International Journal of Engineering Inventions
e-ISSN: 2278-7461, p-ISSN: 2319-6491
Volume 13, Issue 10 [October. 2024] PP: 116-127

Research on quality control and cost optimization in production process based on dynamic programming model

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ABSTRACT: *This paper focuses on the quality control of spare parts in modern manufacturing industry, mainly studies the quality management and cost optimization problems in the production process, and proposes a model solution based on dynamic programming, covering sampling inspection, defective rate evaluation, multi-stage decision-making and finished product disassembly, etc. aiming to help enterprises optimize decision-making, ensure product quality, reduce total cost and enhance market competitiveness.*

In response to the first problem, this paper designs a sampling inspection plan, establishes a hypothesis, and uses a statistical model to describe the defective rate problem using binomial distribution, and uses binomial distribution and normal distribution approximate models to simplify the calculation when the sample size is large. By calculating the sample size at 95% and 90% confidence levels, the sample size is 866 and 609 respectively, ensuring efficient detection of the defective rate of spare parts, and formulating reasonable acceptance or rejection decision criteria. The optimal sample size plan at different confidence levels is obtained to help companies improve decision-making accuracy while reducing the number of inspections.

For the second problem, this paper establishes a dynamic programming model to optimize the decision-making in the production process. The model comprehensively considers the defective rate of spare parts and finished products. Enterprises need to decide whether to conduct inspections and how to deal with unqualified products. Through the dynamic programming model, combined with cost-benefit analysis, enterprises can find a balance between the cost of inspection and the possible losses caused by defective products entering the market, to make the best production decision and minimize the cost of replacement and disassembly.

In response to question 3, this paper further expands the production scenarios of multiple processes and multiple parts and components, expands the complexity of the production process, and constructs a more complex dynamic programming model to optimize the inspection decisions of each process and parts and components. Under the premise of considering the mutual influence between processes, the model provides an optimization plan for the inspection and disassembly decisions at each stage, ensuring that the defective rate is minimized in the multi-process production chain and optimizing the decision points of the entire production process.

For question 4, sampling inspection and Bayesian inference methods can reduce uncertainty and form a more accurate estimate of the defective rate, which in turn affects production decisions. The core formula of Bayesian inference involves posterior probability, likelihood function, and prior probability. In the specific calculation, if the prior distribution follows the binomial distribution, the posterior distribution will follow the Beta distribution. The updated defective rate estimate will guide enterprises to re-formulate production, inspection, and disassembly strategies and optimize the inspection cost calculation.

Date of Submission: 14-10-2024 Date of acceptance: 29-10-2024

I. Restatement of the problem

1.1 Background of the Problem

In modern manufacturing, product quality is one of the key factors in a company's competitiveness. For companies that produce electronic products, the quality of spare parts directly affects the performance and reliability of the final product. Therefore, companies need to be very cautious when purchasing spare parts to ensure that the purchased spare parts meet quality standards. In addition, companies need to decide whether to test spare parts and how to treat assembled finished products and unqualified finished products. These decisions need to be based on the defective rate of spare parts and finished products, as well as the cost of testing, disassembly, and replacement. The company's goal is to minimize total costs while ensuring that product quality meets market requirements.

In short, this topic describes a complex quality management problem, which involves sampling, cost analysis, and decision making. Enterprises need to consider various factors and develop a quality control strategy that is both economical and effective. By establishing a mathematical model, enterprises can make decisions more scientifically, thereby improving product quality, reducing costs, and enhancing market competitiveness [1].

1.2 Problem Statement

Question 1: Design a sampling inspection plan to help companies decide whether to accept spare parts from suppliers with the least number of inspections possible. Under the condition that the nominal defective rate is 10%, determine whether the defective rate exceeds or falls below this standard at a confidence level of 95% and 90% respectively.

Problem 2: Based on known defective rates of parts and finished products, make decisions for each stage of the production process. This includes whether to test parts, how to handle the test results of finished products, the disassembly strategy for unqualified finished products, and how to deal with unqualified products returned by customers.

Question 3: Question 2 is expanded to consider the situation of multiple processes and multiple parts. It is necessary to formulate a decision plan for this complex situation and provide a basis for decision-making and relevant indicators. It is necessary to consider the situation of 2 processes and 8 parts and provide a basis for decision-making and relevant indicators.

Question 4: Considering that the defective rate data in Questions 2 and 3 may come from sampling inspection, re-evaluate, and formulate decision-making plans for Questions 2 and 3.

