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Development of Mathematical Modeling Using the Application of Local Magnetic Field to Describe the Dynamic of Blood Flow During Surgery

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Abstract. *In this research work, a mathematical model for an unsteady, magnetohydrodynamic (MHD) non-Newtonian blood flow, through porous blood vessels, using an oscillatory harmonic pressure gradient to describe the dynamic of blood flow during surgery was studied. Blood was modeled as a fourth grade non-Newtonian fluid. The governing model equation was solved semi analytically, using modified homotopy perturbation method (MHPM) with the help of MATHEMATICA SOFTWARE to simulate the velocity profile. The MHPM, was based on the application of Laplace transform combined with the homotopy perturbation method (HPM). The effect of magnetic field, porosity, body acceleration, pressure gradient, third and fourth-grade non-Newtonian fluid parameters and Womerseley parameter on the flow behavior velocity was examined. The main important results obtained are; the velocity of the blood decreases as the value of both the parameters representing the magnetic field, porosity, and third grade non-Newtonian fluid increases. Also, the velocity of the blood increases as the value of body acceleration, pressure gradient and fourth grade non-Newtonian fluid parameter increases.*

***Keywords**: Local magnetic field, Pressure gradient, Porous medium, Fourth grade non-Newtonian fluid.*

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I. Introduction

The study of bio-fluid dynamics has become quite interesting to many researchers from theoretical, experimental as well as clinical point of view (Mandal, Mukhophadhyay & Layek, 2012). Blood is considered to be one of the most important multi-component mixture in nature that delivers necessary substances such as nutrient and oxygen from one body cell to the other body cells through arteries and veins (Akbarzadeh, 2012). Blood is composed of plasma (contain water, glucose, dissipated protein, mineral ion, hormones, and carbon dioxide), blood cells (red blood cells, white blood cells and platelets) Srivasta, (2014). Blood is considered as either Newtonian or non-Newtonian fluid but it depend on the hypothesis (Morales, Larrabide, Geers, Aguilar, Martha & Alejandro, 2013). Many researchers had studied blood flow in an artery, by considering blood as a Newtonian fluid. Among the researches conducted are (see Das & Saha, 2009; Jain, Sharma & Singh, 2009; Mariamma & Majhi, 2000; Shit & Roy, 2011; Tanwar, Varshney & Agarwal, 2016), all were confined themselves to Newtonian blood flow models by considering Navier Stoke's equation in describing blood flow problem. Blood as whole behaves as a non-Newtonian fluid, it is commonly accepted that the hematocrits (the volume percentage of red blood cells in blood which is normally 45% for men and 40% for women), exhibit shear thinning behavior, and for this reason blood can be modeled as a non-Newtonian fluid Akbarzadeh, (2012). Using non-Newtonian model to describe the rheological characteristics or behavior of blood can be found in the work of (Majhi & Nair, 1994; Massoudi & Phuoc, 2008; Mosayebidorcheh, Hatami & Ganji, 2016; Ponalagusamy & Priyadharshini, 2017; Tan & Masuoka, 2005) and so forth.

The study of hemodynamics and hemorheology, particularly on flow of blood through an artery with the application of magnetic field has drawn the attention of many researchers for a long time due to its great importance in medical sciences. The idea of electromagnetic fields in medical research was firstly given by Hartmann (1937) and later Korchevskii and Marochnik discussed the possibility of regulating the movement of blood in human system by applying magnetic field. Blood can be regarded as magnetic fluid, in which red blood cells are magnetic in nature. Liquid carriers in the blood contain the magnetic suspension of the particle. Human body experiences magnetic fields of moderate, to high intensity in many situations of day to day life. Scientists are using magnetic field in controlling the flow of blood during surgery. For this reason, the use of application of bio magnetic fluid dynamics in blood flow is very important, more especially in dealing or treatment of some cardiovascular diseases. As reported in Sharma & Nasha, (2013), scientists are using magnetic field in controlling the flow of blood during surgery. For this reason the application of magnetohydrodynamic (MHD) in physiological problem is of growing interest (Cherry & Eaton, 2014). The blood consist of a suspension of red blood cells containing hemoglobin, which contain iron oxide, it is quite apparent that blood is an electrically conducting fluid, and exhibit (MHD) flow characteristics (Eldabe, Agoor & Alame, 2014). Agarwal et el. (2014) devel-

oped a mathematical model to study the (MHD) oscillatory blood flow through stenosed artery under the effect of slip velocity, the blood was assumed to be Newtonian. Analytical expression for velocity profile, flow rate, wall shear stress and resistive impedance have been obtained.
Jamil et el. (2018) Studied the unsteady Newtonian blood flow in the stenosed porous artery subjected to a magnetic field. Oscillating pressure gradient and periodic body acceleration were imposed on the flow field. The governing non-linear partial differential equation that governed the flow problem was solved analytically using a regular perturbation method.
Cedril et.al (2021) studied the stability analysis of non-Newtonian blood flow, conveying hybrid magnetic nanoparticles as largest drug delivery in presence of inclined magnetic field, and thermal radiation with application for cancer diagnosis, and therapy as magnetic nanoparticles can be used as a therapeutic agent in presence of thermal radiation, and an inclined magnetic field. The model governing equation of the flow problem was solved numerically by the spectral collocation method.
Jamil et.al (2021) studied the analysis of non-Newtonian magnetic blood flow in an inclined stenosed artery. The Casson fluid was used to model the blood that flows under the influences of uniformly distributed magnetic field and oscillating pressure gradient. The governing fractional differential equations were expressed using the Caputo Fabrizio fractional derivative without singular kernel.
Adrian et.al (2022) investigated the effect of non-Newtonian bio magnetic power law fluid in a channel undergoing external localized magnetic fields. The governing equations are derived by considering both the effect of Ferro hydrodynamic (FHD) and Magneto hydrodynamic (MHD).

