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Graphical Representation of Addition and Subtraction of Fuzzy Numbers with Composition Table

Imran Hassan[1] and Suman Kar[2]
[1,2]*Department of Basic science, Faculty of Science and Engineering,*
World University of Bangladesh, Dhaka, Bangladesh.

Abstract:
A Fuzzy number is just an ordinary number whose precious value is somewhat unpredictable. Fuzzy number are used in experimental science, engineering (especially communication), mathematical model etc. The arithmetic operation on fuzzy numbers are fundamental concepts in fuzzy mathematics. The operation of interval are discussed. The paper presents addition and subtraction of Fuzzy numbers with constructing composition table. Fuzzy arithmetic operations are frequently used for solving mathematical equations that contains Fuzzy numbers. The extension principle and the α cut approach using different t- norms. Such approach causes difficulties in calculations and is a reason for arithmetical paradoxes. The paper shows how the addition operation and subtraction operation can be connected on Fuzzy numbers. This paper also shows when operating with Fuzzy numbers, then the result of our calculations strongly depend on shape of the membership functions of these numbers.
Keywords: *Fuzzy numbers, Fuzzy arithmetic, Membership function.*

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I. Introduction:

The fuzzy numbers and arithmetic operations are rich in methods for defining fuzzy operations with many desirable properties that are not always present in the implementations of classical extension principle. The fuzzy calculations are not performed immediately and in many cases they require to solve mathematically or computationally difficult sub problems for which a closed form is not available. Fuzzy numbers are complementary to probability and statistical modeling uncertainty, imprecision and ambiguity of data and information. In particular, fuzziness is essentially connected with incompleteness and uncertainty in the boundaries of sets and numbers, while granularity defines the scale or level of detail at which the domain of the values of the variable or object of interest described and coded. The arithmetic operations on fuzzy numbers can be approached through direct use of the membership functions or equivalently through the use of the αcuts representation. The arithmetic operations and more general fuzzy calculations are normal when dealing with Fuzzy logic and systems, where variables and data are described by Fuzzy numbers and sets. The essential uncertainties are generally modeled in the initial definition of the variables, but it is very important to pay close attention to how they propagate during the calculations. A difficult consequenceof Fuzzy theory and practice is that calculations cannot be performed by using the same rules as in arithmetic with real numbers and indeedfuzzy calculus will not always satisfy the similar properties. If not performed by taking into account existing dependencies between the data, Fuzzy calculations will produce over propagation of initial uncertainty. By the α- cuts approach, it is possible to define a graphical representation of addition and subtraction of Fuzzy numbers that allow a large variety of possible shapes and is very simple to implement, with the advantage of obtaining a much wider family of Fuzzy numbers.

II. Some basic concept on Fuzzy numbers:

A. Intervals:

Definition 1: $\mathrm{C} \subseteq \mathrm{R}$ is an interval in $\mathbf{R}$ if $\forall$ x, y, z $\in$ C and $x < z < y$ *imply* $z \in C$.
Definition 2: For $\bar{a}, \underline{a} \in \mathbf{R}$, the interval $\mathrm{J} = \{x \in \mathrm{R}: \bar{a} < x \leq \underline{a}\}$has lower end point $\underline{a}$ and upper end point$\bar{a}$, and is denoted by $[\underline{a}, \bar{a}]$. It is proper interval $[\underline{a}, \bar{a}]$ will be denoted by [a].
For instance, if interval is denoted by as$A = [a_1, a_3], a_1, a_3 \in \mathbf{R}, a_1 < a_3$. We may regard this as one kind of sets.
Expressing the interval as membership function is shown in the following figure 1.2

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ 1, & a_1 \le x \le a_3 \\ 0, & x \ge a_3 \end{cases}$$

If $a_1 = a_3$, this interval indicates a point. That is $[a_1, a_1] = a_1$.