II. Problem Analysis

2.1 Regarding Question 1

Companies are faced with the challenge of designing a sampling inspection program to assess whether the defective rate of spare parts provided by suppliers exceeds a predetermined standard. To effectively control the inspection cost, companies cannot conduct comprehensive inspections on all spare parts. Instead, they should use sampling inspections to infer the defective situation of the entire batch with a smaller sample size. The key is to develop a reasonable sampling plan to ensure that a high-confidence estimate of the defective rate can be obtained with a smaller sample size. By rejecting batches that exceed the nominal defective rate (for example, 10%) at a 95% confidence level, companies can effectively prevent unqualified spare parts from entering the production process, thereby improving the quality of the final product [2].

2.2 Regarding Question 2

It is required to make the best decisions at different stages of the production process, including deciding whether to test spare parts and finished products, and how to deal with unqualified products. The defective rate of spare parts directly affects the quality of finished products, so the testing decision will directly affect the qualified rate of the final product. Testing can help companies screen out defective products and reduce subsequent replacement losses, but this will increase the cost of testing. On the contrary, although choosing not to test reduces costs, it may cause defective products to flow into the market, causing returns or damaging brand reputation. How to find a balance between the two is a key issue that needs to be optimized in the production process. In addition, the handling strategy of unqualified finished products (such as disassembly, disposal or recycling and reuse) will also affect the total cost. It is necessary to comprehensively consider the value of the finished product and the disassembly cost to optimize the decision of the entire production link.

2.3 Regarding Question 3

The complexity of the production process is increased, involving multiple parts and processes. Each process may produce defective products, and the combination of different parts and components will also affect the quality of the finished product. This requires companies to make comprehensive optimization decisions for each process and parts to minimize the defective rate throughout the production chain. Companies not only need to decide whether to conduct inspections in each process, but also need to consider the mutual influence between processes and optimize decision points in the production process. In the case of multiple processes, the quality of spare parts affects not only individual parts, but also the subsequent assembly links and the quality of semi-finished products. Therefore, Problem 3 requires a mathematical model to calculate the optimal production plan under multi-process conditions to balance cost and quality.

2.4 Regarding Question 4

Based on questions 2 and 3, the defective rate obtained through sampling inspection is considered, which means that the defective rate is an estimate with a confidence interval, thus introducing uncertainty. This uncertainty requires that when making decisions, enterprises must consider the risks brought by the error in the estimation of the defective rate in addition to the testing cost and the quality of the finished product. For example, if the sampling test results show that the defective rate is high, the enterprise may take more stringent testing measures; if the test results show that the defective rate is low, the enterprise may reduce testing to reduce costs. Decisions based on sampling inspections require dynamic adjustments to strategies in the production process. Enterprises must comprehensively consider the relationship between the uncertainty of the test results and the testing costs to achieve the optimal production decision.

III. Model Assumptions

Assume that the number of finished products produced in Problem 2 is 10,000 pieces.
Assume that all users who purchased substandard products apply for replacement.
If the testing cost does not exceed expectations, the finished product testing coverage, and the utilization rate of spare parts after disassembly remain unchanged.
Parts and processes are independent of each other, making them easy to analyze separately without considering interaction effects [3].
The defective rate and cost factors are fixed and not affected by external factors.
Operations are carried out according to the optimal process, and the equipment is always in good condition to ensure continuous and stable production.
No operational errors, consistent employee skills and efficiency, and reduced production errors.
The production environment is constant, eliminating the impact of environmental factors on product quality.
The supply chain is timely and sufficient, and inventory management is optimal to avoid production interruptions and inventory problems.
Accurately forecast market demand and match production plans with demand to ensure product competitiveness.

IV. Explanation of symbols

symbol	illustrate
n	Refers to the sample size of the sampling
X	Indicates the number of defective items found in the sampling
p	Representative sample defective rate
p^	Estimate of the defective rate in the sample
$Z_{\frac{\alpha}{2}}$	Critical value under standard normal distribution
δ	Allowable error, that is, the difference between the expected inspection defective rate and the actual defective rate
P	Nominal defective rate 10%
N	Number of parts to be tested
C_0	Testing cost per component
p_1	Assembly defective rate
m	Represents the number of finished products that need to be tested
C_1	Represents the inspection cost per finished product
C_s	Represents the disassembly cost of each finished product
Z_i	Procurement cost of part i
C_i	Inspection cost of part i

Z_j	Cost of semi-finished product j
C_j	Testing cost of semi-finished product j
P_i	Defective rate of part i
p_j ^	Corrected defective rate of semi-finished product j

V. Model establishment and solution

5.1 Model building and solution of problem 1

5.1.1 Model Construction

1. Scenario 1 involves a rejection decision, while Scenario 2 involves an acceptance decision. For these two situations, we need to design different inspection criteria based on different confidence requirements to decide whether to accept or reject spare parts. In the process of building a hypothesis testing framework, we can define the following assumptions:

Null hypothesis H 0:

Assume that the defective rate p of spare parts does not exceed the upper limit declared by the supplier, that is, p≤0.1.

