International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 12, Issue 4 [April. 2023] PP: 41-56

Analytical Study of Ion-Acoustic Solitons in Plasma having Ultra –Relativistic Degenerate Electrons and Positrons

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Abstract
Ion-acoustic solitons (IAS) have been theoretically investigated in four component plasma consisting of inertialess ultra-relativistic degenerate (URD) electrons and positrons, cold- mobile- inertial ion fluid, and negatively charged static dust particles using a new analytical method. The degenerate pressure of electron-positron fluid has been assumed in terms of the density of ultra-relativistic limit of Chandrasekhar. Some necessary and sufficient conditions are derived for the existence of IAS in URD plasma. The expressions of critical values of positron density and positron temperature for the existence of IAS have been obtained and discussed graphically. The limiting values of soliton- speed for the IAS have also been calculated and graphically discussed. It is seen that IAS would be excited in URD plasma and the electrons and positrons have significant effects on the IAS. Our results are new and may be applicable for the study of nonlinear wave processes in relativistic degenerate dense plasmas of astrophysical objects, namely, in white dwarfs and neutron stars.
Key Words: *Analytical Study, Ion-Acoustic Solitons, Ultra-Relativistic Degenerate Electrons and Positrons, Necessary and Sufficient Conditions.*

Date of Submission: 24-03-2023 Date of acceptance: 07-04-2023

I. Introduction

In relativistic degenerate dense plasma, propagation of waves is studied following the works of Chandrasekhar [1,2] instead of Das and Paul [3]. The equation of state for degenerate electrons in the interstellar compact objects has been mathematically explained by Chandrasekhar [1, 2] for two limits, namely, non-relativistic and ultra-relativistic limits. The degenerate electron equation of state suggested by Chandrasekhar is $P_e \propto n_e^{5/3}$ for the non-relativistic limit, and $P_e \propto n_e^{4/3}$ for the ultra-relativistic limit, where P_e is the degenerate electron pressure and n_e is the degenerate electron number density. Masood and Eliasson [4] have studied electrostatic solitary waves in quantum plasma with relativistic degenerate electrons and cold ions. They derived the Korteweg-deVries (KdV) equation for finite small amplitude waves and studied the properties of localized ion acoustic solitons for parameters relevant for dense astrophysical objects such as white dwarf stars. Later, Chandra et al. [5] have studied electron-acoustic solitary waves in a relativistic degenerate quantum plasma with two-temperature electrons using the Quantum Hydro-Dynamic (QHD) model and degenerate electron pressure $P_e \propto n_e^{4/3}$ .It is seen that that degeneracy parameter significantly influences the conditions of formation and properties of solitary structures. Later, Chandra et al. [6] have investigated the nonplanar ion-acoustic waves in relativistic degenerate quantum plasma using QHD model and deriving a nonlinear Spherical Kadomtsev–Petviashvili (SKP) equation using reductive perturbation method. It is shown that electron degeneracy parameter significantly affects the linear and nonlinear properties of ion-acoustic waves in quantum plasma.But, for the study of nonlinear waves in compact astrophysical objects Chandrasekhar Model is more applicable in ultra-relativistic degenerated (URD) dense plasma. Mamun and Shukla [7] have been studied the planner and nonplanar electrostatic solitary waves using reductive perturbation method in ultra-relativistic degenerate dense plasma, which is relevant to interstellar spherical compact objects like white dwarfs. Later, Esfandyari-Kalejahi et al [8] have studied arbitrary amplitude ion-acoustic solitary waves using Sagdeev-Potential approach in electron-positron-ion plasma with ultra-relativistic or non-relativistic degenerate electrons and positrons and numerically investigated the matching criteria of existence of such solitary waves It is seen that the relativistic degeneracy of electrons and positrons has significant effects on the amplitude and the Mach-number range of solitary waves. Recently, Roy et al [9] investigated the nonlinear propagation of waves (specially solitary waves) in an ultra-relativistic degenerate dense plasma containing ultra-relativistic degenerate electrons and positrons, cold mobile inertial ions, and negatively charged static dust using reductive perturbation method. Rahman et al [10] have investigated the linear and nonlinear properties of ion acoustic excitations adopting a reductive perturbation method, the K-dV equation is derived, which admits a localized wave solution in the form of a small-amplitude weakly super-acoustic pulse-shaped soliton. The analysis is extended to account for arbitrary amplitude solitary waves, by deriving a pseudo energy-balance like equation, involving Sagdeev-type pseudo-potential. They have shown that the two approaches agree exactly in the small amplitude weakly super-acoustic limit. The range of allowed values of the pulse soliton speed (Mach number), wherein solitary waves may exist, is determined. The effects of plasma parameters, namely, the electron relativistic degeneracy parameter, the ion (thermal) to the electron (Fermi) temperature ratio, and the positron-to-electron density ratio, on the soliton characteristics and existence domain, have been studied in detail.

In this paper, using a new analytical method the IAS has been studied in four component plasma consisting of ultra-relativistic degenerate (URD) electrons and positrons, cold- mobile- inertial ion fluid, and negatively charged static dust particles considering the ultra-relativistic limit of Chandrasekhar [1,2]. The necessary and sufficient conditions are derived for the existence of IAS in the URD plasma. The critical values of positron density, positron temperature and ion temperature for the existence of IAS have been obtained and discussed. The Limiting values of soliton- speed for the IAS are calculated and graphically discussed. It is seen that URD electrons and positrons in plasma have significant contribution to the excitation of IAS in plasma.

II. Basic Equations

We consider a four-component component plasma consisting of inertialess URD electrons and positrons, cold- mobile- inertial ion fluid, and negatively charged static dust particles . The degenerate pressure of electron-positron fluid are expressed in terms of density of ultra-relativistic limit of Chandrasekhar [1,2]. The nonlinear dynamics of the electrostatic wave in the URD dense plasma is described by the equations given below [7-10]:

$$\frac{\partial n_s}{\partial t}+\frac{\partial}{\partial x}(n_s u_s)=0, \tag{1}$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}=-\frac{\partial \varphi}{\partial x}, \tag{2}$$

$$n_e\frac{\partial \varphi}{\partial x}-\frac{3}{4}\beta_e\frac{\partial n_e^{4/3}}{\partial x}=0, \tag{3}$$

$$n_p\frac{\partial \varphi}{\partial x}+\frac{3}{4}\beta_p\frac{\partial n_p^{4/3}}{\partial x}=0, \tag{4}$$

$$\frac{\partial^2 \varphi}{\partial x^2}=\alpha_e n_e-\alpha_p n_p-n_i+\alpha_d, \tag{5}$$