One of the significant factor that affect the flow of blood is pressure gradient. Under normal condition, blood flow in the human circulatory system is caused by the pumping action of the heart (Uddin, Mohamad, Kamardan, Hakim, Sufahani & Rozaini, 2019). The heart is a muscular organ in humans and other animals, which produce a pulsatile pressure gradient throughout the system (popularly known as a pressure pulse which physicians check at the wrist) (Jamil, Roslan, Abdulhameed, Che-Him, Sufahani, Mohamad, & Kamardan, 2008). Thus, several researchers have made excellent studies on pulsatile flow of blood in a blood vessels. Some of these studies can be found in (Chaturani & Palanisamy, 1991; Majhi & Nair, 1994; Siddique & Awasthi, 2017; Siddique, Verma, Mishra & Gupta, 2009). For instance, Das et al. (2009) observed and investigate a mathematical model for pulsatile flow of blood through a stenosed porous medium with periodic body acceleration under the influence of a uniform transverse magnetic field by considering the blood to be a Newtonian and incompressible fluid. The governing equations that described the problem are solved analytically using the finite Hankel and Laplace transform.
Shit et al. (2011) investigated the study of pulsatile blood flow through a constricted porous channel in the presence of an external magnetic field by considering blood as an incompressible Newtonian fluid model. A perturbation method was employed to solve the governing differential equation.
Eldesoky (2012) developed a mathematical model to study the unsteady pulsatile flow of Newtonian blood flow through a porous medium in a time dependent constricted porous channel subjected to time dependent suction \ injection at the walls of the channel. The blood flow was subjected to a constant transverse magnetic field. Perturbation analysis is used to solve the systems of equations governing the flow.

The flow properties of non-Newtonian fluids are quite different from Newtonian fluids. Therefore, the main distinguishing features of many non-Newtonian fluids is that, they exhibit both viscous and elastic properties, and the relation between the shear stress and rate of shear are non-linear (Larson 1999). The non-Newtonian fluids exhibit numerous strange features such as shear thinning, shear thickening and display elastic effect. These non-Newtonian fluids have been modeled by different constitutive equations that vary greatly in complexity (Rajagopal, 1995). Therefore, because of the complex diversity in the physical structure of non-Newtonian fluids, there is no single constitutive equation in the literature that capture all the flow properties of non-Newtonian fluids. For this reason various rheological models have been proposed in order to describe the non-Newtonian flow behavior. These rheological models are classify under the following type; rate type, differential type, and integral type (Akbarzadeh, 2016; Rivlin, 1955). Amongs all these types, fluids of differential type have received special attention from different researchers (Hatami & Ganji, 2014; Rajagopal, 1995), to describes the several non-Newtonian flow behavior. Therefore due this, fluid of differential type have became the subject of many investigations covering various facets, for example thermodynamical aspect (Ellahi & Riaz, 2010; Hatami & Ganji, 2014; Majhi & Nair, 1994; Massoudi & Phuoc, 2008) etc. For instance Majhi et el. (1994) employed a mathematical model for pulsatile blood flow, subjected to externally imposed periodic body acceleration, by considering blood as a third grade non-Newtonian fluid, where the effect of the body acceleration on the velocity, flow rate and the wall shear stress were also obtained.
Hatami et el. (2014) studied the heat transfer and flow analysis for a non-Newtonian third grade nanofluid flow in a porous medium of a hollow vessel in the presence of a magnetic field. Blood was considered as the third grade non-Newtonian fluid and gold nanoparticles (A_u) are added to it.
Abdulhameed et el. (2014) studied the unsteady flow of a non-Newtonian fluid in a metallic wire coating process inside a cylindrical roll die. The constitutive equation of the fluid is modeled for a fourth grade non-Newtonian fluid with non homogeneous boundary condition. Analytical solution for the axial velocity field have been obtained in an explicit form by modified homotopy perturbation transform method (MHPTM).
Akbarzadeh (2016) studied the unsteady MHD blood flow through porous arteries concerning the in-

fluence of externally imposed periodic body acceleration and a periodic pressure gradient. Blood was considered as a third grade non-Newtonian fluid. The governing equation that described the flow problem was solved numerically using a finite difference technique and analytically using a regular perturbation. Ponalagusamy (2017) studied the pulsatile flow of Herschel-bulkley fluid through a bifurcated arterial stenosis in a porous medium with magnetic field and periodic body acceleration. The governing equations involving the shear stress were solved numerically using finite difference schemes.

The most interesting and important task that we need to address when dealing with the flow problems of non-Newtonian fluids is that, the governing equations of those models are non linear, and much more complex as compared with Newtonian fluids models. As a result of the non-linearity in the models of non-Newtonian fluids, the exact solutions of those problems are very difficult to obtain. Due to these, numerical and approximate analytical techniques (semi-analytical) have been proposed to handle such difficulties. Some of these techniques can be found in work reported in (Abdulhameed, Roslan & Bin Mohamad, 2014; Jamil, Roslan, Abdulhameed & Hashim, 2018; Massoudi & Phuoc, 2008).

The motivation of this research is to develop a mathematical modeling using the application of local magnetic field to describe the dynamic of blood flow in an artery during surgery. In this present study, we intend to broaden the work Akbarzadeh, (2016) to study the more general problem by incorporating the local magnetic field. Furthermore, the pulsatile flow is studied by considering into account a periodic pressure gradient. Also, to considered the generalized form of fluid of differential type which is (Fourth-grade fluid model). The fourth grade non-Newtonian fluid is one of the sub-class of fluid of differential type that describe the rheological behavior of fluid such as shear thinning, shear thickening and viscoelasticity, which are characteristics of non-Newtonian fluids. The semi-analytical method (homotopy perturbation transform method) is employed to solved the strongly non-linear governing equation. Consequently, analytical expression for the velocity profile was obtained. The result of modified homotopy perturbation transform method is compared with the existing result available in the literature to check the validity and effectiveness of the present study. In addition, the effect of some of the studied governing parameters in the model such as pressure gradient, porous medium, fourth grade fluid and local magnetic field on the velocity profile are examined.