Alternative hypothesis H 1:

Assume that the defective rate p of spare parts exceeds the upper limit declared by the supplier, that is, p>0.1.

2. When dealing with the problem of defective rate detection, we usually use the binomial distribution model to describe the detection results, because the appearance of defective products is a typical binomial event:

X B (n, p)

Where X: represents the number of defective products found in the sampling.

n: refers to the sample size of the sampling.

p: represents the sample defective rate.

For larger sample sizes n, the binomial distribution can be approximated by a normal distribution [4] , because according to the central limit theorem, the sum of the binomial distribution approaches a normal distribution when the sample size is large enough, where p^ is an estimate of the sample defect rate. This approximation helps to simplify the calculation process, especially when the sample size is large.

p^ ~ N (p,)

where p^ is the estimate of the defective rate in the sample.

3. To make accurate judgments at the 95% and 90% confidence levels, we must calculate the required sample size respectively.

According to the calculation formula of confidence interval:

is the critical value under the standard normal distribution, for a 95% confidence level, ≈ 1.96,

For a 90% confidence level, ≈ 1.645.

P is the nominal defective rate 10%

δ is the allowable error, which is the difference between the expected inspection defective rate and the actual defective rate.

When determining the sampling size, we first need to define an acceptable error range δ, which should be set based on the specific needs of the enterprise.

4. Scenario Analysis

Scenario 1: Rejection decision at 95% confidence level

In this scenario, the company's goal is to ensure that spare parts can be rejected if the defective rate exceeds 10%. To do this, we will define the rejection criteria through hypothesis testing, that is, when the defective rate observed from the sample is significantly higher than the nominal value, choose to reject.

Scenario 2: Acceptance decision at 90% confidence level

In this scenario, the company hopes to receive spare parts when the defective rate remains below 10%. This requires us to confirm through inspection whether the defective rate in the sample is maintained within an acceptable range, to decide whether to accept this batch of spare parts.

5.1.2 Model Solution

Assuming that our allowed error E is 0.02 and the sample defect rate p = 0.1, we can calculate the sample size.

Sample size at 95% confidence level:

After rounding, we get n = 866.

Sample size at 90% confidence level:

After rounding, we get n = 609.

5.1.3 Model Results

Decision-making plan

Through the above analysis, we can more accurately calculate the required sample size to meet the decision-making needs of enterprises at different confidence levels. The decision-making solutions for the two situations in question 1 are as follows:

At a 95% confidence level, if the defective rate among the 866 sampled spare parts is found to be significantly higher than 10%, the company should choose to reject the entire batch of spare parts.

At a 90% confidence level, if the defective rate of the 609 sampled spare parts does not exceed 10%, the company should consider accepting this batch of spare parts.

Specific results

Following the above steps, we can develop an efficient sampling inspection plan for enterprises, allowing them to accurately judge the defective rate of spare parts at 95% and 90% confidence levels. Such a plan ensures the accuracy of inspection while effectively controlling the inspection cost.

The specific sample sizes are as follows:

At the 95% confidence level, the sample size is 865.

At a 90% confidence level, the sample size is 609.

At both confidence levels, if the decision is to reject the null hypothesis, this means that the defective rate is considered to exceed the nominal value declared by the supplier. To further reduce the number of necessary tests, the sequential probability ratio test (SPRT) can be used, which is a method that dynamically decides whether to continue sampling based on the results of each test until a decision to accept or reject the null hypothesis can be made. In SPRT, if two hypothetical values of the defective rate p 0 and p 1 are set, the decision rule of the test can be defined as:

accepting the null hypothesis H 0:

rejecting the null hypothesis H 0:

Conditions for continued sampling:

Here V and U represent the error probability ratio associated with the significance level and test power, respectively.

By using SPRT, the number of tests can be dynamically adjusted to minimize the required sample size while ensuring that the established decision criteria are met.

5.2 Model establishment and solution of problem 2

5.2.1 Model construction and solution

To achieve the goal of minimizing costs and maximizing benefits in the production process, companies need to formulate strategies at each key link. This includes the procurement and testing of spare parts, the assembly and quality inspection of finished products, and the disassembly and reuse of unqualified finished products. We need to establish an effective dynamic programming model to fully consider the impact of these decisions on costs and benefits. In the process of simplifying and clarifying the dynamic programming model, we should consider how to divide the problem into the fewest stages while ensuring that all key decision points are covered and that costs and benefits are easy to calculate [5]. Based on this principle, we divide the production process into the following three main stages:

Phase 1: Parts procurement and testing decisions

Phase 2: Finished product assembly and inspection decisions

Phase 3: Dismantling of unqualified finished products and market circulation decisions

Through this division, we can more clearly analyze and optimize decisions in the production process to maximize cost-effectiveness.

