the absorption coefficients. In the works done by many researchers the absorption coefficient is found to be a frequency dependent [6, 7, 8]. In another work it was recorded that the absorption increases when the concentration increases [8]. In various research works the absorption coefficient is found to be changed with many coefficients [9,10]. In this work the most important purpose is to find an expression for the absorption coefficient by following a theoretical basis by managing the Maxwell Equation of the electric field intensity E, in a medium having conductivity σ. The work is divided into many sections in studying the absorption coefficient and its dependence as shown in sections (II) and (III). Sections (IV) and (V) to go on a head through the discussion and conclusion.

II. BULK MATTER ABSORPTION COEFFICIENT

According to Maxwell's equations [11] the equation of the electric field intensity E, in a medium having conductivity σ is given by the following equation:

$$\nabla^2 E - \epsilon\mu\frac{\partial^2 E}{\partial t^2} - \mu\sigma\frac{\partial E}{\partial t} = 0 \quad (1)$$

Where ε , $\boldsymbol{\sigma}$ and μ stands for electric permittivity, conductivity and magnetic permeability, consequently. The solution of equation (1) due to the general properties of E can be suggested to be:

$$\boldsymbol{E = E_o e^{i(kz-wt)}} \quad \textbf{(2)}$$

Where E_o is the amplitude, k and w stands for the wave number and angular frequency. Differentiating equation (2) for the first time with respect to space (position) gives the following formula:

$$\boldsymbol{\nabla^2 E - \frac{\partial^2 E}{\partial z^2} = -k^2 E} \; \textbf{(3)}$$

While the time differentiation of equation (2) could be resulting in the following expressions:

$$\boldsymbol{\frac{\partial E}{\partial t} = -iwE \;, \frac{\partial^2 E}{\partial t} = -iw^2 E} \quad \textbf{(4)}$$

Inserting equation (3) in equation (1) this can yield in $(-k^2 + w^2\mu\varepsilon + iw\mu\sigma)E = 0$ thus as E eliminated equation(4) should be written as the following:

$$\boldsymbol{-k^2 + w^2\mu\varepsilon + iw\mu\sigma = 0} \quad \textbf{(5)}$$

From equation (5) it is clear that the third complex valued term ($\boldsymbol{iw\mu\sigma}$), forces to define k to be together with σ in a complex form, where we can represent both of k and $\boldsymbol{\sigma}$ as a complex terms as following:

$$\boldsymbol{k = k_1 + ik_2} \quad \textbf{(6)}$$

$$\boldsymbol{\sigma = \sigma_1 + i\sigma_2} \quad \textbf{(7)}$$

With a direct substitution of equations (6) and (7) in equation (5) this can result in

$$\boldsymbol{-(k_1 + ik_2)^2 + w^2\mu\varepsilon + iw\mu(\sigma_1 + i\sigma_2) = 0} \; \textbf{(8)}$$

By substituting equation (6) in equation (2) this can be resulting in the following expression as shown in equation (9)

$$\boldsymbol{E = E_o e^{-k_2 z} e^{i(k_1 z - wt)}} \quad \textbf{(9)}$$

Therefore the wave intensity *I* due to all of these equations should be given by

$$\boldsymbol{\mathrm{I} = |E|^2 = E\bar{E} = E_0^2 e^{-2k_2 z} = I_0 e^{-\propto z}} \; \textbf{(10)}$$

Where the initial intensity I_0 takes the form $\mathrm{I}_0 = \mathrm{E}_0^2$, while the absorption coefficient $\propto$ is given by

$$\boldsymbol{\propto = 2\mathrm{k}_2} \; \textbf{(11)}$$

In a general view of equation (9) the wave number k_1 should be given by $k_1 = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{w}{v} = w\sqrt{\mu\varepsilon}$

Hence it is suitable to write the value of the wave number k_1 as shown in equation (12) blow:

$$\boldsymbol{\mathrm{k}_1^2 = w^2\mu\varepsilon} \; \textbf{(12)}$$

Analyzing the first term in equation (8) this can gives

$$\boldsymbol{-\mathrm{k}_1^2 - 2ik_1k_2 + k_2^2 + w^2\mu\varepsilon + iw\mu\sigma_1 - w\mu\sigma_2 = 0} \quad \textbf{(13)}$$