where, *s = e, p, i* denote for electron, positron and ion; velocity u_s is normalized by ion-acoustic speed $C_s=(k_B T_e/m_i)^{1/2}$ where k_B is Boltzmann constant, T_e is the temperature of electrons and m_i is the mass of an ion; density n_s is normalized by the equilibrium ion density; the electrostatic potential φ is normalized by $m_e c^2/e$; space variable x is normalized by $(m_e c^2/4\pi n_{i0}e^2)^{1/2}$ and time variable t is normalized by ion plasma period $(\omega_{pi})^{-1}$; $\omega_{pi}=(4\pi n_{i0}e^2/m_i)^{1/2}$;

$$\left.\begin{aligned}&\alpha_1=\lambda_c n_{e0}^{1/3},\alpha_2=\lambda_c n_{p0}^{1/3},\lambda_c=hc/m_e,\\&\beta_e=K_0\alpha_1,\beta_p=K_0\alpha_2,K_0=(\hbar/m_e c),\\&\beta_p=\gamma\beta_e,\gamma=(n_{p0}/n_{e0})^{1/3},\\&\alpha_e=(n_{e0}/n_{i0}),\alpha_p=(n_{p0}/n_{i0}),\alpha_d=(Z_d n_{d0}/n_{i0}).\end{aligned}\right\} \tag{6}$$

where: n_{e0}, n_{p0}, n_{i0} are the equilibrium densities of electrons, positrons and ions; Z_d is number of negative charges of dust particles. Other parameters have their usual meanings.
The charge neutrality condition is

$$\alpha_e + \alpha_d = 1 + \alpha_p, \text{ i.e. } i)\alpha_e = 1 + \alpha_p - \alpha_d, \text{ or } ii)\alpha_p = \alpha_e + \alpha_d - 1. \quad (7)$$

III. Analytical Study

To obtain a solution of IAS, we introduce the single independent variable η defined by the relation $\eta = x - Vt$, where V is the velocity of the solitary wave. The boundary condition is

$$n_i \to 1`, u_i \to u_{io}, \phi \to 0 \text{ at } |\eta| \to \infty \quad (8)$$

Using above transformation in (1) we obtain

$$n_i = \frac{V - u_{i0}}{V - u_i} \quad (9a)$$

Using (9a) in Eq.(2) we get

$$n_i = \frac{1}{\sqrt{1 - \frac{2\varphi}{V^2}}} \quad (9b)$$

Again using (9a) in (2) we get

$$\varphi(u_i) = V(u_i - u_{i0}) - \frac{1}{2}(u_i^2 - u_{i0}^2) \quad (10)$$

From (3) an(4) we obtain

$$n_e = \left(1 + \frac{\varphi}{3\beta_e}\right)^3 \quad (11)$$

$$n_p = \left(1 - \frac{\varphi}{3\beta_e}\right)^3 \quad (12)$$

Using Eqs. (9) ,(11) and (12) we obtain from Eq. (5),

$$\frac{d^2\phi}{d\eta^2} = G(u_i) \quad (13)$$

Where,

$$G(u_i) = \alpha_e \left(1 + \frac{\varphi}{3\beta_e}\right)^3 - \alpha_p \left(1 - \frac{\varphi}{3\beta_p}\right)^3 - \frac{V - u_{i0}}{V - u_i} \quad (14)$$

3.1. Analytical Study of Ion-Acoustic Solitons

Integrating Eq. (13), we get

$$\left(\frac{d\phi}{du_i}\right)^2 \left(\frac{du_i}{d\eta}\right)^2 = H(u_i) - K \quad (15)$$

where, K is a constant of integration and

$$H(u_i) = 2\int G(u_i) \frac{d\phi}{du_i} du_i \quad (16)$$

Now, from Eq. (10) we obtain

$$\frac{d\phi}{du_i} = (V - u_i) \quad (17)$$

Using Eqs**.** (11) and (12), the Eq. (15) yields

$$H(u_i)=\left[\frac{1}{4}\left\{3\alpha_e\beta_e\left(1+\frac{\varphi}{3\beta_e}\right)^4-3\alpha_p\beta_p\left(1-\frac{\varphi}{3\beta_p}\right)^4\right\}-(V-u_{i0})u_i\right] \quad (18)$$

It may be noted that the function $H(u_i)$ is highly nonlinear and contains a number of parameters including soliton velocity, nonthermal parameter of the electrons and ion temperature. These parameters enter into the function in a complicated way and thus restrict the range of possible solitary wave solutions.

3.2. Necessary and Sufficient Conditions

For ion-acoustic solitary wave in a nonthermal electron plasma with warm positive ion, the physically admissible solution of Eq. (14) is obtained with some special observations:

(i) $\left(\frac{d\phi}{du_i}\right)^2\left(\frac{du_i}{d\eta}\right)^2$ must be non-negative, (19)

and

(ii) u_i and $\frac{du_i}{d\eta}$ must be bounded. (20)

From Eqs. (14) and (17), it is easy to make the following observations:

***Observation 1*:** $G(u_{i0})=1>0$ (21)

***Observation 2*:** $\phi(u_{i0})=0$ (22)

***Observation 3*:** $\phi^{/}(u_i)>0$ or < 0 for according as

$u_i<V$ or $u_i>V$ (23)

Therefore, for physical solution of Eq. (13) we need following requirements:

***Requirement 1*:** There exists $u_{\max}$ or $u_{\min}$ such that

$H(u_{i0})=H(u_{\max})=K$ for $u_{i0}\le u_{\max}<u'$ (24)

Or $H(u_{i0})=H(u_{\min})=K$ for $u'\le u_{\min}<u_{i0}$ (25)

***Requirement 2*:** $H(u_i)\rangle K$ for either $u_{i0}\le u_i<u_{\max}$ or $u_{\min}<u_i\le u_{i0}$ (26)

Our next task is to obtain simple conditions (either necessary or sufficient) for the requirement to be fulfilled. We establish the following Theorems:

Theorem 1. A necessary condition is that the Eq.(15) will admit a real and bounded solution if $H''(u_{i0})>0.$

Proof: If requirement 2 is fulfilled, one has $H(u_i)>H(u_{i0})$ for $u_i=u_{i0}+\varepsilon$ $(\varepsilon>0)$ however small, an arbitrary number. It immediately follows that $H''(u_{i0})>0.$

Theorem 2. A sufficient condition is that the Eq.(15) will admit a real and bounded solution determined by $H(u_i')-H(u_{i0})<0.$

Proof: For a physically admissible solution it is obvious that $H'(u_i)\ge 0$ for $u_i=u_{i0}+\varepsilon$ $(\varepsilon>0)$ however small, an arbitrary number.

