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Stochastic Analysis in Investment Project Appraisal

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ABSTRACT: *The use of stochastic methods such as the Monte Carlo method is essential to include the concept of risk and uncertainty in investment evaluation today. The Monte Carlo method helps to draw conclusions about the identification of factors that create business risk (selling price, unit capacity, production costs, excise duty) and how an industrial investment project is affected by external environmental factors.*
KEY WORDS*: investment, risk, uncertainty, investment project appraisal, Monte Carlo simulation, stochastic analysis, NPV, IRR.*

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I. INTRODUCTION AND LITERATURE REVIEW

Today the Monte Carlo method is one of the useful tools in risk assessment and investment evaluation. In the process of evaluating investment plans, it becomes necessary to study stochastic variables, pointing out that the determination of just rational values is not sufficient to make the right decision. According to Patris (2008), the Monte Carlo simulation method is recommended in those cases where sufficient historical data is not available or risks that are difficult to predict must be included in the assessment process.

The Monte Carlo method provides reliable results but presents some peculiarities. An appropriate sample must be defined, and several replicates performed. It is not capable of formulating strategy, and it is not possible to include variables that cannot be expressed in units of money. The method provides satisfactory evidence in the evaluation of an investment (NPV - Net Present Value - and IRR - Internal Rate of Return - from the perspective of the national economy, investors, and shareholders) but cannot formulate a strategy. In conclusion, it is a method that dynamically describes the business risk and gives reliable results.

II. LITERATURE REVIEW

The Monte Carlo method consists of algorithms for simulating the behavior of mathematical or physical systems. It examines the set of factors that affect the returns of an investment proposition through probability distributions and random numbers. The method estimates a price distribution of the NPV of the considered investment proposal. The main stages of the Monte Carlo simulation method according to Fotis (2015) and Murray-Smith (2015) are as follows:

In the first stage, the empirical model that characterizes the investment proposal is determined, i.e. the mathematical relationships of the model:

$$NCF_t = (REV_t - COST_{tot})(1 - ART_t) + DEPT_t * ART_t + dt \quad (1)$$

For each factor affecting the NCF_t (Net Cash Flows) we define a subjective probability distribution for which the mean value and standard deviation are calculated.

In the second stage, a stochastic function is selected which describes the changes in the factors affecting the NCF_t of the investment proposal. The differential equation that describes geometric Brownian motion is usually used:

$$dS(t) = \nu S(t)dt + \sigma S(t)dz \quad (2)$$

$dS(t)$: change in investment value between t and dt
$S(t)$: the current value of the investment proposal at time t
ν: expresses the average change in the value of the investment.
σ: the standard deviation
z: is a random variable $\{ z \sim N(0,1)\}$, which is the residual of the value of the factor we are considering from its expected value.

In the third stage, a random value of the residue is chosen which is replaced in the above relationship and thus a random value is obtained for the factor under consideration. After the random values for all factors are obtained, a substitution is made in the determination model of NCF_t and thus a random value is obtained, which constitutes the NCF_t of the investment proposal.

In the fourth stage, the second half of the third stage is repeated to construct a distribution of prices, which constitute the NCF_t of the investment proposal. These prices are pre-paid, and the NPV of the investment proposal is calculated.

Criticism of the method

According to Artikis (2002) the advantages and disadvantages of the Monte Carlo simulation method are presented. The advantages of the Monte Carlo method include (Karnavas, 2024):

- the ability to solve complex problems and the fact that her estimates provide important information for the investment proposal under consideration.
- Gives the correlation of the determining factors of the investment with mathematical expressions and their connection with the cash flows.
- All the factors affecting the investment and the uncertainty it contains are determined.
- It is a useful tool for predicting future financial results and planning available resources.
- Also, it is a dynamic model with respect to sensitivity analysis and breakeven analysis. It gives information on the whole range of values of net cash flows and NPV.
- Finally, it calculates the frequency with which the NPV takes extreme values and the factors that cause it.
- Also, the results of the simulation can be presented in diagrams, so that they are easier to understand by the researcher.