By substituting equation (12) in equation (13) this can gives

$$\boldsymbol{-2ik_1k_2 + k_2^2 + iw\mu\sigma_1 - w\mu\sigma_2 = 0} \; \textbf{(14)}$$

Equating the real and imaginary parts of equation (14) this can give

$$\boldsymbol{2k_1k_2 = w\mu\sigma_1 \quad thus \quad k_2 = \frac{w\mu}{2k_1}\sigma_1} \; \textbf{(15)}$$

$$\boldsymbol{\mathrm{k}_2^2 = w\mu\,\sigma_2} \quad \textbf{(16)}$$

$$\boldsymbol{k_2 = \sqrt{w\mu\,\sigma_2}} \quad \textbf{(17)}$$

Returning to equations (15) and (12) it is clear that

$$\boldsymbol{\mathrm{k}_2^2 = \frac{w^2\mu^2\sigma_1^2}{2\mathrm{k}_1^2} = \frac{w^2\mu^2}{2w^2\varepsilon}\sigma_1^2 \,,\; k_2^2 = \frac{\mu\sigma_1^2}{2\varepsilon}} \; \textbf{(18)}$$

$$\boldsymbol{w\mu\,\sigma_2 = \frac{\mu\sigma_1^2}{2\varepsilon} \,, \sigma_2 = \frac{\sigma_1^2}{2\varepsilon w}} \; \textbf{(19)}$$

A direct substitution of equations (19) in (17) gives

$$\boldsymbol{k_2 = \frac{\sqrt{\mu\varepsilon}}{\sqrt{2}\,\varepsilon}\sigma_1 \quad , \quad k_2 = \frac{\sigma_1}{\sqrt{2}\,v\,\varepsilon} = \frac{n_0\sigma_1}{c\sqrt{2}\,\varepsilon}} \; \textbf{(20)}$$

Where $n_o = c/v$ is the refractive index. This conforms with the fact that the absorption coefficient $\propto$ in equation (11) is depending on the refractive index where the absorption coefficient is given by equation (21) as

$$\boldsymbol{\propto = 2k_2 = \frac{\sqrt{2}\,n_0\sigma_1}{c\,\varepsilon}} \; \textbf{(21)}$$

Which is seen to be proportional to the real conductivity. Where no attenuation is observed in free space as $(\sigma_1 = 0)$ and as the conducting medium absorb the electromagnetic wave forcing it to decay and convert it to electric energy. The electric energy density $\boldsymbol{E_e}$ and the electric intensity $\boldsymbol{\mathrm{I}_e}$ are given by the following terms as in equation (22)

$$\boldsymbol{E_e = IV = \frac{V^2}{RAL} = \frac{V^2}{\left(\frac{\varrho\ell}{A}\right)A\ell} = \sigma\frac{V^2}{L^2} = \sigma\,E^2 = \sigma_1 E^2 \quad , \quad \mathrm{I}_e = cE_e = c\,\sigma_1 E^2} \; \textbf{(22)}$$

With A, ℓ, I and v standing for conductor cross section, length, current and potential difference. According to equations (21) and (22) the increasing in conductivity resulting in decreases values of the electromagnetic energy flux which is transformed to electric flux that increases and this conforms strongly as it is quite seen from equation (22), due to all of these factors and facts it is so important to emphasizes that the continious increase in conductivity increases the electric energy density.