It follows that $H(u_i+\varepsilon)>H(u_{i0}).$ Also from the condition $H(u_i')<H(u_{i0})$ there exists a point $u_{i\max}$ (say) ($u_{i\min}$ (say)) between u_{i0} and u_i' such that $H(u_{i\max})=H(u_{i0})$ for $u_{i0}<u_i<u_{i\max}$

Or $H(u_{i\min})=H(u_{i0})$ for $u_{i\min}<u_i<u_{i0}$. Consequently, there exist u^* between u_{i0} and $u_{i\max}$ such that $H(u^*)=0.$

From the above two Theorems, we get the following simple conditions for real and bounded solution of Eq.(15), we obtain

$$V > u_{i0} + \sqrt{\frac{\alpha_e}{\beta_e} + \frac{\alpha_p}{\beta_p}} \quad \text{for} \quad V > u_{i0} \tag{27}$$

and $$u_{i0} > V + \sqrt{\frac{\alpha_e}{\beta_e} + \frac{\beta_e}{\beta_p}} \quad \text{for} \quad V < u_{i0} \tag{28}$$

3.3. Critical Values of Plasma Parameters for Solitary Wave Solitons

In the plasma consisting of URD electrons and positrons the critical values of phase velocity (V) are obtained from (27) and (28) given by

$$V = V_F = u_{i0} + \sqrt{\frac{\alpha_e}{\beta_e} + \frac{\alpha_p}{\beta_p}} \tag{29}$$

$$V = V_S = u_{i0} - \sqrt{\frac{\alpha_e}{\beta_e} + \frac{\alpha_p}{\beta_p}} \tag{30}$$

It is seen from (29) and (30) that the streaming motion (u_{i0}) of ions has significant effects on the critical values of the phase velocity and other plasma parameters. It is interesting to note that two distinct modes (Fast and Slow) of the wave exist in presence of streaming motion of ions in URD plasma consisting of electrons and positrons.

IV. Sagdeev Potential and Ion-Acoustic Solitons

In previous section, we have discussed the conditions for the existence of IAS in terms of ion fluid velocity (u_i) instead of electrostatic potential ϕ or number density (n_i) of ions. However, expressing the term G in Eq. (14) as a sole function of ϕ and integrating the resulting equation one can find out the Sagdeev potential and then one can use it in usual way to discuss the properties of IAS structures.

The Eq. (14) can be written in the form

$$\frac{d^2\phi}{d\eta^2} = \alpha_e \left(1 + \frac{\varphi}{3\beta_e}\right)^3 - \alpha_p \left(1 - \frac{\varphi}{3\beta_e}\right)^3 - \frac{V - u_{i0}}{V - u_i} = G(u_i) = -\frac{d\psi}{d\varphi} \tag{31}$$

where $\psi(\phi)$ is known as the Sagdeev potential and is given by

$$\psi = \left[\frac{1}{4} \left\{ \alpha_e \beta_e \left(1 + \frac{\varphi}{3\beta_e}\right)^4 - \alpha_p \beta_p \left(1 - \frac{\varphi}{3\beta_p}\right)^4 \right\} - (V - u_{i0}) u_i \right] \tag{32}$$

The form of the pseudo-potential $\psi(\varphi)$ would determine whether a soliton like solution of Eq. (32) will exist or not. The conditions for the existence of soliton solution are:

i) $\psi(\varphi) = 0$ at $\varphi = 0$ and $\varphi = \varphi_c$ (33a)

ii) $\dfrac{d\psi}{d\varphi} = 0$ at $\varphi = 0$ (33b)

and

iii) $\dfrac{d^2\psi}{d\varphi^2} < 0$ at $\varphi = 0$ (33c)

An analytical solution for the IAS can be obtained by expanding the right hand side of Eq.(31) in terms of ϕ keeping the terms up to second order, third order or even next higher orders.

Using the values of n_i, n_e and n_p of (9b),(10) and (11) in (5) we obtain

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 + R\varphi^3 + \cdots\cdots \tag{34}$$

where

$$P = \left[\left(\frac{\alpha_e}{\beta_e} + \frac{\alpha_p}{\beta_p} \right) - \frac{1}{(V - u_{i0})^2} \right], \tag{35a}$$

$$Q = \left[\left(\frac{\alpha_e}{3\beta_e^2} - \frac{\alpha_p}{3\beta_p^2} \right) - \frac{3}{(V - u_{i0})^4} \right], \tag{35b}$$

$$R = \left[\left(\frac{\alpha_e}{27\beta_e^3} + \frac{\alpha_p}{27\beta_p^3} \right) - \frac{5}{2(V - u_{i0})^6} \right]. \tag{35c}$$

Now, for small amplitude IAS, the terms up to φ^2 in (34) are taken into consideration, neglecting next higher order terms of φ i.e. φ^3, φ^4, etc. So, we get

$$\frac{d^2\varphi}{d\eta^2} = P\varphi - Q\varphi^2 \tag{36}$$

Using the standard mathematical technique, the solutions of Eq. (36) for the small amplitude IAS are obtained as:

$$\varphi_1 = \frac{3P}{2Q} \sec h^2\theta \tag{37}$$

where $\theta = [(P/4)^{1/2}\eta]$.

The amplitude (φ_{01}) of the IAS is given by

$$\varphi_{01} = \frac{3P}{2Q} \tag{38}$$

and the widths (δ_1) of the IAS is given by

$$\delta_1 = \frac{2}{\sqrt{P}} \tag{39}$$

To obtain the solutions of small amplitude IAS for Slow-mode and Fast-mode in URD plasma, the phase velocity (*V*) is replaced by V_S and V_F of Eq.(29) and Eq.(30) in the Eqs.(37) . So, the solution of Eq. (36) of small amplitude IAS for Slow-mode in URD plasma will be

$$\varphi_{1S} = \frac{3P_S}{2Q_S} \sec h^2\theta_S \tag{40a}$$

and

$$\varphi_{1F} = \frac{3P_F}{2Q_F} \sec h^2\theta_F \tag{40b}$$

The amplitude (φ_{01}) and the width (δ_1) of small amplitude IAS for Slow-mode and Fast-mode in URD plasma are:

$$\varphi_{01S} = \frac{3P_S}{2Q_S}, \varphi_{01F} = \frac{3P_F}{2Q_F} \tag{41a}$$

and

$$\delta_{1S} = \frac{2}{\sqrt{P_S}}, \delta_{1F} = \frac{2}{\sqrt{P_F}} \tag{41b}$$

V. Results and Discussion

5.1. Variation of H with u_i

In Eq. (18) it is seen that the nonlinear function H is highly nonlinear and it depends on the parameters of URD plasma. To see the effects of u_i on H for different values of electron temperature (β_e) , positron temperature (β_p) and ion stream velocity (u_{i0}) numerical estimation are made considering model URD plasma and the graphs shown in Figs. 1, 2 and 3 are drawn.