The disadvantages of the Monte Carlo method include:

- the choice of probability distribution is subjective and may differ between researchers. the complexity in designing the model, as it contains a large amount of information with which the analyst may not be familiar.
- does not have the ability to readjust the initial options depending on the prevailing conditions. In the case of a high NPV, the investor would choose to expand the investment, resulting in an increase in future cash flows, while on the contrary, in negative results, he would proceed to cancel the investment to reduce losses.
- the method presents a distribution of possible values of NPV, whose extreme values deviate from reality. The greater the deviation of the actual values of the determining factors of the investment from the expected ones, the less reliable the result of the method will be.

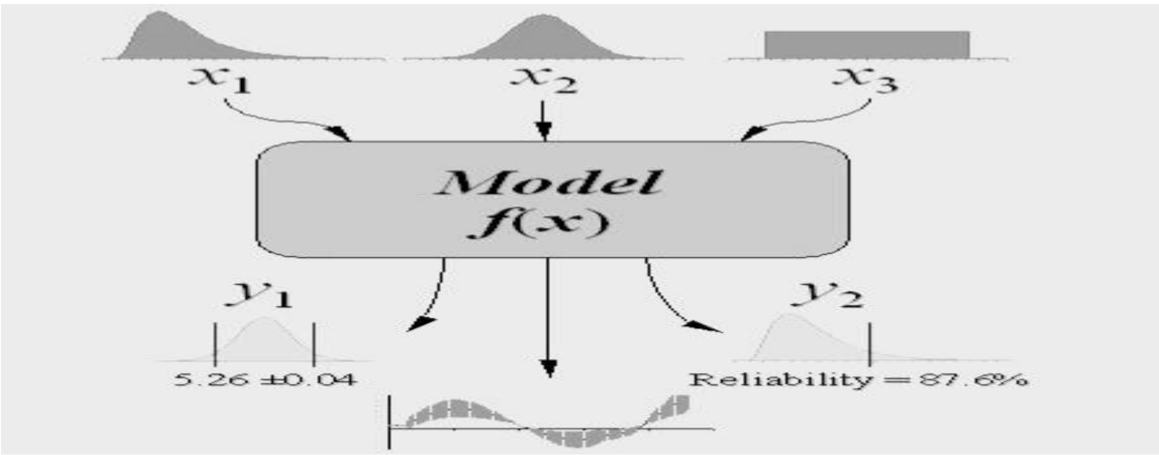

Figure 1: Model Monte Carlo (MC)

Summing up, we conclude that the main advantage of the MC simulation lies in the high accuracy of the calculation of financial indicators regardless of the complexity and specificities of the investment under consideration. It allows the introduction of several variables, which have a simultaneous interaction, taking different values in their range (Carmichael, 2018). The number of variables that can be introduced in an MC simulation model, enables the in-depth analysis of more perspectives of an investment project and makes it a suitable decision-making tool in projects that involve stakeholders with different interests (Jimenez et al, 2005). It is still considered easy by users without a mathematical background. Disadvantages include the complexity of the simulation, the large number of scenarios, which means increased time and cost, and the subjective estimation of the probability distribution that leads to approximate results.

Today the Monte Carlo method is one of the useful tools in risk assessment and investment evaluation. In the process of evaluating investment plans, it becomes necessary to study stochastic variables, pointing out that the determination of just rational values is not sufficient to make the right decision. According to Patris (2008), the Monte Carlo simulation method is recommended in those cases where sufficient historical data is not available or risks that are difficult to predict must be included in the assessment process.

In a simulation the indicators that are usually considered are the NPV and the IRR. According to Hacura et al (2001), the NPV because of the simulation, is the most reliable indicator in the evaluation of an investment since all cash flows are considered for its calculation. A group of researchers do not consider the use of IRR reliable in the evaluation of investments. However, Brounen et al (2004) and Osborne (2010) use IRR equally with NPV in the investment evaluation process probably because of the ease of comparison with the cost of capital.

III. RESEARCH METHODOLOGY

In Monte Carlo simulation the variables can take random values through the probability distributions. This is achieved by sampling or generating values for the set of variables for each year of the lifetime, and then calculating the NPV for each of these sets.