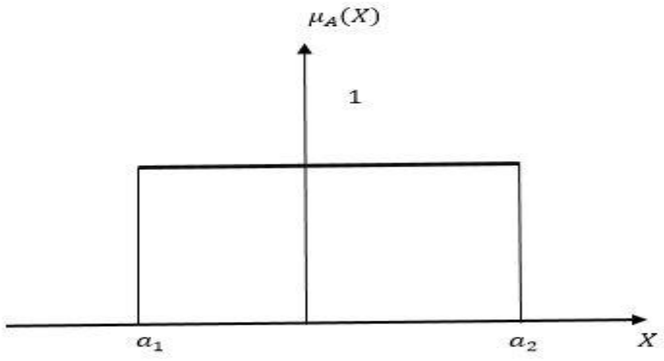

Figure: Interval $A = [a_1, a_3]$

B. Interval arithmetic:

Let $*$ denote any of the four arithmetic operations on closed intervals: addition (+), subtraction (-), multiplication (·), division (/). Then

$$[a, b] * [d, e] = \{f * g \mid a \le f \le b, d \le g \le e\} \text{ except when } 0 \epsilon [d, e].$$

Is a general property of all arithmetic operations on closed intervals, except that $\frac{[a,b]}{[d,e]}$ is not defined when $0 \in [d, e]$.

C. Arithmetic operations with closed intervals:

(1) $[a, b] + [d, e] = [a + d, b + e]$

(2) $[a, b] - [d, e] = [a - e, b - d]$

(3) $[a, b] \cdot [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]$

(4) $\frac{[a,b]}{[d,e]} = [\min\left(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e}\right), \max(\frac{a}{d}, \frac{a}{e}, \frac{b}{d}, \frac{b}{e})]$

D. Fuzzy Number:

If a fuzzy set is convex and normalized, and its membership function is defined in R and piecewise continuous, it is called fuzzy number. So fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy.

To qualify a fuzzy number, a set A on R must possess at least the following three properties:

1. A must be a normal fuzzy set.
2. A^{α} must be a closed interval for every $\alpha \epsilon$ (0,1].
3. The support of A, A^{0+} must be bounded.

E. Extension Principle and Fuzzy Arithmetic:

Now we wish to use the extension principle to perform algebraic operations on fuzzy numbers. We define a normal, convex set on the real line to be a fuzzy number and denote it $S_{\tilde{x}}$.

Let $S_{\tilde{x}}$ and $S_{\tilde{y}}$ be two fuzzy numbers with $S_{\tilde{x}}$ defined on the real line in the universe X and $S_{\tilde{y}}$ defined on the real line in universe Y, and let symbol $*$ denote a general arithmetic operation, that is, $* \equiv \{+, -, \cdot, /\}$. An arithmetic operation between these two numbers denoted $S_{\tilde{x}} * S_{\tilde{y}}$ will be defined on universe Z, and can be accomplished using the extension principle by

$$\mu_{S_{\tilde{z}}}(z) = \sup_{z=x*y} \{\mu_{S_{\tilde{x}}}(x) \wedge \mu_{S_{\tilde{y}}}(y)\}$$

III. Addition of Fuzzy arithmetic with composition table:

$$Let\ z = F(x,y) = x + y. Then\ \tilde{x} + \tilde{y}\ with$$
$$S_{\tilde{z}} = \{z \in Z \mid z = x + y, x \in S_{\tilde{x}}, y \in S_{\tilde{y}}$$
$$And\quad \mu_{S_{\tilde{z}}}(z) = \sup_{z=x*y}\{\mu_{S_{\tilde{x}}}(x) \wedge \mu_{S_{\tilde{y}}}(y)\}$$
$$By\ \alpha - cut\ notation, (S_{\tilde{z}})^{\alpha} = F\left((S_{\tilde{x}})^{\alpha}, (S_{\tilde{y}})^{\alpha}\right) = (S_{\tilde{x}})^{\alpha} + (S_{\tilde{y}})^{\alpha}.$$