2 Problem Formulation

In this study, we consider the problem of unsteady, pulsatile, laminar flow of an incompressible non-Newtonian blood flow, through a porous blood vessel or artery, in the presence of magnetic field and body acceleration. Blood is modeled as a fourth grade non-Newtonian fluid. Also, for the mathematical model of the problem, we considered an artery to be a long cylindrical tube with coordinates (r, θ, z), where r, z denotes the radial and axial coordinates and θ is the azimuthal. In this problem, blood flows in the z-direction through a fully porous vessel of radius R with an axial velocity of $V = (0, 0, u(r, t))$. The flow is assume to be stable and axisymmetric, with no radial and azimuthal component of velocity. It is suppose that there is no slip condition $(u = 0)$ on the outer wall $(r = R)$. The schematic diagram of the problem is depicted in below figure.

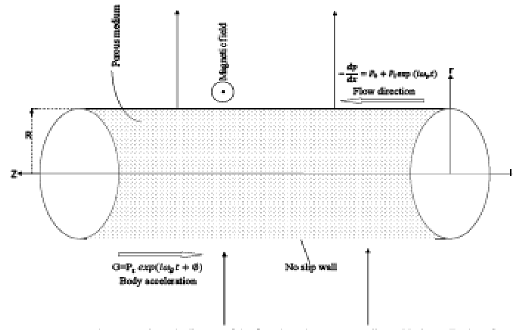

Fig 3.2.1: Schematic diagram of the flow through a porous medium with the application of magneto hydrodynamic (MHD)

2.1 Pressure Gradient

In human being blood flow is driven by the pressure gradient $\frac{\partial p}{\partial z}$, produced by the pumping action of the heart. Since blood flow in a human circulatory system is in general pulsatile, the pressure gradient component is assumed to be given as reported in the work of Abdulhameed, Vieru, Roslan & Shafie,

(2015) as follow

$$-\frac{\partial p}{\partial z} = P_0 + P_1 \cos(\omega_p t), \tag{1}$$

where P_0 is the constant or steady state part of the pressure gradient, P_1 is the amplitude or the oscillatory part of the pressure fluctuation giving rise to systolic and diastolic pressure, $\omega_p = 2\pi f_p$ is the heart pressure frequency, f_p is the pulse rate frequency.

2.2 Body Acceleration

The body acceleration G is assumed to be given by a harmonic formula as follows

$$G(t) = P_g \cos(\omega_g t + \varphi), \tag{2}$$

where P_g is the amplitude or oscillatory part of the body acceleration, $\omega_g = 2\pi f_g$ is the frequency and φ is the lead angle of the body acceleration with respect to the pressure gradient or heart action.

2.3 Magnetic Field

In this study, we consider the Maxwell's equation and generalized Ohm's law appropriate to describes the flow problem with the effect of magnetic field as reported in many studies such as (Eldesoky, 2012; Rossow, 1958; Verma & Parihar, 2009).

$$\nabla \cdot B = 0, \qquad \nabla \times B = \mu_m J, \qquad \nabla \times E = -\frac{\partial B}{\partial t}, \tag{3}$$

$$J = \sigma(E + V \times B), \tag{4}$$

where $V = (0, 0, u)$, J, μ_m, E, σ, B are velocity, current density, magnetic permeability, electric field, electric conductivity, and total magnetic field respectively. The total magnetic field B may be written as

$$B = B_0 + b, \tag{5}$$

where B_0 is the applied magnetic field due to external current, and b is the induced magnetic field due to induced current in the fluid (see Rossow 1958). As reported in rossow (1958), $b \ll B_0$ hence b can be neglected comparing to B_0, which lead to

$$B \approx B_0. \tag{6}$$

For small magnetic Reynold number, the linearized magnetohydrodynamic force $J \times B$ can be expressed as

$$J \times B = \sigma(E + V \times B_0) \times B_0 = -\sigma B_0^2 u. \tag{7}$$

2.4 Constitutive Equation for a Fourth Grade non-Newtonian Fluid

Constitutive equation for a fourth grade fluid model is a relation between the stress and the local properties of a fluid, which is used to describe the rheological characteristics or behavior of the fluid. It is one of the popular subclass of differential type non-Newtonian fluid model. The shear stress tensor for an axisymmetric, incompressible non-Newtonian fourth grade fluid as reported in Abdulhameed, (2014) was given in form of

$$\tau_{rz} = -pI + \sum_{i=1}^{4} S_i, \tag{8}$$

where S_i, are the other stress tensors defined by

$$S_1 = \mu A_1, \tag{9}$$

$$S_2 = \alpha_1 A_2 + \alpha_2 A_1^2, \tag{10}$$

$$S_3 = \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (tr A_1^2) A_1. \tag{11}$$

$$\begin{aligned} S_4 &= \gamma_1 A_4 + \gamma_2 (A_3 A_1 + A_1 A_3) + \gamma_3 A_2^2 + \gamma_4 (A_2 A_1^2 + A_1^2 A_2) + \gamma_5 (tr A_2) A_2 \\ &+ \gamma_6 (tr A_2) A_1^2 + [\gamma_7 tr A_3 + \gamma_8 tr (A_2 A_1)] A_1. \end{aligned} \tag{12}$$

where p is the pressure, I is the identity tensor, μ is the dynamic viscosity and $\alpha_i\, (i = 1, 2)$ are the material constants for second grade fluid, $\beta_i\, (i = 1, 2, 3)$ are the material constants for third grade fluid, $\gamma_i\, (i = 1 - 8)$ are the material constants for fourth grade fluid. A_1, A_2, A_3 and A_4 are the Rivlin Ericksen tensors which maybe defined through the following equations (Ellahi & Riaz, 2010)

$$A_1 = (\nabla V) + (\nabla V)^T, \tag{13}$$

$$A_n = \frac{\partial}{\partial t}(A_{n-1}) + A_{n-1}(\nabla V) + (\nabla V)^T A_{n-1}, \qquad n > 1. \tag{14}$$