III. WAVE GUIDE SOLUTION AND ABSORPTION COEFFICIENT

The absorption coefficient can be found to depend on micro or nano size mediums by using the notion of a wave guide. The solution of equation (1) thus can be rewritten in a form which express absorption coefficient dimension wise by treating the medium as aware guide. This requires E to take the form:

$$E = E_0\, X(x)\, Y(y) e^{+\gamma z} e^{-iwt} = E_0\, X(x)\, Y(y) e^{\gamma z} e^{-iwt} \quad (23)$$

By differentiating equation (23) with respect to space coordinates gives

$$\nabla^2 E = \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2}$$

$$\nabla^2 E = E\left[y e^{\gamma z}\left[\frac{\partial^2 x}{\partial x^2}\right] + X e^{\gamma z}\frac{\partial^2 y}{\partial y^2} + \gamma^2 XY e^{\gamma z}\right] e^{\gamma z} e^{-iwt} \quad (24)$$

$$\nabla^2 E = E_0\left[y\frac{\partial^2 x}{\partial x^2} + X\frac{\partial^2 y}{\partial y^2} + \gamma^2 XY\right] e^{\gamma z} e^{-iwt}$$

By using equations (3) and (24), equation (1) takes the form

$$E_0\left(y\frac{\partial^2 x}{\partial x^2} + X\frac{\partial^2 y}{\partial y^2} + \gamma^2 XY\right) e^{\gamma z} e^{-iwt} + E_0(w^2\varepsilon\mu + iw\mu\sigma)\, XY e^{\gamma z} e^{-iwt} = 0 \quad (25)$$

$$Y\frac{\partial^2 x}{\partial x^2} + X\frac{\partial^2 y}{\partial y^2} + \gamma^2 XY + (w^2\varepsilon\mu + iw\mu\sigma)XY = 0 \quad (26)$$

Returning to the above equation (26) and by dividing both sides by XY this can results and yields as:

$$\frac{1}{X}\frac{\partial^2 x}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 y}{\partial y^2} + \gamma^2 + \mu\varepsilon w^2 + iw\mu\sigma = 0 \quad (27)$$

Since the first two terms are functions of x and y only, thus it is suitable to be written as

$$\frac{1}{X}\frac{\partial^2 x}{\partial x^2} = -N^2 \quad (28)$$

$$\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -M^2 \quad (29)$$

Thus according to equations (28), (29) and (27), it is clear that equation (27) should be reported as: $-N^2 - M^2 + \gamma^2 + \mu\varepsilon w^2 + i\mu\sigma w = 0$ so this can be rearranged in ordering the equation terms to be written as:

$$\gamma^2 = N^2 + M^2 - \mu\varepsilon w^2 - i\mu\sigma w \quad (30)$$

The values of N and M can be found by suggesting X and Y in equations (28) and (29) to be

$$X = \sin \propto x \quad (31)$$

$$Y = \sin \beta\, x \quad (32)$$

By inserting a direct substitution of equation(31) in equation (28) this can give the following expressions

$$-\propto^2 = -N^2 \quad , \quad \propto = N \quad (33)$$

Also by inserting equation (32) in equation (29) gives

$$-\beta^2 = -M^2 \quad , \quad \beta = M \quad (34)$$

For a given wave guide that having dimensions $(x = a\ ,\ y = b)$ thus the electric field inside such conductors at $x = a$ and $y = b$ will vanishes. This means that and according to equations (31) and (32) that $X(a) = sin \propto a = 0\ ,\ Xa = 2n\pi$ where ;

$$\propto = \frac{2n\pi}{a} \;,\; \mathrm{N} = 0 , 1 , 2 , 3 \ldots\ldots (35)$$

$$Y(b) = sin\beta b = 0$$

$$\beta b = 2m\pi \;,\; \beta = \frac{2m\pi}{b} \;,\mathrm{M} = 0 , 1 , 2 , 3 \ldots\ldots$$

thus from equations (33) and (34) we can find that

$$N = \frac{2n\pi}{a} \;,\; M = \frac{2m\pi}{b} \quad (36)$$

To find an expression for γ as in equation (23), it is better to split this equation into real part to be related to absorption and imaginary part that related to the wave number as in equation (37):

$$\gamma = ik + \gamma_0 \quad (37)$$

Inserting equation (37) in equation (30) this can yields

$$-k^2 + 2\gamma_0 ki + \gamma_0^2 = N^2 + M^2 - \mu\varepsilon w^2 - i\mu\sigma w \quad (38)$$