Example: Let $\tilde{x}$ and $\tilde{y}$ be such that $S_{\tilde{x}} = [-5,1], S_{\tilde{y}} = [-5,12]$ with associate Membership functions

$$\mu_{S_{\tilde{x}}}(x) = \begin{cases} \frac{x}{3} + \frac{5}{3}, -5 \le x \le 2 \\ \frac{-x}{3} + \frac{1}{3}, -2 \le x \le 1 \end{cases} \quad and\ \mu_{S_{\tilde{y}}}(y) = \begin{cases} 0, -5 \le y \le -3 \\ \frac{y}{7} + \frac{3}{7}, -3 \le y \le 4 \\ \frac{-y}{8} + \frac{12}{8}, 4 \le y \le 12 \end{cases}$$

We obtain $\mu_{S_{\tilde{z}}}(z) = \mathrm{Sup}_{z=x*y}\{\mu_{S_{\tilde{x}}}(x) \wedge \mu_{S_{\tilde{y}}}(y)\}$

From interval [-5, 1] and [-5, 12], we take some integer and make a composition table is follows:

+	-5	-4	-3	-2	-1	0	1
-5	-10	-9	-8	-7	-6	-5	4
-3	-8	-7	-6	-5	-4	-3	-2
-1	-6	-5	-4	-3	-2	-1	0
0	-5	-4	-3	-2	-1	0	1
2	-3	-2	-1	0	-1	2	3
4	-1	0	1	2	3	4	5
6	1	2	3	4	5	6	7
8	3	4	5	6	7	8	9
10	5	6	7	8	9	10	11
12	7	8	9	10	11	12	13

From the above table let's think when z=2. Addition to make z=2 is possible for following cases:
(6 + (-4), 4 + (-2), 2+0,⋯⋯⋯)

So $\mu_{S_{\tilde{z}}}(2) = \sup_{x+y=2}\{\mu_{S_{\tilde{x}}}(6) \wedge \mu_{S_{\tilde{y}}}(-4), \mu_{S_{\tilde{x}}}(4) \wedge \mu_{S_{\tilde{y}}}(-2), \mu_{S_{\tilde{x}}}(2) \wedge \mu_{S_{\tilde{y}}}(0), \cdots\cdots\cdots\}$

$= \sup_{x+y=2}\{0.33 \wedge 0.75, 1 \wedge 1, 0.33 \wedge 0.71, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\}$.

$= \sup_{x+y=2}\{0.75, 1, 0.71, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\} = 1$.

Again let's think when z=8. Addition to make z=8 is possible for following cases:
(10+ (-2), 8+0, …….).

So $\mu_{S_{\tilde{z}}}(8) = \sup_{x+y=8}\{\mu_{S_{\tilde{x}}}(10) \wedge \mu_{S_{\tilde{y}}}(-2), \mu_{S_{\tilde{x}}}(8) \wedge \mu_{S_{\tilde{y}}}(0), \cdots\cdots\cdots\}$

$= \sup_{x+y=8}\{1 \wedge 0.25, 0.33 \wedge 0.50, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\}$.

$= \sup_{x+y=2}\{0.25, 0.33, \cdots\cdots\cdots\} = 0.25$.

From this kind of method, if you come by membership function for all $z \in S_{\tilde{x}} + S_{\tilde{y}}$. For convenience, we can express it is a fuzzy number by approximating where $S_{\tilde{z}} = [-10,13]$.