We assume that the velocity field is unidirectional, the axial velocity of blood is expressed as

$$V(r,t) = (0, 0, u(r,t)). \tag{15}$$

Based on the assumption made in equation (15), we compute the coefficient of equation (8) as follow

$$\tau_{rz} = -pI + \left[\mu + 2\beta_3\left(\frac{\partial u}{\partial r}\right)^2\right]\frac{\partial u}{\partial r} + \gamma_1\frac{\partial^4 u}{\partial r\partial t^3} + 2(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8)\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r\partial t}. \tag{16}$$

2.5 Momentum Equation

The momentum equation for an incompressible, unsteady, laminar, axisymmetric and fully developed fourth grade non-Newtonian blood flow, through a porous blood vessels in the presence of magnetic field, and oscillatory pressure gradient can be expressed as (Akbarzadeh, 2016)

$$\rho\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\{r\tau_{rz}\} - \frac{\mu\phi}{K}u + J \times B. \tag{17}$$

Substituting equations (1), (2), (7) and (16) into equation (17) we get

$$\begin{aligned}
\rho\frac{\partial u}{\partial t} &= P_0 + P_1(\cos\omega_p t) + P_g\cos(\omega_g t + \varphi) + \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\mu\frac{\partial u}{\partial r} + 2\beta_3\left(\frac{\partial u}{\partial r}\right)^3\right)\right] \\
&+ \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\gamma_1\frac{\partial^4 u}{\partial r\partial t^3} + 2(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8)\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r\partial t}\right)\right] \\
&- \frac{\mu\phi}{K}u - \sigma {B_0}^2 u
\end{aligned} \tag{18}$$

Now, by introducing the following dimensionless parameters as follows;

$$\bar{r} = \frac{r}{R} \Rightarrow r = \bar{r}R \qquad \bar{u} = \frac{u}{U_o} \Rightarrow u = \bar{u}U_0 \qquad \bar{t} = \frac{\omega_p t}{2\pi} \Rightarrow t = \frac{2\pi\bar{t}}{\omega_p}, \tag{19}$$

where, U_0 denotes the reference velocity. Substitute the dimensionless quantities given in equation (19) into an equation (18), the non-dimensional form of the momentum governing equation after dropping bars for simplicity, leads to the following relation:

$$\begin{aligned}
\alpha^2\frac{\partial u}{\partial t} &= B_1(1 + e\cos 2\pi t) + B_2\cos(2\pi\omega t + \varphi) + \left[\frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right] \\
&+ \Lambda\left[\frac{1}{r}\left(\frac{\partial u}{\partial r}\right)^3 + 3\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r^2}\right] + \gamma_a\left[\frac{1}{r}\frac{\partial^4 u}{\partial r\partial t^3} + \frac{\partial^5 u}{\partial r^2\partial t^3}\right] \\
&+ \chi_b\left[\frac{1}{r}\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r\partial t} + \left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^3 u}{\partial r^2\partial t} + 2\left(\frac{\partial u}{\partial r}\right)\frac{\partial^2 u}{\partial r^2}\frac{\partial^2 u}{\partial r\partial t}\right] \\
&- (P + M^2)u.
\end{aligned} \tag{20}$$

after naming some of the parameters as follows;
$\alpha^2 = \frac{\rho R^2\omega_p}{2\pi\mu}$ is the Womersley number, $B_1 = \frac{P_0 R^2}{\mu U_0}$ is the pressure gradient parameter, $\Lambda = \frac{2\beta_3 U_0^2}{\mu R^2}$ is the third grade non-Newtonian parameter, $P = \frac{\phi R^2}{K}$ is the porosity parameter, $M^2 = \frac{\sigma B_0^2 R^2}{\mu}$ is the magnetic parameter, $e = \frac{P_1}{P_0}$, and $\omega = \frac{\omega_g}{\omega_p}$ is the frequency ratio, $\gamma_a = \frac{\gamma_a(\omega_p)^3}{\mu(2\pi)^3}$ and $\chi_b = \frac{2\gamma_b U_0^2\omega_p}{2\pi\mu R^2}$ where $\gamma_b = \frac{2(3\gamma_2+\gamma_3+\gamma_4+\gamma_5+3\gamma_7+\gamma_8){U_0}^2 R^2}{2\pi\mu R^2}$
The domain is $0 < r < 1$, and boundary condition in dimensionless form that enable to solve equation (20) is given as follows;

$$\begin{aligned}
(r &= 1), \quad u = 0, \\
(r &= 0), \quad \frac{\partial u}{\partial r} = 0.
\end{aligned} \tag{21}$$

It should be noted that, the Womersley number (α^2) is a dimensionless parameter in a bio-fluid mechanics. It is a dimensionless expression of the pulsatile flow frequency in relation to viscous effect. Since the vessels diameter in the human body differ up to three orders of magnitude, the Womersley number will depend predominantly on diameter (Akbarzadeh, 2016). The Womersley number of human blood flow has been estimated as according to Table 1.

Table 1: Womersley numbers in different human blood vessels (Fung, 1997)

Blood vessel	Diameter(mm)	α^2
Ascending aorta	25	13.2
Descending aorta	23.3	11.5
Abdominal aorta	19.5	8
Femoral artery	12.9	3.5
Arterioles	1.37	0.04
Capillaries	0.48	0.005

