# Cost With Discounts in Inventory Control 

ADEBISI SUNDAY ADESINA<br>Faculty of Science, Department of Mathematics, University of Lagos, Akoka,Yaba, Nigeria, West Africa


#### Abstract

In inventory control the Economic Order Quantity (EOQ) assumes constant price per unit stock. But discount is given for quantity purchase very often. In the course of this findings, we shall be able to see how the discount upon every purchase of goods and services guarantees reduction in the ordering cost, stock holding cost and purchase price apart from giving room for higher average stock. Inventory can be regarded as a stock of goods and can be considered an idle but usable resource. It includes; labor, raw materials, finished goods and equipment stock is reserved to provide a flow of supply.


Keywords: Inventory control, stockouts, discounts, variable cost, ordering cost
Date of Submission: 14-03-2023
Date of acceptance: 29-03-2023

## I. INTRODUCTION

The concept of logistics has greatly been considerably extended very wide in scopes, as it has really been recognized majorly as one of the important and real essential tools which are developed in general for competitiveness. (Please, see [2]) Competitive advantage means the company has the ability to differentiate itself, in the customer's eyes, and also is operating at a lower cost and greater profit. Logistics, in general, makes getting products and services easy and available as and when they are needed, expected and desired by the customer. It also helps in economic transactions, serving as a major enabler of growth of trade and commerce in an economy. Every organization can be said to derive concrete shapes as a result of its structure. From inception, the suppliers in every distribution activity seems to spread across the complete structure. This without doubt, would result in various forms of overlapping activities and then, generally, in authorities and responsibilities which can not by any means accountable for. Today, the focus is completely been shifted to process, from mere functions. This is feasible whenever we have process driven organization. In such kind of situations, the process now possesses as its essential part, the logistics.

## II. LOGISTICS AND TECHNICALITIES

Of recent, corporations are earnestly looking for sustainable and competitive forms of advantages. This is not only for growth, but as well for reasonable survival. Most often, the interest on competition is now having major focuses on the product rather than just on the supply chain. Essentially, logistics management is concerned mostly with the system and the aspect of the concept and the approach, from the perspective of cost. No doubt, warehousing, transportation and handling of material are the major logistics activities. The same is that of inventory management and order processing. This actually, impacts the operation as well as the customer cost. Moreover, enhancement of customer services and its level is mostly been supported by Integrated logistics. This also helps the process of taking the cost out of the existing supply chain. ( see [8])

All over the world, business organizations seems to be struggling for competitiveness. This is not majorly for growth but serves as well for the survival. Moreover, the customers have become more demanding and look for value added services from prospective suppliers, as he wants value for the money he is spending. Situation of this kind, many business organizations all over the world at large, have started to do the review of their business processes. In this wise, there exists the realization of the cost cutting and the discrepancies of value delivery ad well as the solutions to the current problems Outsourcing is the transfer of a function previously per-formed in-house to an outside provider. Outsourced providers are often referred to as con-tractors or "third parties". (please, see [8])

## III. INVENTORY CONTROL

Inventory is a stock of goods. An inventory is an idle but usable resource. It includes; labor, raw materials, finished goods and equipment stock is reserved to provide a flow of supply. Furthermore, as concerns the design, planning, as well as the inventory control, it could be deduced that the applications of queuing theory can also be found to be highly useful, in the performance evaluation and improvement. Logistics and transportation, and other areas of supply chain are also involved [4]. It is illustrative, that, a newly established business would willingly decide concerning the number of telephone lines to be installed in such a cost-effective manner. Identify the elements of the underlying process of the telephone answering system and
indicate the specific data that need to be collected to establish the parameters of the system. Also identify the performance measures of interest..( see [8])
Basically, there exists three major types of Inventory. These are :
Raw Material, Work in Progress, and Finished Goods (see [2]).

## Raw Material

Here, everything that are purchased, which includes the parts and direct materials which go through to the end product. Such materials have value which are thus added to them, as they flows together in forms of subassemblies, assemblies eventually, into the product which are very much shippable.

## Work-in-Process

Refers to the inventory waiting in the process for being assembled into final products.

## Finished goods

Such goods are the inventory. They are ready for delivery to some distribution centers. These include : the retailers, and wholesalers. The customers may as well be involved directly.
The process of reducing the inventories, and the working capital requirements, may be increasing whenever there is time of relatively low interest rates. Using a finite source of capital, which might be more efficient but and could yield high rates of return for employing the essential idle capital in some businesses. Take for instance, the reduction of inventories can likely provide the necessary capital to finance such things. This includes, new product development, expanded marketing and sales, modernization, business process redesign. It can as well be for improved supply chain management, expansion, the acquisitions and debt reduction, to mention, but a few.