By returning to the wave number k it is clear that the value of k is given from equation (38) so

$$k^2 = \frac{w^2}{v^2} = w^2\mu\varepsilon = \mu\varepsilon w^2 \quad (39)$$

This simplifies that equation (38) should be written in the form

$$2\gamma_0 ki + \gamma_0^2 = N^2 + M^2 - i\mu\sigma w \quad (40)$$

Equating real parts from the equation and the also imaginary parts this can give:

$$2\gamma_0 k = -\mu\,\sigma\,w \quad (41)$$

$$\gamma_0 = \pm\sqrt{N^2 + M^2} \quad (42)$$

$$\gamma_0 = \sqrt{\left(\frac{2n\pi}{a}\right)^2 + \left(\frac{2m\pi}{b}\right)^2} \quad (43)$$

However if the conductivity is a complex quantity as it should be written in the form

$$\sigma = \sigma_1 + i\sigma_2 \quad (44)$$

Then by inserting equation (44) in equation (40) the imaginary part gives

$$2\gamma_0 \mathbf{k} = -\mu\sigma_1 w \quad (45)$$

While the real part of equation (40) gives

$$\gamma_0^2 = N^2 + M^2 + \mu\sigma_2 w \quad (46)$$

$$\gamma_0 = \pm\sqrt{N^2 + M^2 + \mu\sigma_2 w} \quad (47)$$

By substituting the values of N and M from equation (36) into equation (47) one gets:

$$\gamma_0 = \pm\sqrt{\left(\frac{2n\pi}{a}\right)^2 + \left(\frac{2m\pi}{b}\right)^2 + \mu\sigma_2 w} \quad (48)$$

According to equation (45) the wave number should be written as

$$k = -\frac{\mu\sigma_1 w}{2\gamma_0} \quad (49)$$

Where k is positive, this requires that γ_0 to be negative in equation (48) to be

$$\gamma_0 = -\sqrt{\left(\frac{2\pi n}{a}\right)^2 + \left(\frac{2\pi m}{b}\right)^2 + \mu\sigma_2 w} = -\beta_0 \quad (50)$$

$$\mathbf{k} = \frac{\mu\sigma_1 w}{\beta_0} = \frac{w}{v} \quad (51)$$

$$v = \frac{\beta_0}{\mu\sigma_1 w} \quad (52)$$

Thus according to equations (37), (50) and (23) one gets

$$E = E_0\, X\, Y e^{-\beta_0 z} e^{i(kz - wt)} \quad (53)$$

Which represents a decaying electric field vector, where the intensity I is given by

$$\mathrm{I} = |E|^2 = E_0^2 X^2 Y^2 e^{-\beta_0 z} = \mathrm{I}_0 e^{-\propto z} \quad (54)$$

Where the absorption coefficient $\propto$ is given by

$$\propto = 2\beta_0 \quad (55)$$

The term β_0 is equal to $-\gamma_0$ that representing the meaning of the absorption coefficient $\propto$ as a decaying quantity throughout such a medium.

IV. DISCUSSION

For continuous media the absorption coefficient $\propto$ in this study is found to be depending on the refractive index as well as the conductivity σ_1 of the medium, where it increases upon increasing them. Thus in order to increase absorption value in a medium one needs a good conductor with high refractive index. However it should be suitable to assemble the absorbing medium in the form of isolated wave guides that having dimensions a , b. the values of these dimensions can affect in controlling absorption coefficient value by changing the values of a and b. It is very important to mention that the absorption process can be increased by decreasing the wave guide dimensions. These values can be more useful in the fields of engineering, construction of buildings, isolating systems and other field in the lifestyle, this can easily be done by controlling the absorption coefficient value by determining the factors belonging to the materials and the values of such parameters that the absorption coefficient is depending on.

V. CONCLUSION

For continuous medium the absorption of electromagnetic waves can be increased by increasing the conductivity and the refractive index. However if the medium is assembled as isolated rectangular wave guide one can increase absorption by decreasing the dimensions of the wave guide to be as small as possible provided that the dimensions should not be smaller than $300nm$ where quantum effects become important.

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