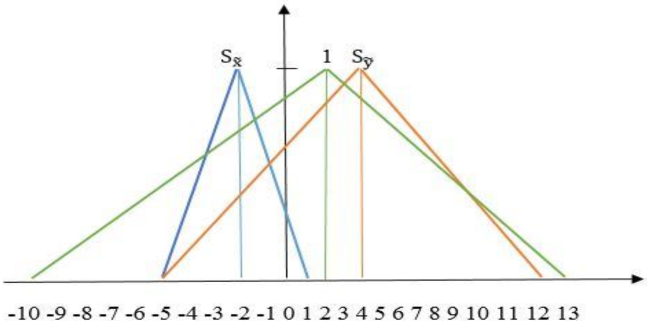

Figure: Addition of Two Fuzzy numbers

Now we perform this operation by using $\alpha -$ cut of $S_{\tilde{x}}$is obtained by,
Letting, $\alpha = \frac{x}{3} + \frac{5}{3}$ and $\alpha = -\frac{x}{3} + \frac{1}{3}$ respectively, which give $x_1 = 3\alpha - 5$ and $x_2 = -3\alpha + 1$.
Hence, the projection interval is, $(S_{\tilde{x}})^{\alpha} = [x_1, x_2] = [3\alpha - 5, -3\alpha + 1]$.
Similarly, $(S_{\tilde{y}})^{\alpha} = [y_1, y_2] = [7\alpha - 3, -8\alpha + 12]$
So that $(S_{\tilde{z}})^{\alpha} = (S_{\tilde{x}})^{\alpha} + (S_{\tilde{y}})^{\alpha} = [3\alpha - 5 + 7\alpha - 3, -3\alpha + 1 - 8\alpha + 12]$
$= [10\alpha - 8, -11\alpha + 13] = [-8, 13]$for $\alpha = 0$.
Setting $x_1 = 10\alpha - 8$ and $x_2 = -11\alpha + 13$ gives $\alpha = \frac{z}{10} + \frac{8}{10}$ and $\alpha = -\frac{z}{11} + \frac{13}{11}$,
Which yield the membership function. Taking $S_{\tilde{z}} = [-8,2,13]$ into account, we finally arrive at

$$\mu_{S_{\tilde{z}}} = \begin{cases} 0, -10 \leq z \leq -8 \\ \frac{z}{10} + \frac{8}{10}, -8 \leq y \leq 2 \\ -\frac{z}{11} + \frac{13}{11}, 2 \leq z \leq 13 \end{cases}$$

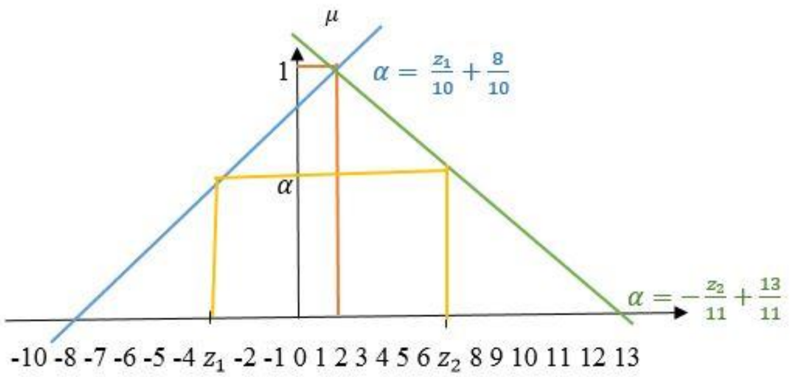

Figure: The resulting membership function.

IV. Subtraction of Fuzzy arithmetic with composition table:

$$Let\ z = F(x, y) = x - y. Then\ \tilde{x} - \tilde{y}\ with$$
$$S_{\tilde{z}} = \{z \in Z \mid z = x - y, x \in S_{\tilde{x}}, y \in S_{\tilde{y}}$$
$$And \quad \mu_{S_{\tilde{z}}}(z) = \sup_{z = x*y} \{\mu_{S_{\tilde{x}}}(x) \wedge \mu_{S_{\tilde{y}}}(y)\}$$
$$By\ \alpha - cut\ notation, (S_{\tilde{z}})^{\alpha} = F\left((S_{\tilde{x}})^{\alpha}, (S_{\tilde{y}})^{\alpha}\right) = (S_{\tilde{x}})^{\alpha} - (S_{\tilde{y}})^{\alpha}$$