In the production process, for each decision point (such as whether to test spare parts or finished products, whether to disassemble unqualified finished products, etc.), the company will face corresponding costs, which will affect the final profit or loss. To optimize the decision-making of these links, the cost-benefit analysis model can be applied. The main goal of this model is to seek cost minimization, and its calculation formula is as follows:

M total cost = M inspection + M assembly + M disassembly + M replacement N sales in:

The cost required to test spare parts or finished products;

Cost of assembling the finished product;

Profits from sales of qualified finished products.

Phase 1: Parts procurement and testing decisions

In the production management process of an enterprise, the quality inspection of spare parts is a crucial link. Enterprises need to carefully decide whether to conduct quality inspection on spare parts. If inspection is implemented, unqualified spare parts will be eliminated to ensure that all spare parts entering the assembly process meet the quality standards. On the contrary, if inspection is not carried out, there is a risk that unqualified spare parts will directly enter the assembly process, thereby affecting the overall quality of the finished product.

Assuming the defective rate is p 0 and the assembly defective rate is p 1, the expected loss without inspection can be calculated by the following formula:

E loss = N p 0 M assembly loss + n p 0 p 1 M replacement loss in,

M assembly loss represents the loss caused by defective spare parts entering the assembly process;

M exchange loss refers to the exchange loss caused by customers returning unqualified finished products.

By comparing the testing costs with E losses, companies can develop appropriate testing decision criteria.

If the detection cost is lower than the expected loss, that is: < E Loss

The enterprise should choose to conduct testing to ensure the quality of spare parts and avoid greater losses caused by defective products entering the assembly process. On the contrary, if the testing cost is higher than the expected loss, the enterprise will choose not to conduct testing to save testing costs, but will bear the potential loss risk caused by defective products entering the assembly process.

Phase 2: Finished product assembly and inspection decisions.

The core purpose of finished product testing is to screen out unqualified products to prevent them from entering the market, thereby avoiding economic losses and damage to corporate reputation caused by returns and exchanges.

The detection cost can be calculated by the following formula:

M finished product inspection = m × C_1

Among them, m represents the number of finished products that need to be tested;

C1 represents inspection cost of each finished product.

However, if the company chooses not to conduct finished product testing, unqualified finished products may flow into the market, leading to customer returns and exchanges, which in turn damages the company's reputation. This potential loss can be calculated using the following formula:

M exchange loss = m × p finished product defect rate × M exchange

Among them, p finished product defect rate represents the defective rate of finished products;

Exchange represents the exchange loss for each defective finished product.

To make the decision whether to conduct finished product testing, companies can use the following guidelines:

If M finished product inspection < M exchange loss, then choose to test;

If M finished product inspection > M exchange loss, choose not to conduct inspection.

In this way, companies can make informed decisions based on a comparison of costs and potential losses, ensuring product quality while minimizing economic losses.

Phase 3: Dismantling of unqualified finished products and market transfer decisions

When faced with unqualified finished products, companies need to weigh two strategies: directly discard them or dismantle them to recover qualified parts. The dismantling process will incur certain costs, and the cost calculation formula is as follows:

M dismantling = n unqualified × Cs

Where Unqualified represents the number of unqualified finished products;

Cs represents the disassembly cost of each finished product.

If the disassembled parts can be reused, it will bring certain benefits. The calculation formula of expected benefits is:

M exchange loss = m × p finished product defect rate × M exchange

Among them, p qualified parts represent the probability of obtaining qualified spare parts after disassembly;

M Parts Recovery is the value of each recycled spare part;

Assembly is the cost of assembling the new finished product.

If the disassembly cost is lower than the disassembly benefit, the enterprise should choose disassembly. To comprehensively evaluate the decisions at each stage of the entire production process, a total cost expectation model can be constructed:

By calculating the expected total cost, companies can determine whether to conduct inspection or disassembly at each stage, thereby optimizing the cost structure of the entire production process and ensuring the economy and efficiency of the production process.

If the disassembled product still has residual value, the enterprise can calculate it according to the following formula:

E return disassembly = M logistics + M disassembly - M disassembly × E value of spare parts after disassembly

Where Logistics represents the logistics costs incurred during the return process, including transportation, warehousing, and management related costs;

M dismantling refers to the cost of dismantling processing;

E The value of spare parts after disassembly refers to the expected profit from qualified spare parts or materials recovered from the disassembly process.

Using this formula, companies can assess the economic feasibility of dismantling and make informed decisions.