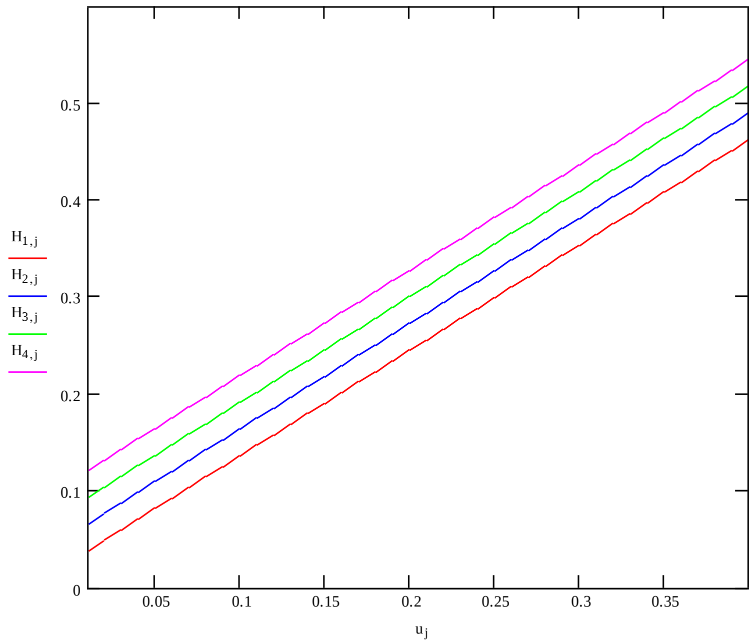

Fig.1. Variation of H for different values of u_i and electron temperature (β_e) .. The red, blue, green and magenta lines for β_e = 0.1, 0.2, 0.3, 0.4 The values of other parameters of the plasma are: u_{i0} = 0.5, φ =0.01, V =1.6, α_p =0.1, β_p =0.5.

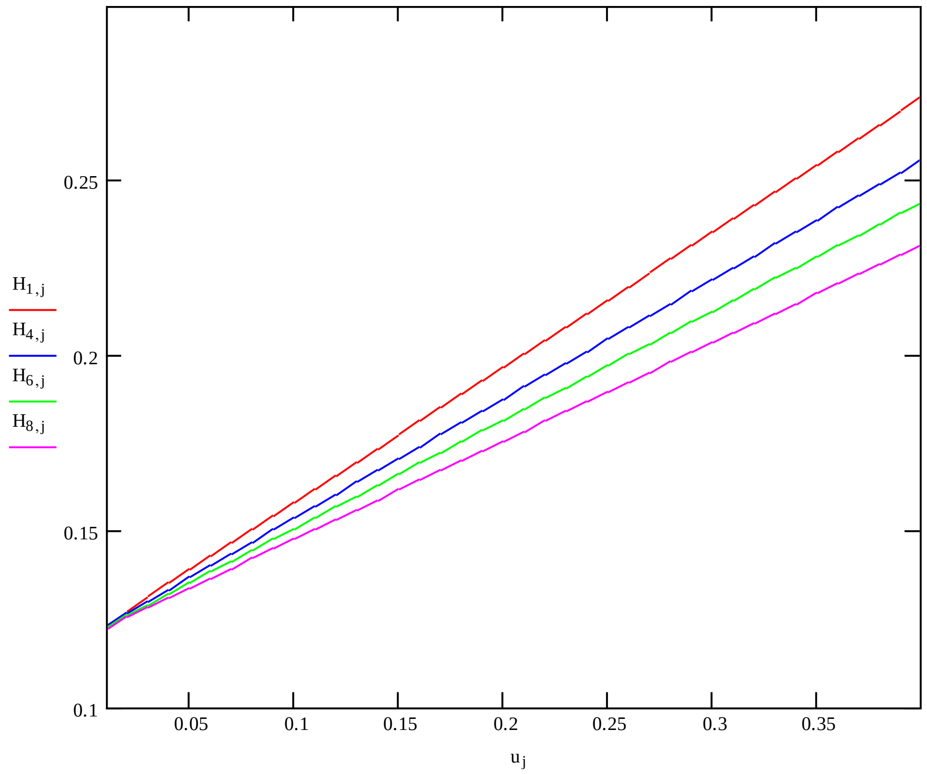

Fig.2. Variation of H for different values of u_i and positron temperature (β_p) ,The red, blue, green and magenta lines for β_p = 0.1, 0.4, 0.6, 0.8 The values of other parameters of the plasma are: u_{i0} = 1.2, φ =0.01, V =1.6, α_p =0.6, α_e =1.6.

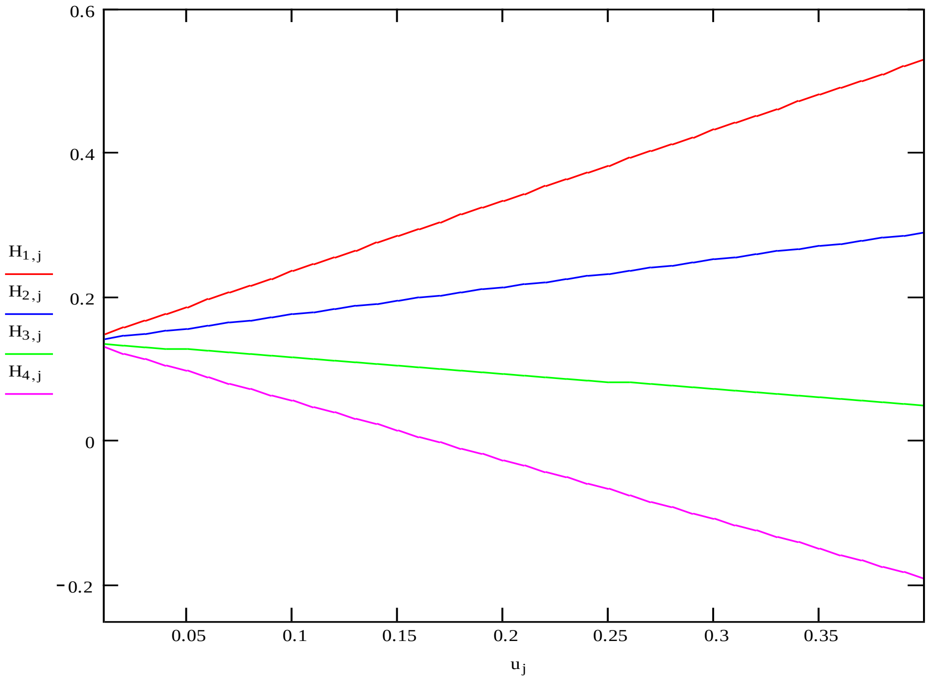

Fig.3. Variation of H for different values of u_i and ion stream velocity (u_{i0}).. The red, blue, green and magenta lines for u_{i0} = 0.6, 1.2, 1.8, 2.4. The values of other parameters of the plasma are: φ =0.01, V =1.6, β_e =0.5, β_p =0.8, α_p =0.1.