Example: Let $\tilde{x}$ and $\tilde{y}$ be such that $S_{\tilde{x}} = [-5,1], S_{\tilde{y}} = [-5,12]$ with associate
Membership functions

$$\mu_{S_{\tilde{x}}} = \begin{cases} 0, 0 \le x \le 7 \\ \frac{x}{7} - 1, 7 \le x \le 7 \\ -\frac{x}{5} + \frac{19}{5}, 14 \le x \le 19 \\ 0, 19 \le x \le 20 \end{cases} \text{ and } \mu_{S_{\tilde{y}}} = \begin{cases} 0, 0 \le y \le 3 \\ \frac{y}{2} - \frac{3}{2}, 3 \le y \le 5 \\ -\frac{y}{5} + 2, 5 \le y \le 10 \end{cases}$$

We obtain $\mu_{S_{\tilde{z}}}(z) = \text{Sup}_{z=x-y}\{\mu_{S_{\tilde{x}}}(x) \wedge \mu_{S_{\tilde{y}}}(y)\}$

From interval [0, 20] and [0, 10], we take some integer and make a composition table is follows:

-	0	4	8	12	16	20
0	-10	-6	-2	2	6	10
2	-8	-4	0	4	8	12
4	-6	-2	2	6	10	14
6	-4	0	4	8	12	16
8	-2	2	6	10	14	18
10	0	4	8	12	16	20

From the above table let's think when z=2. Subtraction to make z=2 is possible for following cases: (4-2, 8-6, 12-10, ……)

So $\mu_{S_{\tilde{z}}}(2) = \underset{x-y=2}{\text{Sup}} \{\mu_{S_{\tilde{x}}}(4) \wedge \mu_{S_{\tilde{y}}}(2), \mu_{S_{\tilde{x}}}(8) \wedge \mu_{S_{\tilde{y}}}(6), \mu_{S_{\tilde{x}}}(12) \wedge \mu_{S_{\tilde{y}}}(10), \cdots\cdots\cdots\}$

$= \underset{x-y=2}{\text{Sup}} \{0 \wedge 0, 0.14 \wedge 0.8, 0.71 \wedge 0, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\}$.

$= \underset{x-y=2}{\text{Sup}} \{0, 0.14, 0, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\} = 0.14$

Again let's think when z=4. Subtraction to make z=4 is possible for following cases:
(4-0, 8-4, 12-8,…..)

So $\mu_{S_{\tilde{z}}}(4) = \underset{x-y=4}{\text{Sup}} \{\mu_{S_{\tilde{x}}}(4) \wedge \mu_{S_{\tilde{y}}}(0), \mu_{S_{\tilde{x}}}(8) \wedge \mu_{S_{\tilde{y}}}(4), \mu_{S_{\tilde{x}}}(12) \wedge \mu_{S_{\tilde{y}}}(8), \cdots\cdots\cdots\}$

$= \underset{x-y=4}{\text{Sup}} \{0 \wedge 0, 0.14 \wedge 3.5, 0.71 \wedge 0.4, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\}$.

$= \underset{x-y=4}{\text{Sup}} \{0, 0.14, 0.40, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\} = 0.40$

From this kind of method, if we come by membership function for all $z \in S_{\tilde{x}} - S_{\tilde{y}}$.For convenience, we can express it is a fuzzy number by approximating where $S_{\tilde{z}} = [-10,20]$.