3 Algorithm of Modified Homotopy Perturbation Method

Modified homotopy perturbation method (MHPM) is a combine form of the Laplace transform with the homotopy perturbation method HPM. The MHPM help in finding the solutions of non-linear differential equations, integral equations, fractional differential equations without any discretization or restrictive assumptions as reported in (Khan & Wu, 2011). To solve the strongly non linear differential equations, MHPM will be applied. Below, we give a summary of MHPM procedure as outlined in the report in (Abdulhameed, Roslan & Bin Mohamad, 2014). Consider a general nonlinear non-homogenous differential equation of the form

$$A(u(r,t)) = g(r) \qquad r \geq 0, \qquad t \geq 0. \tag{22}$$

Generally speaking, A is a differential operator which can be decomposed into linear parts L, Q and non-linear part N. The operator L is the second order linear differential operator, Q is the linear differential operator of less order than L, and $g(r)$ is a source term. Therefore, equation (22) can be rewritten as follow:

$$L(u(r,t)) + Q(u(r,t)) + N(u(r,t)) = g(r). \tag{23}$$

Based on homotopy perturbation idea as reported in Khan & Wu, (2011), we construct a homotopy as follow:

$$L(u(r,t)) + qQ(u(r,t)) + qN(u(r,t)) = g(r). \tag{24}$$

Considering the linear operator L, Q in (23), the concept of homotopy perturbation method with embedding parameter $q \in [0,1]$ is used to generate a series of expansion for L and Q as reported in Khan & Wu, (2011) as follows:

$$L(u(r,t)) + Q(u(r,t)) = L\left[\sum_{i=0}^{\infty} q^i u_i\right] + Q\left[\sum_{i=0}^{\infty} q^i u_i\right], \text{ where } u_i = u_i(r,t). \tag{25}$$

$$L\left[\sum_{i=0}^{\infty} q^i u_i\right] + Q\left[\sum_{i=0}^{\infty} q^i u_i\right] = L(u_0) + q^1 L(u_1) + \cdots + Q(u_0) + q^1 Q(u_1) + \ldots, \tag{26}$$

for the non linear operator N in equation (23), we generate the He's polynomial H_n Abdulhameed, Roslan & Bin Mohamad, (2014) as follows:

$$N(u(r,t)) = \sum_{n=0}^{\infty} q^n H_n(u). \tag{27}$$

$$\sum_{n=0}^{\infty} q^n H_n(u) = H_0(u) + qH_1(u) + q^2 H_2(u) + \ldots, \tag{28}$$

where the He's polynomials H_n are defined as

$$H_n(u) = \frac{1}{n!}\frac{d^n}{dq^n} N\left[\sum_{i=0}^{\infty} q^i u_i\right]_{q=0}, \quad n = 0,1,2,..., \tag{29}$$

(see Ghorbani, (2009)) for more details. The first few component of He's polynomials, for example are given by

$$\begin{aligned} H_0(u) &= N(u_0), \\ H_1(u) &= \frac{d}{dq} N\left[\sum_{i=0}^{1} q^i u_i\right]_{q=0}, \\ H_2(u) &= \frac{1}{2!}\frac{d^2}{dq^2} N\left[\sum_{i=0}^{2} q^i u_i\right]_{q=0}, \\ H_3(u) &= \frac{1}{3!}\frac{d^3}{dq^3} N\left[\sum_{i=0}^{3} q^i u_i\right]_{q=0}. \end{aligned} \tag{30}$$

Substituting equation (25) and (27) into (24) we have

$$L\left[\sum_{i=0}^{\infty} q^i u_i\right] + Q\left[\sum_{i=0}^{\infty} q^{i+1} u_i\right] + \sum_{i=0}^{\infty} q^{i+1} H_i = g(r). \tag{31}$$

Taking the Laplace transform of both sides of equation (31) we obtained

$$\ell\left\{L\left[\sum_{i=0}^{\infty} q^i u_i\right]\right\} + \ell\left\{Q\left[\sum_{i=0}^{\infty} q^{i+1} u_i\right]\right\} + \ell\left[\sum_{i=0}^{\infty} q^{i+1} H_i\right] = \ell(g(r)). \tag{32}$$

Applying linearity of the Laplace transform to equation (32) gives

$$\sum_{i=0}^{\infty} q^i \ell\left[L(u_i)\right] + \sum_{i=0}^{\infty} q^{i+1} \ell\left[Q(u_i)\right] + \sum_{i=0}^{\infty} q^{i+1} \ell(H_i) = \ell(g(r)). \tag{33}$$

Using equation (33), we introduce a recursive relation as follows

$$\ell\left[L(u_0)\right] = \ell(g(r)), \tag{34}$$

which implies

$$\sum_{i=0}^{\infty} q^i \ell\left[L(u_i)\right] + \sum_{i=0}^{\infty} q^{i+1} \ell\left[Q(u_i)\right] + \sum_{i=0}^{\infty} q^{i+1} \ell(H_i) = 0. \tag{35}$$

From the recursive equation deduced from equation (35) we have zero order (q^0), first order (q^1), second order (q^2), third order (q^3) up to k^{th} order (q^k), as follows

ZERO ORDER

$$q^0 : \ell\{L(u_0)\} = \ell\{g(r)\}. \tag{36}$$

FIRST ORDER

$$q^1 : \ell\{L(u_1)\} + \ell\{Q(u_0)\} + \ell\{H_0\} = 0. \tag{37}$$

SECOND ORDER

$$q^2 : \ell\{L(u_2)\} + \ell\{Q(u_1)\} + \ell\{H_1\} = 0. \tag{38}$$

THIRD ORDER

$$q^3 : \ell\{L(u_3)\} + \ell\{Q(u_2)\} + \ell\{H_2\} = 0. \tag{39}$$

k^{th} ORDER

$$q^k : \ell\{L(u_k)\} + \ell\{Q(u_{k-1})\} + \ell\{H_{k-1}\} = 0. \tag{40}$$

Taking q as a small parameter we assume a power series solutions of (36-40) in the form

$$\ell\left[u(r,t;q)\right] = \sum_{k=0}^{\infty} q^k \ell\left[u_k(r,t)\right]. \tag{41}$$

Taking the inverse Laplace of equation (41), we have

$$u(r,t;q) = \sum_{k=0}^{\infty} q^k u_k(r,t), \tag{42}$$

where $u_k(r,t)$ are unknowns function of r, t. Now letting $q \to 1$, equation (42) yield the approximate solution of $u(r,t)$ in the following form

$$\begin{aligned} u(r,t) &= \sum_{k=0}^{\infty} u_k(r,t), \\ &= u_0(r,t) + u_1(r,t) + u_2(r,t) + u_3(r,t) + \ldots, \end{aligned} \tag{43}$$

higher order term of the series in (43), can be neglected because the magnitude decreases as the order increases as reported in (Abdulhameed, Roslan & Bin Mohamad, (2014); Khan & Wu 2011). We shall adopt the MHPM to solve the governing equation that will be developed to described blood flow problem through a porous artery.