From Fig. 1 it is seen that H increases with u_i and β_e and *H* is large for large values of β_e. On the other hand, Fig. 2 shows that *H* sharply increases with the increase of u_i and β_p. The variations of *H* with u_i for different values of u_{i0} are shown in Fig.3. It is seen that *H* sharply increases with the increase of u_i and decreases with increase of u_{i0}.

5.2. Critical Values of Plasma Parameters

For the existence of IAS in URD plasma consisting of degenerate electrons and positrons, the critical values of phase velocity (*V*),degenerate electrons and positrons should be considered.

i). Critical Value of Phase Velocity

Let us first numerically estimated the critical values of phase velocities of fast- and slow- modes of the ion-acoustic wave for different values of ion stream velocity and degenerate electrons and positrons numerically considering model plasma. The variation of phase velocities with the plasma parameters are graphically shown in Figs 4(a).4(b).5(a) and 5(b).

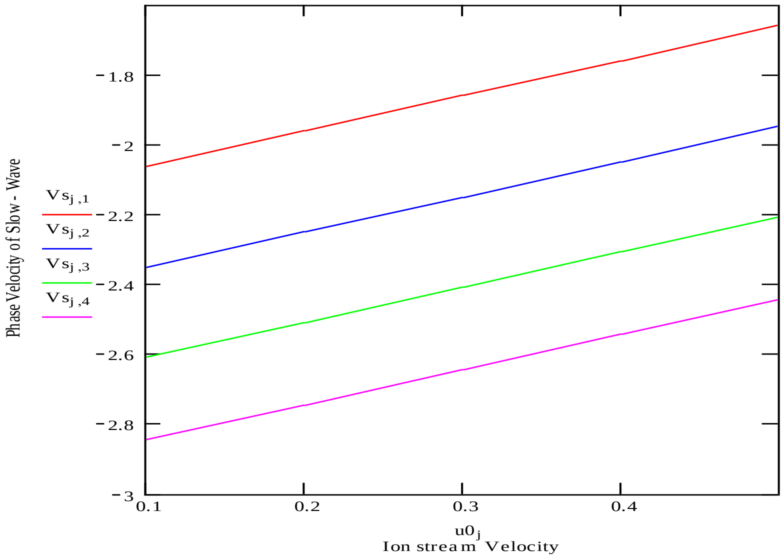

Fig 4(a) Variation of phase velocity (V_s) of Slow – mode of ion- acoustic wave with ion stream velocity (u_{i0}) for different values of URD positron density (α_p) .The red, blue, green and magenta lines correspond α_p =0.1, 0.2, 0.3 and 0.4 respectively; β_e =0.3, β_p =0.1.

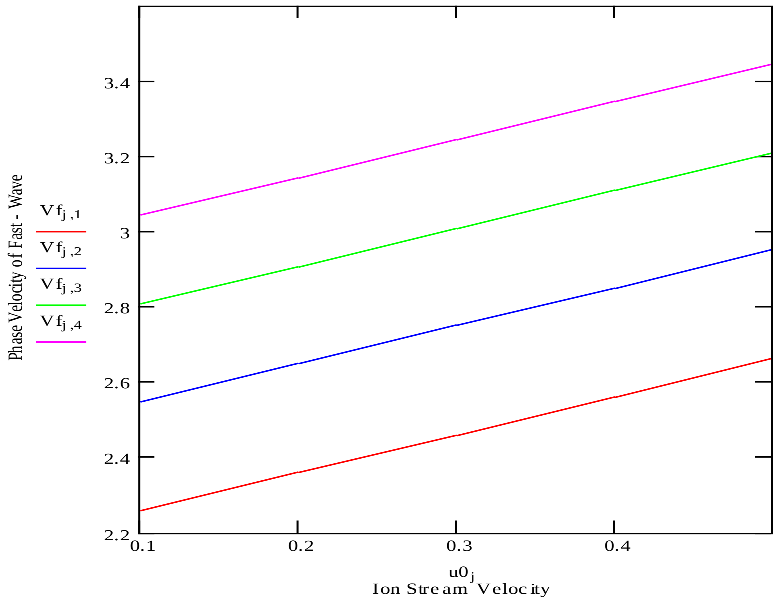

Fig 4(b) Variation of phase velocity of Fast- mode of ion- acoustic wave in URD plasma with ion stream velocity (u_{i0}) for different values of URD positron density (α_p).The red, blue, green and magenta lines correspond α_p =0.1, 0.3, 0.3 and 0.4 respectively; β_e =0.3, β_p =0.1.

Fig.4(a) shows the variations of V_S with ion drift velocity (u_{i0}) for different values of positron density (α_p).It is seen that V_S is always negative and it decreases with the increase of α_p and increases with increases of u_{i0} in URD plasma when other plasma parameters have fixed values β_e =0.3 and β_p =0.1. The negative value of V_S means that the Slow-mode of the wave is non-propagating in URD plasma.

But, it is seen from Fig. 4(b) that V_F is always positive and it increases with the increase of α_p in URD plasma. The positive value of V_F means the Fast-mode of the wave would be propagated through the URD plasma.