Need for inventory control: (see [2]).

1. Increase in the size of manufacturing units: if there is the increase in the manufacturing units, there is definitely, a necessity to have sufficient inventory control, so that the increase in inventories may not become non-value added expenditure. Increasing inventory can likely erode the profits of the company and the possibilities of inventory control.

## 2. Wide variety and complexity of the requirements: The modern

 industry have the requirements to necessitate the need for conscious inventory management.3. High idle time cost of machine and men: If men and machines are kept idle, it is highly uneconomical for the firm. The Inventory levels have to be properly managed.
4. Liquidity: Stress is often being increased on liquidity in many organizations. In such cases, it becomes a necessity to keep maintaining the liquidity at the levels of up to ten through twenty per cent of the total capital which is invested in finished goods.

## Inventory Control Problems

## Definitions ( see [8] )

a. Ordering frequency: This is the rate at which stock is ordered. It could be annually, biannual and monthly.
b. $\quad$ Ordering Quantity $(\mathrm{Q})$ : This is the size of stock ordered at a given time.
c. Average Stock: This is equal to a half of the offering quantity. It is denoted by $\left(\frac{Q}{2}\right)$.
d. Ordering Cost This is fixed per order irrespective of size of the order. It is the fixed charge each time a stock is ordered.
e. Stock holding cost: This is the cost of keeping the average stock for a given period. It is expressed as a given percentage of the stock value. Hence, if a stock cost $\$ 150$ per unit and it cost $15 \%$ to hold a stock, then the stock holding cost of a unit of the stock is given by $15 \%$ of $\$ 150=\$ 22.50$. Therefore, if an average of 1000 stock is held within the period then, the total stock holding cost would be given by $1000 \times \$ 22.5=22,500$
f. Economic Order Quantity (EOQ) The $E O Q$ is the quantity that sets the holding cost and the ordering cost of stock in equilibrium. Stock may be ordered at varying quantity. The $E O Q$ can also be referred to as $(E B Q)$ the Economic Batch Quantity. This is when it is manufactured within the system.
g. Total Variable Cost $(\mathrm{V})$ This is equal to the stock holding cost plus the ordering cost. Now, suppose that D is the annual demand for stock, C represents the fixed ordering cost per order, P is the cost per unit stock,
Q is the size of order and r is the stock holding cost in percentage. We have that the Total stock holding cost is given by the product of the cost of holding one stock and the average stock
$=\mathrm{rPx} \frac{Q}{2}=\mathrm{rP} \frac{Q}{2}$
Ordering cost $=$ the product of the number of orders and ordering cost per order $=\frac{D}{Q} \mathrm{XC}=\frac{C D}{Q}$

Therefore total variable cost is $\mathrm{V}=\mathrm{rP} \frac{Q}{2}+\frac{C D}{Q}$ ..(*)
By applying differentiate on (*), wrt Q , we have that $\frac{d V}{D Q}=\frac{r P}{2}-\frac{C D}{Q^{2}}$
Obtaining the second derivative again, we have $\frac{d^{2} V}{d Q^{2}}=\frac{2 C D}{Q^{3}}>0$
At minimum part of V , we have that $\frac{d V}{d Q}=0 \Rightarrow \frac{C D}{Q^{2}}=\frac{r P}{2} \Rightarrow \mathrm{Q}^{2}=\frac{2 C D}{r P} \Rightarrow \mathrm{Q}=\sqrt{\frac{2 C D}{r P}}$
This is the quantity of stock to be ordered that will minimize the total variable cost.

## IV. QUANTITY DISCOUNT

The Economic Order Quantity (EOQ) assumes constant price per unit stock. But discount is given for quantity purchase very often. The discount guarantees reduction in the ordering cost, stock holding cost and purchase price apart from giving room for higher average stock.

## A case example

If the total cost is $\$ 39,000$ and the supplier is ready to give a discount of $6 \%$ on ordering of 200 units or more, we'll see if it worth it to take the discount. We have that, without the discount, the total cost is $\$ 39,000$. Now, to enjoy or benefit the discount, at least 200 units of stock must be ordered. For a batch of size 200 units, total cost $=$ ordering cost + stock holding cost + purchase price. If it costs $\$ 60$ per unit of stock, with $6 \%$ discount, the unit purchase price would be given by $\frac{94}{100} \mathrm{X} \$ 60=\$ 56.40$. Total cost $=\frac{C D}{Q}+\frac{r P Q}{2}+\mathrm{DP}$, where for now, $\mathrm{C}=\$ 500, \mathrm{D}=600, \mathrm{r}=25 \%$, and $\mathrm{P}=\$ 56,40$. We have, $\mathrm{T}=\frac{500 \times 600}{200}+0.25 \mathrm{X} 56.40 \mathrm{X} 200+600 \mathrm{X} 56.40=35620$. Total cost (without discount)- Total cost (with discounts $=\$(39,000-35,620)=\$ 3,380$. Hence, greater quantity of stock with $6 \%$ discount can be ordered at a lower cost. Thus, the discount is worth talking.