4 Implementation of Modified Homotopy Perturbation Method to Solve the Model Governing Equation

To solve the formulated model governing equation for the velocity of blood in the previous section, we apply the new algorithm formulated in section 3. By applying the Laplace transform with respect to time (t) of equation (20) and (21) we get the following problem

$$\begin{aligned} F(s)+\left[\frac{1}{r}\frac{\partial \bar{u}}{\partial r}+\frac{\partial^2 \bar{u}}{\partial r^2}\right] &= (P+M^2)\bar{u}+\alpha^2 s\bar{u}-\Lambda\left[\frac{1}{r}\left(\frac{\partial \bar{u}}{\partial r}\right)^3+3\left(\frac{\partial \bar{u}}{\partial r}\right)^2\frac{\partial^2 \bar{u}}{\partial r^2}\right] \\ &- \gamma_a s^3\left[\frac{1}{r}\frac{\partial \bar{u}}{\partial r}+\frac{\partial^2 \bar{u}}{\partial r^2}\right] \\ &- \chi_b s\left[\frac{1}{r}\left(\frac{\partial \bar{u}}{\partial r}\right)^2\frac{\partial \bar{u}}{\partial r}+\left(\frac{\partial \bar{u}}{\partial r}\right)^2\frac{\partial^2 \bar{u}}{\partial r^2}+2\left(\frac{\partial \bar{u}}{\partial r}\right)\frac{\partial^2 \bar{u}}{\partial r^2}\frac{\partial \bar{u}}{\partial r}\right] \quad (44) \end{aligned}$$

Subject to boundary condition as follows;

$$\begin{aligned} (r &= 1), \quad \bar{u}=0, \\ (r &= 0), \quad \frac{\partial \bar{u}}{\partial r}=0. \quad (45) \end{aligned}$$

where $\bar{u}(r,s)=\int_0^\infty u(r,t)e^{-st}\mathrm{d}t$ is the Laplace transform of the function $u(r,t)$ and $F(s)=\int_0^\infty f(t)e^{-st}\mathrm{d}t$ is the Laplace transform of the function $f(t)$.
where $f(t)=B_1(1+e\cos 2\pi t)+B_2\cos(2\pi\omega t+\varphi)$
Substituting the recursive equation (32) into equation (44), leads to the following equation

$$\begin{aligned} F(s)+\sum_{n=0}^{\infty}q^n\left[\frac{1}{r}\frac{\partial \bar{u_n}}{\partial r}+\frac{\partial^2 \bar{u}_n}{\partial r^2}\right] &= \sum_{n=0}^{\infty}q^{n+1}\left\{(P+M^2)\bar{u}_n+\alpha^2 s\bar{u}_n-\Lambda\left[\frac{1}{r}\left(\frac{\partial \bar{u}_n}{\partial r}\right)^3+3\left(\frac{\partial \bar{u}_n}{\partial r}\right)^2\frac{\partial^2 \bar{u}_n}{\partial r^2}\right]\right\} \\ &- \sum_{n=0}^{\infty}q^{n+1}\gamma_a s^3\left[\frac{1}{r}\frac{\partial \bar{u}_n}{\partial r}+\frac{\partial^2 \bar{u}_n}{\partial r^2}\right] \\ &- \sum_{n=0}^{\infty}q^{n+1}\chi_b s\left[\frac{1}{r}\left(\frac{\partial \bar{u}_n}{\partial r}\right)^2\frac{\partial \bar{u}_n}{\partial r}+\left(\frac{\partial \bar{u}_n}{\partial r}\right)^2\frac{\partial^2 \bar{u}_n}{\partial r^2}+2\left(\frac{\partial \bar{u}_n}{\partial r}\right)\frac{\partial^2 \bar{u}_n}{\partial r^2}\frac{\partial \bar{u}_n}{\partial r}\right] \quad (46) \end{aligned}$$

we have zero order, first order of the differential equation from equation 46) as follows

ZERO ORDER

$$q^0 : F(s)+\left[\frac{1}{r}\frac{\partial \bar{u}_0}{\partial r}+\frac{\partial^2 \bar{u}_0}{\partial r^2}\right]=0. \quad (47)$$

$$\left[\frac{1}{r}\frac{\partial \bar{u}_0}{\partial r}+\frac{\partial^2 \bar{u}_0}{\partial r^2}\right]=-F(s). \quad (48)$$

Subject to boundary conditions as follows:

$$\begin{aligned} (r &= 1), \quad \bar{u_0}=0, \\ (r &= 0), \quad \frac{\partial \bar{u_0}}{\partial r}=0. \quad (49) \end{aligned}$$

FIRST ORDER

$$\begin{aligned} q^1 : \left[\frac{1}{r}\frac{\partial \bar{u}_1}{\partial r}+\frac{\partial^2 \bar{u}_1}{\partial r^2}\right] &= (P+M^2)\bar{u}_0+s(\alpha^2)\bar{u}_0-\Lambda\left[\frac{1}{r}\left(\frac{\partial \bar{u}_0}{\partial r}\right)^3+3\left(\frac{\partial \bar{u}_0}{\partial r}\right)^2\frac{\partial^2 \bar{u}_0}{\partial r^2}\right] \\ &- \gamma_a s^3\left[\frac{1}{r}\frac{\partial \bar{u}_0}{\partial r}+\frac{\partial^2 \bar{u}_0}{\partial r^2}\right] \\ &- \chi_b s\left[\frac{1}{r}\left(\frac{\partial \bar{u}_0}{\partial r}\right)^2\frac{\partial^2 \bar{u}_0}{\partial r^2}+\left(\frac{\partial \bar{u}_0}{\partial r}\right)^2\frac{\partial^2 \bar{u}_0}{\partial r^2}+2\left(\frac{\partial \bar{u}_0}{\partial r}\right)\frac{\partial^2 \bar{u}_0}{\partial r^2}\frac{\partial \bar{u}_0}{\partial r}\right] \quad (50) \end{aligned}$$