5.2.2 Model Results

The results of the first two stages of solving problem 2 are shown in Figure 1

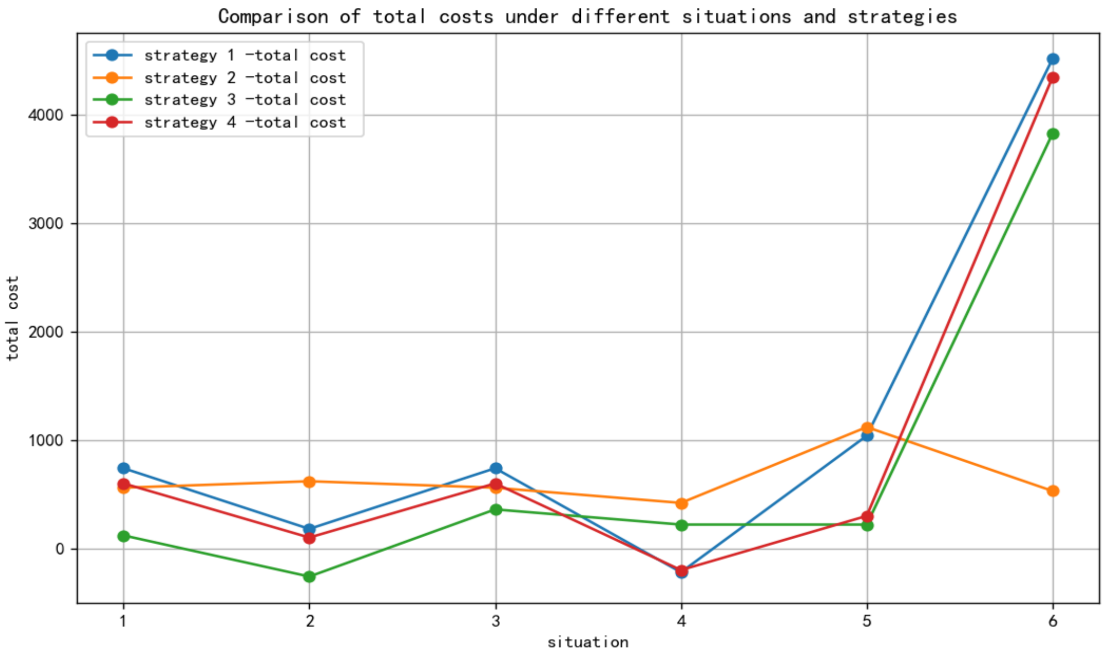

Figure 1 Comparison of total costs of different strategies in the first two stages

Strategy 1: Test component 1, test component 2, test finished products, and disassemble unqualified finished products

Strategy 2: Test component 1, do not test component 2, test finished products, do not disassemble unqualified finished products

Strategy 3: Do not test component 1, do not test component 2, do not test finished products, and disassemble unqualified finished products

Strategy 4: Do not test component 1, test component 2, test finished products, and disassemble unqualified finished products

The results of the last two stages of solving problem 2 are shown in Figure 2

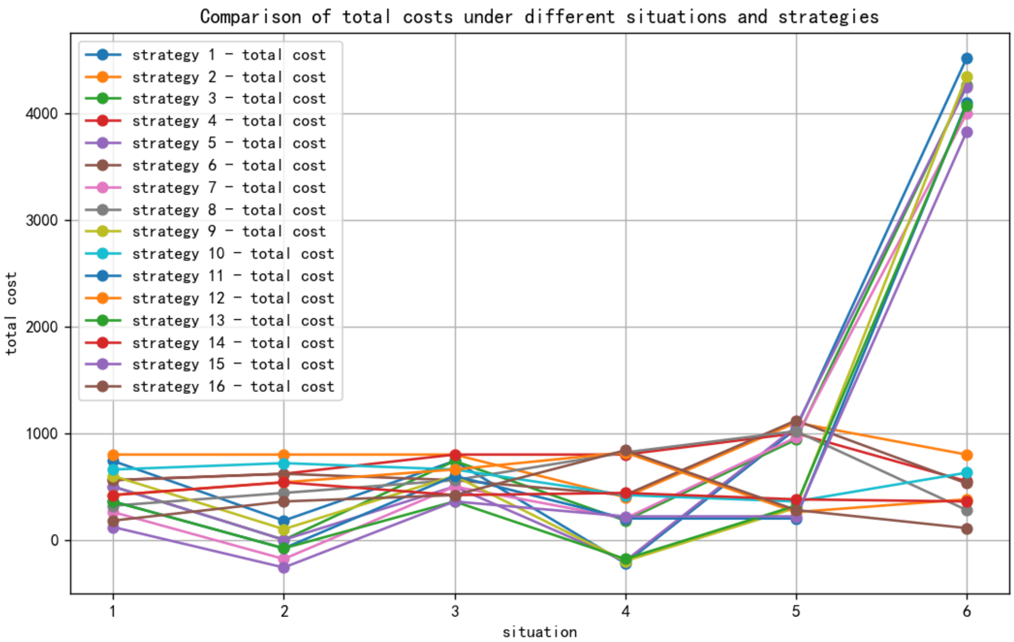

Figure 2 Comparison of the total costs of different strategies in the last two stages

1. First, let's explore a specific situation, an assembly process involving 8 parts and 2 main operations.
Phase 1: The parts are assembled into semi-finished products.
Phase 2: These semi-finished products are further assembled into the final finished product.
2. State definition

Stage 1 state variables: $(X_{11}, X_{12}, \ldots, X_{18})$, where $X_{1i} \in \{0,1\}$ indicates whether part i is detected (0 means no detection, 1 means detection), i=1,2,…,8.