## V. STOCKOUTS

A situation in which companies are run out of stock can be referred to as STOCKOUTS. It is mostly common when stocks are produced within a particular company. Here, the EOQ (Economic Ordered Quantity) is replaced by Economic Batch Quantity (EBQ). Also, a set up cost replaces the ordering cost of the EOQ. In EOQ, stock is replenished without any time lag, but the replenishment takes some time under the EBQ. Moreover, the average stock held under the EBQ is usually greater than $\frac{1}{2} \mathrm{Q}$ held under the EOQ. This is as a result of the fact that the batch quantity produced is for internal use. Now, given that Q is the batch of stock produced, t is the time to produce $\mathrm{Q}, \mathrm{R}$ is the rate of production and d is the stock sold per unit time. We have that the stock sold in time $\mathrm{t}=\mathrm{td}$. Hence, maximum stock level is given by: Q-td. But, $\mathrm{t}=\frac{Q}{R}$. Thus, maximum stock level $=\mathrm{Q}-\frac{Q d}{R}=\mathrm{Q}\left(1-\frac{d}{R}\right)$. And average stock $=\frac{Q}{2}\left[1-\frac{d}{R}\right]$. We have that $:$ total cost $=$ stock holding cost + set up cost. i.e. $\mathrm{V}=\frac{r P Q}{2}\left(1-\frac{d}{R}\right)+\frac{C d}{Q}$ (1) Here, $\mathrm{C}=$ set up cost per batch, and $\mathrm{P}=$ variable cost of producing one unit. Applying differentiation (with respect to Q ) on both sides of (1), we have $\frac{d V}{d Q}=\frac{r P}{2}\left(1-\frac{d}{R}\right)+\frac{C d}{Q^{2}}$. on second derivative, we have $\frac{d^{2} V}{d Q^{2}}=\frac{2 C d}{Q^{2}}$, which must be positive, i.e. $\frac{d^{2} V}{d Q^{2}}=\frac{2 C d}{Q^{2}}>0$. Hence, V must be at minimum whenever $\frac{d V}{d Q}=\frac{r P}{2}\left(1-\frac{d}{R}\right)+\frac{C d}{Q^{2}}=0 . \Rightarrow \frac{C d}{Q^{2}}=\frac{r P}{2}\left(1-\frac{d}{R}\right) \Rightarrow \mathrm{Q}^{2}=\frac{2 C d}{r P\left(1-\frac{d}{R}\right)} \Rightarrow \mathrm{Q}=\sqrt{\frac{2 C d}{r P\left(1-\frac{d}{R}\right)}}$.

## Sample Case

Considering a wood processing company with a steady demand of 200 units in two weeks, with a production rate of 200 units per week. The variable cost for each production is $\$ 200$ per unit. The set up cost for each production run is estimated to be $\$ 1000$. Also, the company's capital cost is about $20 \%$. It's also assumed that ten working weeks is feasible within a year.
We have that : (i)the Economic Batch Quantity $Q=\sqrt{\frac{2 C d}{r P\left(1-\frac{d}{R}\right)}}$. Here, $\mathrm{d}=10 \mathrm{X} 100=1000$ units, $\mathrm{R}=200 \mathrm{X} 10=2000$ units, $\mathrm{r}=20 \%=0.2, \mathrm{P}=\$ 200, \mathrm{C}=\$ 1000$. Then, $\mathrm{Q}=\sqrt{\frac{2 \times 1000 \times 1000}{0.2 \times 200\left(1-\frac{1000}{2000}\right)}}=\sqrt{\frac{2,000,000}{40\left(\frac{1}{2}\right)}}=\sqrt{\frac{2,000,000}{20}}=$ $\sqrt{100,000} \cong 320$ units. We now have that $\mathrm{V}=r P\left(1-\frac{d}{R}\right)+\frac{C d}{Q}=\frac{0.2 \times 200 \times 320}{2}(1-0.5)+\frac{1000 \times 1000}{320}$ $3200+3125=\$ 6325$.

## VI. CONCLUSION

We have been able to see that discounts giving helps more in giving privileges to buyers of goods and services of which the valuable impacts and importance can not be glossed over emphasized or underestimated

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