Subject to boundary conditions as follows:

$$\begin{aligned} (r &= 1), \quad \bar{u_1}=0, \\ (r &= 0), \quad \frac{\partial \bar{u_1}}{\partial r}=0. \quad (51) \end{aligned}$$

The solutions of the recursive equations (48) and (50) is shown only for two terms that is, zero (u_0) and first (u_1) order, subject to their boundary conditions which can be written as

$$\bar{u}(r,s) = \bar{u}_0 + \bar{u}_1. \tag{52}$$

Using the MATHEMATICA Software, we solved (52) in the form

$$u(r,t) = u_0 + u_1, \tag{53}$$

by taking the inverse Laplace transform. The solutions for u_0 and u_1 are truncated after u_1 because the higher order terms as smaller in line with the report in (Abdulhameed, Roslan & Bin Mohamad, 2014).

5 Graphical Results and Discussion

In this section, the unsteady pulsatile laminar flow of an incompressible fourth grade non-Newtonian blood flow through a porous blood vessels or artery with the application of magnetic field to control the movement of blood flow during surgery and imposed periodic body acceleration are discussed. To identify the impact of different flow parameters on blood flow velocity, the approximate analytical result obtained for dimensionless velocity $u(r,t)$ was simulated for different values of the model governing equation parameters. These values are consistent with actual clinical scenarios and have been extracted from a various studies available in literature. The results were portrayed graphically as can be seen on figure $1-7$. Fig 1 depict the effect of magnetic field on blood flow characteristic velocity (u) against radial distance (r) for small value of time at ($t=0.3$) respectively. Figure 2 also showcase the effect of Womerseley parameter (α^2) on blood flow velocity for small value of time ($t=0.2$) respectively. Fig 3 exhibit the effect of pressure gradient (B_1) on blood flow velocity against the radial distance. Fig 4 depict the effect of body acceleration on (B_2) on blood flow velocity against radius respectively. Fig 5 identify the impact of porous medium P on blood flow velocity versus radius for different values of P at time ($t=0.2$) respectively. Fig 6 depict the effect of fourth grade non-newtonian fluid parameter (γ_a) on blood flow specification, velocity (u) against radius (r) for small value of time i.e at ($t=0.2$) respectively. Fig 7 exhibit the effect of third grade non-Newtonian fluid parameter (Λ) on blood flow velocity $u(r,t)$ against the radial distance r for different values of Λ at time ($t=0.2$) respectively.

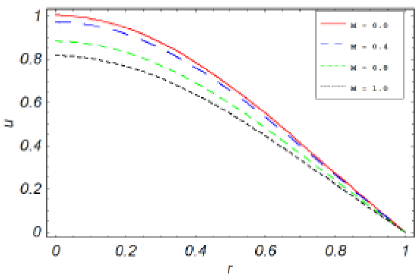

Figure 1: Effect of the magnetic field M on blood flow specification velocity u against radius r for different values of M at $t=0.2$

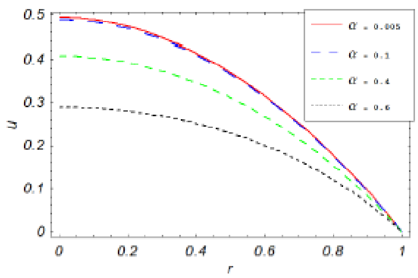

Figure 2: Effect of the Womerseley parameter α^2 on blood flow specification velocity u against radius r for different values of α^2 at $t=0.2$

Figure 1 was used to see the impact of magnetic field M^2 on the flow characteristic velocity u against the radial distance r for different values of magnetic field M^2 and fixing some other parameters with $\alpha^2 = 1.37$, $B_2 = 1.44$, $B_1 = 1.41$, $\Lambda = 0.1$, $\gamma_a = 0.3$, $e = 0.2$, $P = 0.1$, $\chi_b = 2.0$, $\varphi = 0.0$, $r = 0.1$ $\omega = 0.01$ respectively. It can be observe from figure 1 that the axial velocity of the blood is decreases as the magnetic field parameter increases. This happened due to Lorentz force applied to the blood, which lead to decelerate the flow or movement of blood in human arterial system. Therefore, injecting

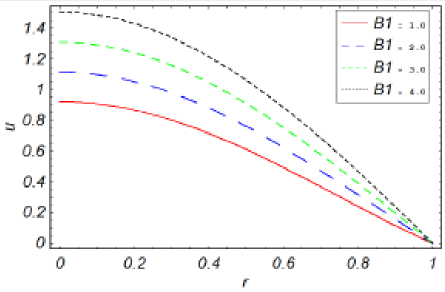

Figure 3: Effect of the pressure gradient parameter B_1 on blood flow specification velocity u against radius r for different values of B_1 at $t = 0.2$

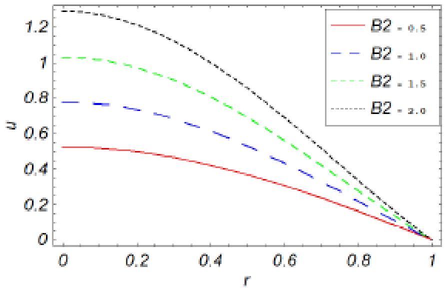

Figure 4: Effect of the Body acceleration parameter B_2 on blood flow specification velocity u against radius r for different values of B_2 at $t = 0.2$

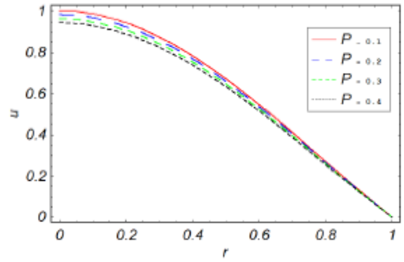

Figure 5: Effect of the Porous medium parameter P on blood flow specification velocity u against radius r for different values of M at $t = 0.2$