Stage 2 state variables: $(X_{21}, Y_{21}, X_{22}, Y_{22}, X_{23}, Y_{23})$, where: $X_{2j} \in \{0,1\}$: indicates whether to detect semi-finished product j (0 means no detection, 1 means detection). $Y_{2j} \in \{0,1\}$: indicates whether to disassemble semi-finished product j after it is detected as defective (0 means no detection, 1 means detection).

Stage 3 state variables: (X_3, Y_3), where: $X_3 \in \{0,1\}$: indicates whether to inspect the finished product (0 means no inspection, 1 means inspection). $Y_3 \in \{0,1\}$: indicates whether to disassemble the finished product after it is detected as defective (0 means no inspection, 1 means inspection).

3.Value Definition

In phases 2 and 3, the decision to dismantle is a common consideration. Therefore, to evaluate the economic effect of dismantling, the reuse value of semi-finished or finished products after dismantling must be clearly defined. This reuse value mainly involves the following two situations: of a semi-finished product when it is disassembled back into individual parts.

When semi-finished products are disassembled back into parts, the evaluation of their reuse value should include the following key factors: Savings on procurement costs: By reusing disassembled parts, the cost of purchasing new parts can be reduced.

Potential benefits of re-testing and reassembly Considering that disassembled parts may need to be re-tested and reassembled, the potential benefits of this process should be evaluated based on the expected quality of the parts.

$$KP = \sum_{i \in Disassembled\ parts} (Z_i - X_{1i} \cdot C_i) \qquad (1)$$

in:C_i
Z_i: The purchase cost of part i.
C_i: Inspection cost of part i.
X_{1i}: Inspection decision for part i (0 means no inspection, 1 means inspection)
when the finished product is disassembled back to the semi-finished state.

When considering disassembling finished products into semi-finished products, the calculation of their reuse value should be based on the following factors: Saving semi-finished product costs By dismantling and reusing semi-finished products, the manufacturing cost of new semi-finished products can be reduced.

Potential benefits from re-testing and reassembly: The benefits that may be brought by the re-testing and reassembly process of disassembled semi-finished products need to *be considered and adjusted based on* the potential quality of the semi-finished products.

$$K_L = \sum_{j \epsilon Disassembled\ parts} \left(Z_j - X_{2j} \cdot C_j\right) \quad (2)$$

in:
Z_j: Cost of semi-finished product j.
C_j: Inspection cost of semi-finished product j.
P_i is the defective rate of part i.
S_{2j}: Inspection decision for semi-finished product j (0 means no inspection, 1 means inspection.
In addition, considering that the defective rate of semi-finished products and finished products is only related to the current production stage, we need to specifically calculate the cumulative defective rate of stage 2 and stage 3. This can be divided into the following two situations:
1. Semi-finished product defective rate: The defective rate of semi-finished products will be affected by the inspection decision of the previous stage (parts):

$$p_j{}^{\wedge} = p_j + \sum_{i \epsilon Parts\ Group\ j} \left(P_i \cdot \left(1 - X_{1i}\right)\right) \quad (3)$$

in:
$p_j{}^{\wedge}$ is the corrected defective rate of semi-finished product j.
p_j is the defective rate of the semi-finished product at link j.
P_i is the defective rate of part i.
X_{1i} Indicates whether the part is detected (0 means not detected, 1 means detected)
2. Finished product defect rate: The finished product defect rate will be affected by the inspection decision of the previous stage (semi-finished product):

$$p_l{}^{\wedge} = p_l + \sum_{j=1}^{3} \left(p_l{}^{\wedge} \cdot \left(1 - X_{2j}\right)\right) \quad (4)$$

Among them: $p_l{}^{\wedge}$ is the corrected defective rate of the finished product. p_l is the defective rate of the finished product in this link.
X_{2j} : Inspection decision for semi-finished products (0 means no inspection, 1 means inspection).
Phase 1: Finished product inspection and disassembly decision
Based on the above information, the following state transfer equation can be constructed:

$$F(3,X_3,Y_3)=\begin{cases} -X_3 \cdot C_f - p_l{}^{\wedge} \cdot X_3 \cdot Y_3 \cdot (H - E_h) + (1 - p_l{}^{\wedge}) \cdot S, X_3 = 1 \\ -p_l{}^{\wedge} \cdot C_d + (1 - p_l{}^{\wedge}) \cdot S \quad , X_3 = 0 \end{cases} \quad (5)$$