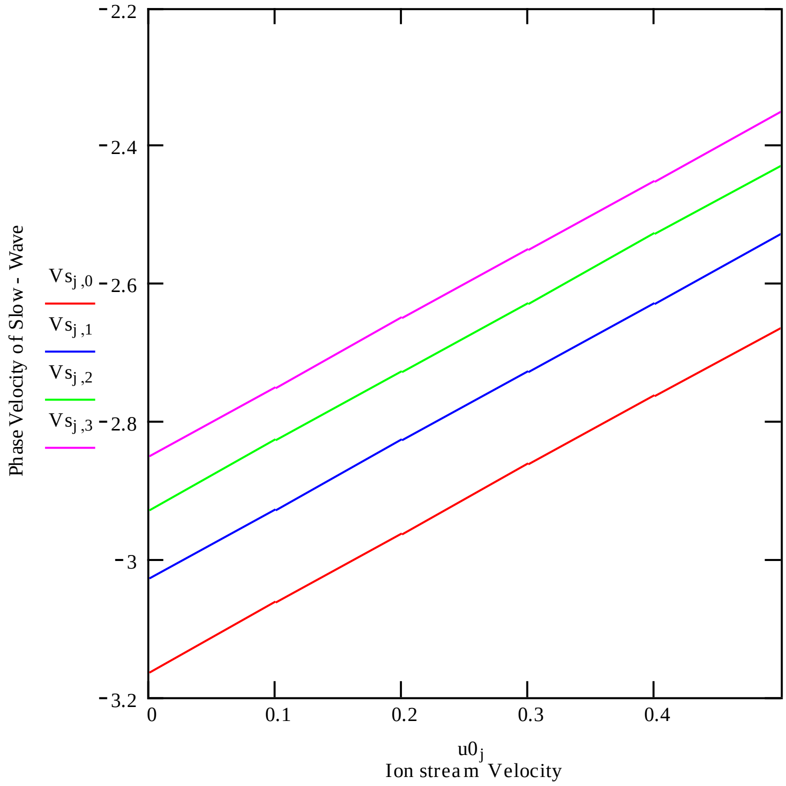

Fig.5(a). Variation of phase velocity of Slow-mode (V_S) of ion- acoustic wave in URD plasma with ion stream velocity (u_{i0}) for different values of degenerate positron temperatures (β_p) .The red, blue, green and magenta lines correspond β_p =0.1, 0.12, 0.14 and 0.16 respectively; β_e =0.3, α_p =0.5.

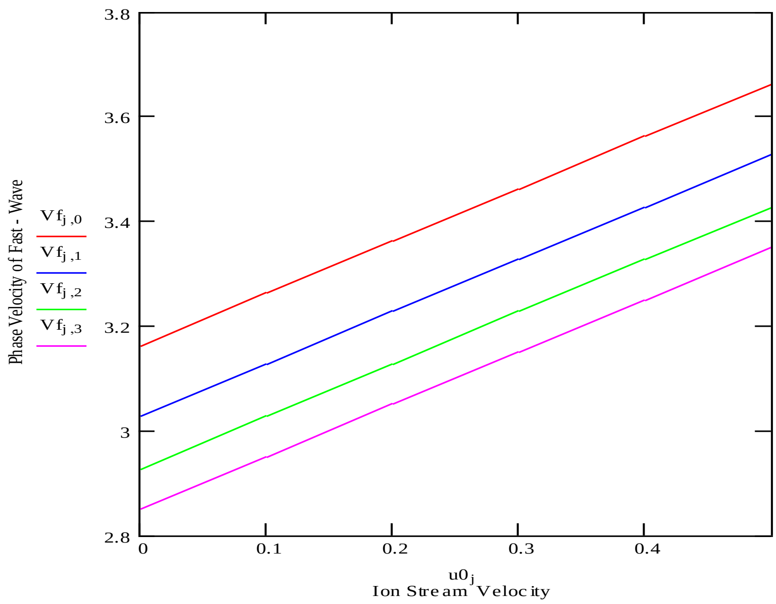

Fig.5(b). Variation of phase velocity of Fast-mode (V_F) of ion- acoustic wave in URD plasma with ion stream velocity (u_{i0}) for different values of degenerate positron temperatures (β_p) .The red, blue, green and magenta lines correspond β_p =0.1, 0.12, 0.14 and 0.16 respectively; β_e =0.3, α_p =0.5.

Similarly, the variations of V_F with ion drift velocity (u_{i0}) for different values of positron temperature (β_p) are numerically estimated and the variation of the phase velocity is shown graphically in Fig. 5(a) and Fig.5(b). It is seen from Fig.5(a) that V_S is always negative and it increases with the increase of β_p and increases with increases of u_{i0} in URD plasma when other plasma parameters have fixed values β_e=0.3 and α_p=0.5. The negative value of V_S means the Slow-mode of the wave is non-propagating in URD plasma. But from Fig.5(b) it is found that V_F is always positive and it increases with the increase of u_{i0} but decreases with increase of β_p. The positive value of V_F indicates that the Fast-mode of the wave would be propagated through in URD plasma.

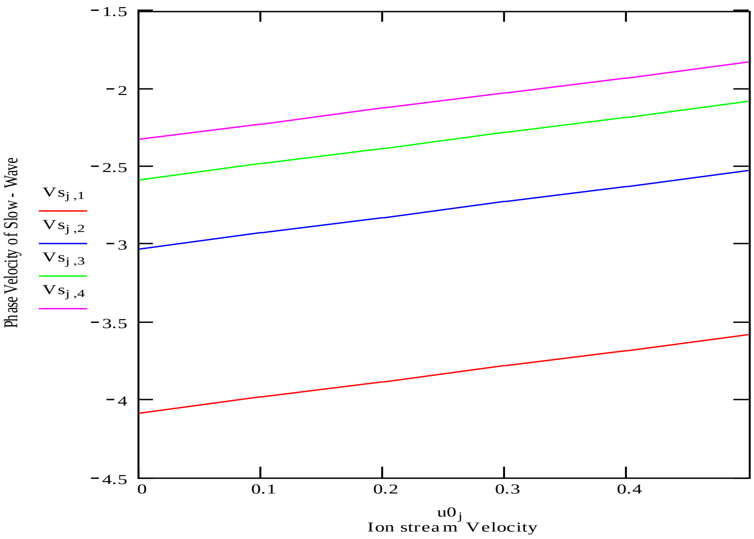

Fig.6(a).Variation of phase velocity of slow-mode (V_S) of ion acoustic wave with ion stream velocity (u_{i0}) for different values of URD electron temperature (α_p) .The red, blue, green and magenta lines correspond α_p =0.1, 0.2, 0.3 and 0.4 respectively; β_e=0.3, β_p=0.5.