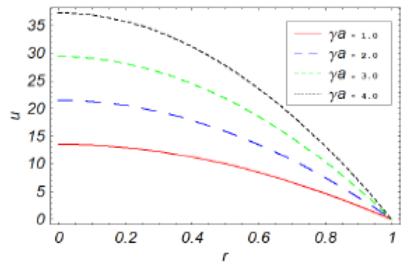

Figure 6: Effect of the fourth grade non-Newtonian fluid parameter γ_a on blood flow specification velocity u against radius r for different values of γ_a at $t = 0.2$

an appropriate amount of magnetic field reduce the flow of blood and thereby it is useful in treatment of certain cardiovascular diseases such as atherosclerosis or stenosis and also, its application is effective during the human surgery.
Figure 2 was used to see the impact of variation of Womerseley parameter α^2 on blood flow, by varying the Womerseley parameter α^2 and fixing some other parameters as follows $M^2 = 0.1$, $\chi_b = 2.0$, $B_2 = 1.44$, $B_1 = 1.41$, $\Lambda = 0.1$, $e = 0.2$, $P = 0.1$, $\varphi = 0.0$, $r = 0.1$, $\gamma_a = 0.3$ respectively. The results displayed in figure 2 shown that the axial velocity of blood decreases with an increase in the Womerseley parameter.
Figure 3 depict the effect of pressure gradient B_1 on the flow velocity u against the radial distance r

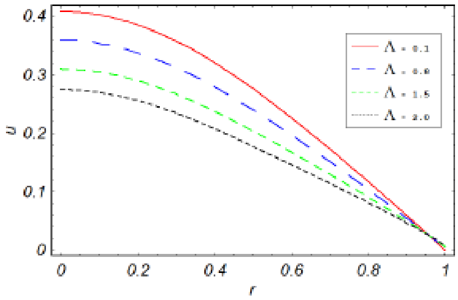

Figure 7: Effect of the third grade non-Newtonian fluid parameter Λ on blood flow specification velocity u against radius r for different values of Λ at $t = 0.2$

by taking the different values of pressure gradient B_1 as (1.0, 2.0, 3.0, 4.0) respectively, and fixing some other parameters as constant. As indicated or shown on figure 3, an increase in the pressure gradient B_1 will definitely causes an increase in the velocity profile.
Figure 4 portray the variation of body acceleration on the flow specification u against the radial distance r when the other governing parameters are held constant with $B_1 = 1.41$, $\gamma_a = 0.3$, $\Lambda = 0.1$, $\alpha^2 = 1.005$, $e = 0.2$, $M^2 = 0.1$, $P = 0.1$, $\omega = 0.2$, $\chi_b = 2.0$, $\varphi = 0.0$, $r = 0.2$ respectively. Here B_2 is chosen as 0.5, 1.0, 1.5, 2.0 respectively. The results from figure 4 shows that by increasing the value of the body acceleration parameter B_2 on the blood flow, the axial velocity of the blood is increases which is commensurate with the study of Akbarzadeh (2016).
Figure 5 exhibit the effect of porosity parameter or porous medium parameter P on the velocity profile u against the radial distance r when the other governing parameters are held constant. Here the value of P was varied as 0.2, 0.4, 0.6 and 0.8 respectively. The results presented in figure 5 depicted that by increasing the values of porosity P, the velocity profile of the fluid decreases. This result is based on constant or specified pressure gradient, when the porosity increases, fluid can easily moves through the porous media and consequently the velocity profile of the fluid increases as reported exactly in the work of Jamil, Roslan, abdulhameed & Hashim, (2018)
Figure 6 showcase the effect of fourth grade non-Newtonian fluid parameter γ_a on velocity profile u against the radial distance r by taking the different values of fourth grade non-Newtonian parameter γ_a as (1.0, 2.0, 3.0 and 4.0). Here, we fixed some other parameters to be $B_2 = 1.44$, $B_1 = 1.44$, $\alpha^2 = 1.005$, $\gamma_a = 0.3$, $\Lambda = 0.1$, $e = 0.2$, $M^2 = 0.1$, $\chi_b = 0$, $P = 0.1$, $\omega = 2.0$, $\varphi = 0.0$ and $r = 0.2$ respectively. It was clearly observed from figure 6 that increases the value of fourth grade non-Newtonian fluid parameter would lead to an increase in the velocity of the blood. This study was correspond to the work of jamil (2019).
Figure 7 portrayed the impact of variation of third grade non-Newtonian fluid parameter Λ on blood flow velocity u against the radial distance r, by varying a third grade non-Newtonian fluid parameter Λ and fixing some other parameters as follows $\alpha^2 = 1.005$, $\gamma_a = 0.3$, $B_2 = 1.44$, $\chi_b = 2.0$, $B_1 = 1.41$, $e = 0.2$, $M^2 = 0.1$, $P = 0.1$, $\omega = 2.0$, $\varphi = 0.0$, $r = 0.2$ respectively. It was explained clearly from figure 7 that when the non-Newtonian third grade fluid parameter Λ increases, the velocity profile decreases slightly. This flow behavior was also observed in the work of Akbarzadeh (2016).

6 Conclusion

In conclusion we find that;

(i) The velocity profile of the fluid is decreases as increases the magnetic field parameter. This happened due Lorenz force applied to the blood, which lead to decelerate the flow of blood in human arterial system.

(ii) The velocity of the fluid decreases as the porosity parameter (P) increases. This was happened because of a constant pressure gradient, when porosity decreases, fluid can easily moves through the porous media and consequently velocity profile increases.

(iii) The axial velocity of the blood increases by increasing the body acceleration parameter (B_2).

(iv) Increasing the pressure gradient (B_1) causes an increase in the velocity profile.

(v) The axial velocity of the blood decreases with increase in the Womersley parameter (α^2).

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Conflict of interest
The author declared that, there is no conflict in this research.

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