Among them: C_f testing cost of finished products.
H. Cost of dismantling finished product.
$p_l{}^{\wedge}$ Corrected defective rate of finished goods.
E_h The potential benefits of disassembling finished products into semi-finished products.
C_d Loss of replacement of finished products.
S finished product market price.
Phase 2: Semi-finished product inspection and disassembly decision

$$F(2,X_{21},Y_{21},X_{22},Y_{22},X_{23},Y_{23})= -\sum_{j=1}^{3}\left(X_{2j} \cdot C_g + p_j{}^{\wedge} \cdot X_{2j} \cdot Y_{2j} \cdot (C_e - R)\right) + \sum_{j=1}^{3}\left(1 - p_j{}^{\wedge}\right) \cdot F(3,X_3,Y_3) \quad (6)$$

C_g Inspection costs of semi-finished products.
C_e The cost of dismantling semi-finished products.
$p_j{}^{\wedge}$ Corrected defective rate of semi-finished products.
Potential benefits of dismantling semi-finished products into parts

Phase 3: Parts procurement and testing decisions

$$\mathbf{F(1,X11,X12...X18)} = -\sum_{i=1}^{8}(X_{1i}\cdot C_i + Z_i) + F(2,X_{21},Y_{21},X_{22},Y_{22},X_{23},Y_{23}) \quad (7)$$

Where: Z_i the purchase cost of part i.

C_i The inspection cost of part i.

$F(2,X_{21},Y_{21},X_{22},Y_{22},X_{23},Y_{23})$ at the semi-finished product stage.

5.3.2 Model establishment in general

1. In a more extensive model construction, we involve a scenario with n parts and m production processes. In this scenario, each process may involve the processing of semi-finished products. Therefore, the entire production process can be divided into the following main stages: Phase 1: involves the procurement and quality testing of parts.
From the second stage to the m+1 stage: including the processing work of each process and the quality inspection of semi-finished products.
Phase m+2: covers the final product inspection, decision-making and product entry into the market process.
2. Establish a dynamic programming model. We define the state variable $F(i, X_i)$ to represent the optimal cost or benefit when in states at stage i. The goal is to minimize the total cost while considering the disposal strategy for defective products.

Phase 4: Parts procurement and testing phase

For each part, companies need to decide whether to test:

$$\mathbf{F(1,X_{11},X_{12}...X_{1n})} = -\sum_{i=1}^{n}(X_{1i}\cdot(C_i + Z_i) + (1 - X_{1i})\cdot Z_i) + \mathbf{F(2,X_{21},X_{22}...X_{2n})} \quad (8)$$

Where: Z_i the purchase cost of part i.

C_i The inspection cost of part i.

Stage 2 to stage m+1: process processing and semi-finished product inspection stage

After each process, the company decides whether to test the semi-finished product:

$$F(k,Xk1,Xk2...X_{kn}) = -\sum_{i=1}^{n}(X_{ki}\cdot C_{ki}) + F(k+1, X_{K+1,1}...X_{K+1,n}) \quad (9)$$

Where C_{ki}: the cost of testing semi-finished product i after the kth process.

Phase m+2: Final product testing and market circulation decision-making stage

In the final stage, the company decides whether to test the finished product:

$$F(m+2, X_{m+2}) = -X_{m+2}\cdot C_f + (1-p_l)\cdot S - p_l\cdot C_d + \text{other processing costs} \quad (10)$$

Where Cd is the inspection cost of a single part

5.3.3 Model Results

See the supporting materials for specific solutions for special cases.

5.4 Model building and solution of problem 4

5.4.1 Introduction of sampling inspection

When the defect rate is based on sampling inspection, it changes from a definite value to an estimate. Increasing the sample size can improve the accuracy of this estimate. Nevertheless, previous analysis shows that dedicated sampling inspections lead to high inspection costs, which is not economically feasible. In the dynamic programming process, the decision itself includes the inspection of some artifacts (such as parts, semi-finished products or finished products). Therefore, the estimation of the defect rate can be integrated into these inspection activities. In addition, changes in the defect rate in the sampling estimate will also affect the decision-making process, ensuring that different artifacts can be inspected.
By sampling and testing the estimated defective rates of spare parts, semi-finished products, and finished products, we can decide on whether to accept the estimate at a certain confidence level. The uncertainty caused by sampling can be reduced by Bayesian inference methods, which combine sampling results and prior information to form more accurate estimates of defective rates [7]. These updated estimates will have an impact on subsequent production decisions.