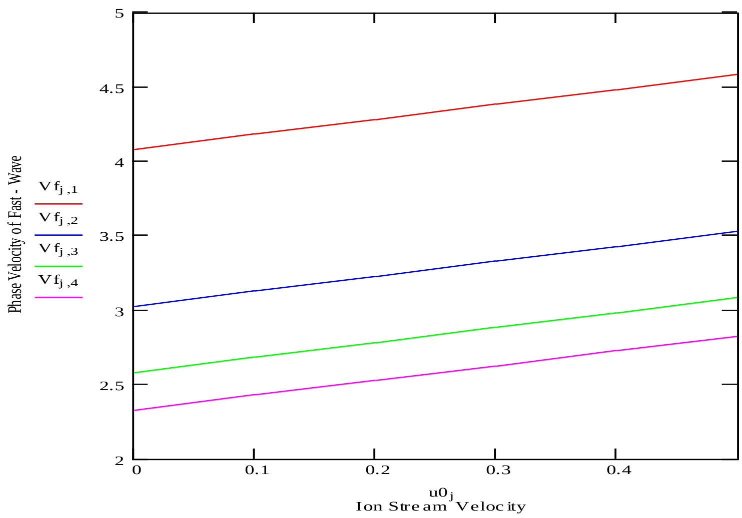

Fig.6(b).Variation of phase velocity of fast – mode (V_F) of ion acoustic wave with ion stream velocity (u_{i0}) for different values of URD electron temperature (α_p) .The red, blue, green and magenta lines correspond α_p =0.1, 0.2, 0.3 and 0.4 respectively; β_e=0.3, β_p=0.5

On the other hand , the variations of V_S and V_F with ion drift velocity (u_{i0}) for different values of electron temperature (α_p) are numerically estimated and the variation of the phase velocities are shown graphically in Fig. 6(a) and Fig.6(b). It is seen from Fig.6(a) that V_S is always negative and it increases with the increase of α_p and increases with increases of u_{i0} in URD plasma when other plasma parameters have fixed values β_e=0.3 and β_p=0.5. The negative value of V_S means the Slow-mode of the wave is non-propagating in URD plasma. But from Fig.6(b) it is found that V_F is always positive and it increases with the increase of u_{i0} but decreases with increase of α_p The positive value of V_F indicates that the Fast-mode of the wave would be propagated through in URD plasma.

5.3. Profiles of IAS for Slow-Mode and Fast-Mode

So, to see the real nature of IAS in URD electrons - positrons. plasma in presence of an ion stream, the values of the phase velocities of slow mode (V_S) and fast-mode (V_F) should be used instead of arbitrary (Fixed) value phase velocity for drawing the profiles of the IAS. Using V_S and V_F for our model of URD plasma, the profiles of small amplitude IAS for the Slow-mode and Fast -mode are drawn for different values of the parameters of plasma parameters e. g,. ion stream velocity, URD electrons and positrons. From Figs.4(a), 5(a) and 6(a) it is seen that the phase velocity V_S of Slow-mode is negative i.e. the Slow-mode of IAS will not propagate through the URD plasma. So, we have drawn the profiles of IAS for the phase velocity V_F of Fast-mode of the wave.

i). Effect of Ion Stream Velocity (u_{i0}) :

The phase velocity Slow-mode IAS in URD plasma for different values of ion stream velocity is negative. So, IAS would not be excited for the Slow mode. But, the phase velocity of Fast-mode V_F is positive for different values of ion stream velocity. So, for the Fast- mode the IAS in URD plasma would be excited .The profiles of small amplitude IAS of Fast-mode are drawn as shown in Fig. 7.

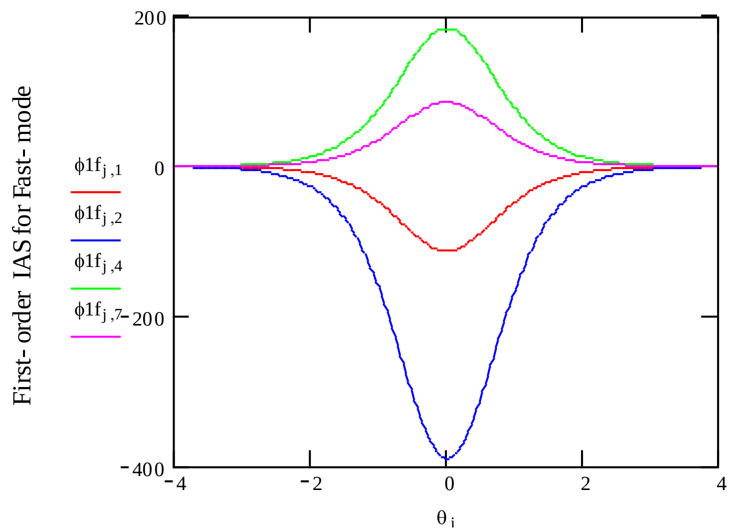

Fig. 7.Profiles of first-order IAS of Fast-mode (V_F) for different values of ion stream velocity (u_{i0}) in URD plasma The red, blue, green and magenta graphs correspond to u_{i0} =0.2,0.4, 0.8 and 1.4. Other values of plasma parameters are α_e=1.12, α_p =0.12, β_e=0.3 and β_p =0.1.

From Fig.7 it is observed that both rarefactive and compressive IAS of Fast-mode (V_F) will be excited in URD plasma depending upon the values of ion stream velocity (u_{i0}) when other plasma parameters have fixed values α_e =1.12, α_p =0.12, β_e =0.3 and β_p =0.1. For u_{i0} =0.2 and 0.4 the IAS will be rarefactive. But for u_{i0} = 0.8 and 1.4 the IAS will be compressive. The amplitude of rarefactive IAS of Fast-mode (V_F) increases with the increase of u_{i0} but the amplitude of compressive IAS will decrease with the increase of u_{i0}.

ii). Effect of Degenerate Electron Temperature (β_e) :

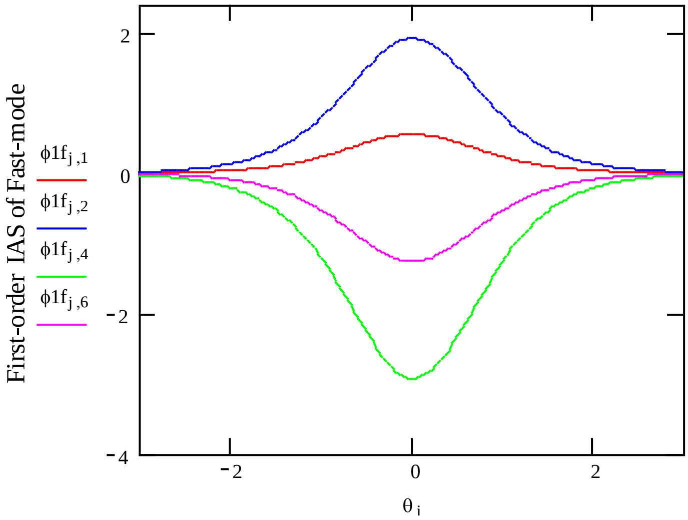

Fig. 8.Profiles of IAS of Fast-mode (V_F) for different values of degenerate electron temperature (β_e) in URD plasma .The red, blue, green and magenta graphs correspond to β_e =0.1,0.2,0.4 and 0.6. Other values of plasma parameters are α_e =1.12, α_p =0.12, u_{i0} =0.2 and β_p =0.1.