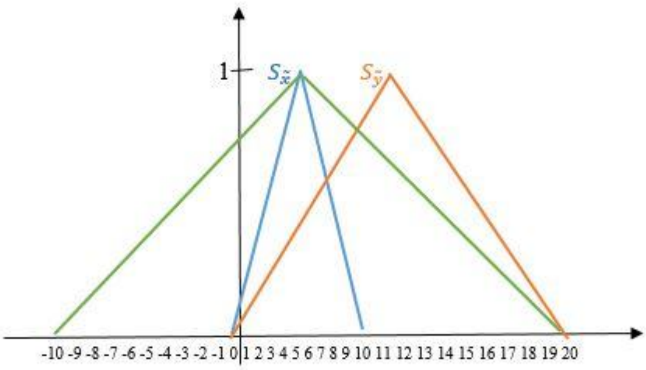

Figure: Subtraction of Two Fuzzy numbers

By $\alpha -$ cut notation, for any α value, the α −cut of $S_{\tilde{x}}$ is obtained by,

Let$\alpha = \frac{x}{7} - 1$ and $\alpha = -\frac{x}{5} + \frac{19}{5}$ respectively, which implies $x_1 = 7\alpha + 7$ and $x_2 = -5\alpha + 19$.

Hence the projection interval is $(S_{\tilde{x}})^{\alpha} = [x_1, x_2] = [7\alpha + 7, -5\alpha + 19]$.

Similarly, $(S_{\tilde{y}})^{\alpha} = [y_1, y_2] = [2\alpha - 3, -5\alpha + 10]$.

$-(S_{\tilde{y}})^{\alpha} = [5\alpha - 10, -2\alpha - 3]$.

So that $(S_{\tilde{z}})^{\alpha} = (S_{\tilde{x}})^{\alpha} - (S_{\tilde{y}})^{\alpha} = [7\alpha + 7 + 5\alpha - 10, -5\alpha + 19 - 2\alpha + 3]$

$= [12\alpha - 3, -7\alpha + 2]$= $[-3, 16]$for $\alpha = 0$.

Setting $z_1 = 12\alpha - 3$ and $z_2 = -7\alpha + 22$ gives $\alpha = \frac{z}{12} + \frac{3}{12}$ and $\alpha = -\frac{z}{2} + \frac{16}{7}$,

Which yield the membership function. Taking $S_{\tilde{z}} = [-3,9,16]$.

Consequently, we have the membership function$\mu_{S_{\tilde{z}}}(z) = \begin{cases} 0, -10 \le z \le -3 \\ \frac{z}{12} - \frac{3}{12}, -3 \le z \le 9 \\ -\frac{z}{7} + \frac{16}{7}, 9 \le z \le 16 \\ 0, 16 \le z \le 20 \end{cases}$

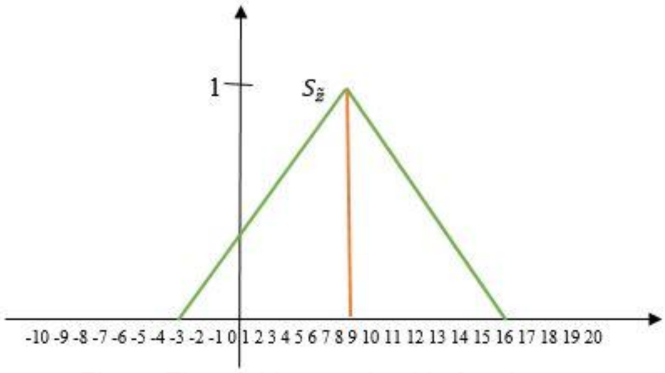

Figure: The resulting membership function

V. Conclusion:

In this paper we investigate a special type of fuzzy set, namely fuzzy numbers with arithmetic. We have methodically reviewed some operations like addition and subtraction of fuzzy numbers with constructing composition table and also showed graphical representation. We know fuzzy numbers are subsets of the set of real numbers satisfying some additional conditions. Arithmetic operations on fuzzy numbers have also been developed and are based mainly on the extension principle and on interval arithmetic. When operating with fuzzy numbers, then the result of our calculations strongly depend on the shape of the membership functions of these numbers andthe addition operation and subtraction operation can be connected on Fuzzy numbers. So finally we say that fuzzy numbers have been widely used in quantitative analysis in recent decades and fuzzy arithmetic is widely used in numerical analysis with dealing decision making under uncertainty with computationally effective way.

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