The core formula of Bayesian inference is

$$P(\theta|M) = \frac{P(M|\theta)P(\theta)}{P(M)} \quad (11)$$

in:

θ represents the defective rate to be estimated;

M represents the observed data of sampling detection;

$P(\theta|M)$ is the posterior probability, i.e., the probability distribution of the defective rate given the test data;

$P(M|\theta)$ is the likelihood function, which represents the probability of observing data when the defective rate is;

$P(\theta)$is the prior probability, which represents the defective rate distribution before detection.

Specific calculation of Bayesian update

Assume that the defective rate of spare parts follows a binomial distribution, that is, X~B(n, θ), where n is the sample size and $\theta$is the defective rate. If m defective products are detected during the sampling process, the likelihood function is: If the prior distribution θfollows the binomial distribution, that is, θ~ B(δ , ∂) , then the updated posterior distribution still follows the Beta distribution:

θ|M ~ B(δ+m, ∂+nm)

in:

δ and $\partial$are the parameters of the prior binomial distribution; m is the number of defective products detected; n is the sample size.

Updated decision-making process.

The estimated defective rate obtained from sampling inspection will affect the decision-making process in the previous problem, and the company will need to re-formulate production, inspection and disassembly strategies based on these updated defective rates.

Detection decision

In the previous problem, the decision to inspect parts and finished products was based on the defect rate and the cost of inspection. Now, because sampling inspection provides an updated estimate of the defect rate, the company can recalculate the expected cost and inspection strategy based on the posterior distribution.

Updated detection cost calculation:

$$E[M_{detection}] = n \times \theta\text{^} \times M_1 + n \times (\theta\text{^}) \times C \tag{12}$$

in:

θ^ is the defective rate obtained by Bayesian updating

5.4.2 Summary

This problem lies in the uncertainty of the defective rate, which requires us to quantify the range of the defective rate through confidence intervals and make optimized production decisions based on this. Through Bayesian updating, we can combine the test results with the company's previous experience data to obtain a new estimate of the defective rate, thereby guiding production decisions more accurately.

VI. Model Evaluation and Promotion

6.1 Advantages of the model

1. High efficiency, greatly reducing computational complexity. For some complex problems, dynamic programming avoids repeated calculations and greatly reduces the number of calculations by saving solutions to sub-problems.

2. Guarantee the global optimal solution. Dynamic programming can ensure the global optimal solution of the problem by solving sub-problems and gradually merging them. This is because it considers all possible situations in each decision step and chooses the best path. When solving problems with multi-stage decision-making processes, dynamic programming can systematically traverse all possible decision sequences to find the optimal decision.

3. Calculation simplicity, simplifying complex calculations. When the number of trials in the binomial distribution is large, directly calculating the probability of the binomial distribution may be very complicated. Using the normal distribution for simulation, you can use its relatively simple probability density function and cumulative distribution function for calculation, which greatly simplifies the calculation process and improves the calculation efficiency.

6.2 Disadvantages of the Model

1. Ignoring non-monetary factors: Cost-benefit analysis focuses mainly on monetized costs and benefits, and may ignore some important non-monetary factors. For example, some social and ethical factors, cultural values, etc. may not be measurable in monetary terms, but these factors may also be very important in decision-making. Cost-benefit analysis is suitable for some projects or decisions where costs and benefits can be quantified, but for some decisions that cannot be quantified, such as those involving human rights, ethics, etc., cost-benefit analysis may not be applicable.

2. Subjectivity of benefit evaluation: Benefit evaluation in cost-benefit analysis often involves value judgments, and different people may have different evaluations of the same benefit. When considering multiple cost and benefit indicators, it is necessary to determine the weight of each indicator. However, the determination of weights is often subjective, and different decision makers may determine weights based on their own preferences and values, which will affect the results of cost-benefit analysis.

6.3 Generalization of the model

1. Extension of multi-stage decision-making problems. Dynamic programming can be applied not only to traditional single-objective multi-stage decision-making problems, but also to multi-objective multi-stage decision-making problems. For example, in resource allocation problems, multiple objectives are considered at the same time, such as maximizing economic benefits and minimizing environmental impacts. The optimal decision is solved by constructing a multi-objective dynamic programming model.

2. Simulation of non-independent binomial distribution. For non-independent binomial distribution, we can consider using conditional normal distribution or copula function to simulate it. For example, in a binomial experiment with correlation, we can construct a suitable copula function to describe the correlation between binomial distributions [7], and then use normal distribution to simulate the correlated binomial distributions.

3. The cost-benefit analysis model can be extended to life cycle cost-benefit analysis: Extend the cost-benefit analysis to the entire life cycle of the project or product. Consider the costs and benefits of each stage of the project from planning, design, construction, operation to decommissioning, and conduct a full life cycle cost-benefit analysis to achieve more comprehensive and sustainable decision-making.

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