The phase velocity V_s of Slow-mode of IAS in URD plasma for different values degenerate electron temperature (β_e) is negative. So, IAS would not be excited for the Slow mode. But, the phase velocity of Fast-mode V_F is positive for different values of degenerate electron temperature (β_e) . So, for the Fast- mode the IAS in URD plasma would be excited .The profiles of small amplitude IAS of Fast-mode are drawn as shown in Fig. 8.

It is seen from Fig.8 that both rarefactive and compressive IAS for fast-mode of the wave will be excited in URD plasma depending upon the values of degenerate electron temperature (β_e) when other plasma parameters have fixed values α_e =1.12, α_p =0.12, u_{i0} =0.2 and β_p =0.1. For β_e =0.1 and 0.2 the IAS will be compressive. But for β_e = 0.4 and 0.6 the compressive IAS will be excited .The amplitude of compressive IAS increases with the increase of β_e but the amplitude of rarefactive IAS will decrease with the increase of β_e.

iii). Effect of Degenerate Positron Temperature (β_p) :

The phase velocity Slow-mode IAS in URD plasma for different values of positron temperature positive. So, IAS would not be excited for the Slow mode in URD plasma. Similarly, the phase velocity of Fast-mode is positive for different values of positron temperature So, for the Fast- mode the IAS in URD plasma would be excited .The profiles of small amplitude IAS of Slow-mode and Fast-mode are drawn for different values of positron density.

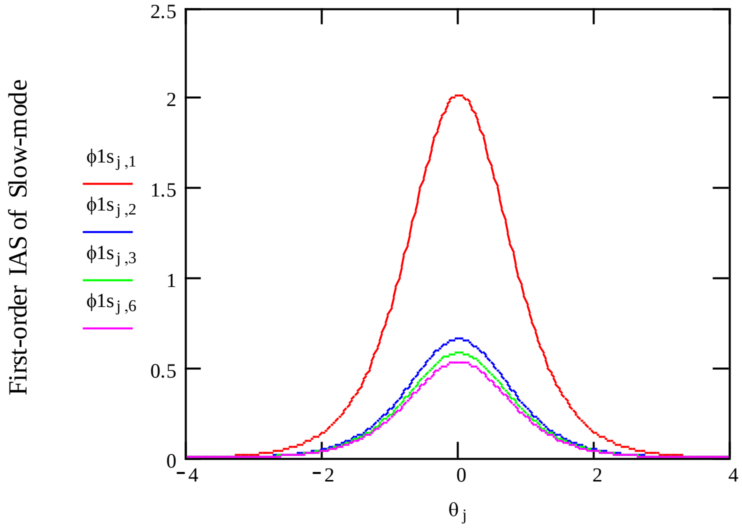

Fig. 9(a). Profiles of IAS of Slow-mode for different values of positron temperature (β_p) in URD plasma .The red, blue, green and magenta graphs correspond to β_p =0.1,0.2,0.3 and 0.6. Other values of plasma parameters are α_e =1.12, α_p =0.12, β_e =0.1 and u_{i0} =4.0.

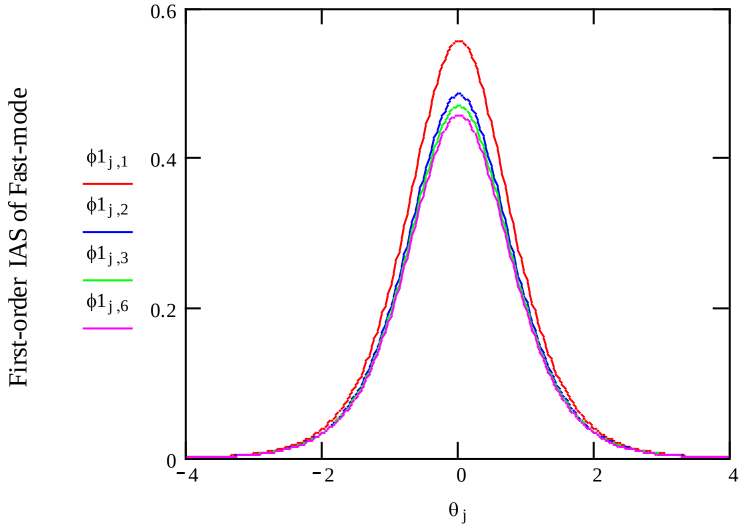

Fig. 9(b). Profiles of IAS of Fast-mode for different values of positron temperature (β_p) in URD plasma .The red, blue, green and magenta graphs correspond to β_p =0.1,0.2,0.3 and 0.6. Other values of plasma parameters are α_e =1.12, α_p =0.12, β_e =0.1 and u_{i0} =0.2.

From Fig.9(a) it is seen that only compressive IAS will be excited in URD plasma for different values of positron temperature (β_p) when other parameters have fixed values α_e =1.12 α_p =0.12, β_e =0.1 and u_{i0} =4.0 The amplitude of the IAS decreases with the increase of β_p. In Fig.9(b) it is also seen that compressive IAS will be excited in same URD plasma for different values of positron temperature (β_p) when other parameters have same values: α_e =1.12 α_p =0.12, β_e =0.1 and u_{i0} =0.2 .The amplitude of the compressive IAS decreases with the increase of β_p.

VI. Summary and Conclusion

We have made an analytical study for the existence of IAS in ultra-relativistic plasma consisting of degenerate electrons and positrons. Some necessary and sufficient conditions for the solutions of IAS of the nonlinear equations governing the dynamics of the ion-acoustic waves are also obtained. We have obtained expressions for the critical values of the phase velocity of ion-acoustic wave for the existence of IAS. The dependence of different plasma parameters of URD plasma has been depicted graphically and discussed. An important observation is that for the presence of streaming ions two distinct wave modes (slow-mode and fast-mode) are possible. To examine the effects of degenerate electrons and degenerate positrons on the nonlinear function H are studied graphically. Moreover, the behaviour of Fast-mode of the IAS are graphically discussed for different parameter of the URD plasma.

The analysis presented here for the model URD plasma consisting of degenerate electrons and degenerate positrons which may be of relevance to some space plasmas such as cometary plasmas.

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