The model to be used is based on the indicators of Net Present Value (NPV) and Internal Rate of Investment (IRR). Three simulations will be performed, one for each scenario (baseline, optimistic, pessimistic). In each scenario, conclusions will be drawn for the indicators for all three perspectives, national economy (NPV1, IRR1), investment bodies (NPV2, IRR2), shareholders (NPV3, IRR3). (Damigos 2006, Polyzos 2018)

To evaluate the investment, the normal and triangular distributions are chosen to represent the variables of the specific model, giving a good approximation of the cash flows of the investment. Random numbers, whose range is between 0 and 1, are generated by random number generators and normal and triangular distributions are applied to these numbers to produce the random observation for the variables.

The following applies to the financial evaluation of investment projects (Karnavas, 2024):

$$NCF_t = (REV_t - COST_{tot})(1 - ART_t) + DEPT_t * ART_t \quad (3)$$

with: $COST_{tot} = QTY_t * COMP_t + STAFF_t + INTERATE_t + INDUST_{COST_t}$

and

$$REV_t = QTY_t * PRICE_t$$

where:

NET_PROFIT_t:	annual net profit
REV_t :	annual gross receipts
$COST_{tot}$	annual total operating cost
$DEPT_t$:	annual depreciation cost
TAX_t :	annual taxes
ART_t :	tax rate
QTY_t :	annual number of sales
$COMP_t$:	annual cost of A and B materials
$STAFF_t$:	annual personnel costs
$INTERATE_t$:	annual interest
$INDUST_{COST_t}$:	annual other industrial costs
$PRICE_t$:	annual selling price per product unit

The results of the indicators obtained from the simulation are:

$$(NPV_{kn})_{MC} = -C + \sum_{t=1}^{N} \frac{(NCF_{kn})_t}{(1 + r_k)^t}$$

$$-C + \sum_{t=1}^{N} \frac{(NCF_{kn})_t}{(1 + (IRR_{kn})_{MC})^t} = 0$$

C	Investment Cost
NCF	Net Cash Flow

$k = 1$	perspective of the National Economy
$k = 2$	perspective of Investment Agencies
$k = 3$	perspective of the Shareholders
$n = 1,2,3, \dots 1000$	Number of iterations
t	Period

The mean values $(ENPV_k)_{MC}$, standard deviations $(\sigma_{NPV_k})_{MC}$ and coefficients of variability $(CV_k)_{MC}$ of the MC simulation are given by the formulas:

$$(ENPV_k)_{MC} = \frac{\sum_{n=1}^{1000}(NPV_{kn})_{MC}}{n} \tag{4}$$

$$(\sigma_{NPV_k}^2)_{MC} = \frac{\sum_{n=1}^{1000}[(ENPV_k)_{MC} - (NPV_{kn})_{MC}]^2}{n} \tag{5}$$

$$(CV_k)_{MC} = \frac{(\sigma_{NPV_k})_{MC}}{(ENPV_k)_{MC}} \tag{6}$$

Then the application of the methodology to the construction of an industrial brewery unit will be presented.

DATA ANALYSIS

Next, the data that will be used in the application of the methodology are given.

Table 1: Data of Investment Project

	1	2	3	4	5	6	7	8	9	10
NET_PROFIT	833.594	833.594	746.526	1.023.255	1.032.335	1.280.714	1.291.110	1.597.995	1.609.897	4.442.233
REV_t :	15.575.000	15.575.000	15.575.000	17.800.000	17.800.000	20.025.000	20.025.000	22.250.000	22.250.000	22.250.000
$COST_{tot}$	13.273.190	13.273.190	13.273.190	15.019.186	15.019.186	16.858.001	16.858.001	18.554.857	18.554.857	18.554.857
$DEPT_t$:	506.500	506.500	506.500	506.500	506.500	426.500	426.500	426.500	426.500	426.500
TAX_t :	654.966	654.966	586.556	803.986	811.121	1.006.276	1.014.444	1.255.567	1.264.919	3.490.326
ART_t :	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44	0,44
QTY_t :	70.000	70.000	70.000	80.000	80.000	90.000	90.000	100.000	100.000	100.000
$COMP_t$:	143	143	143	143	143	143	143	143	143	143
$STAFF_t$:	655.200	655.200	655.200	704.340	704.340	846.300	846.300	846.300	846.300	846.300
$INTERATE_t$:	0	0	155.477	140.323	124.109	106.759	88.194	68.331	47.076	24.334
$INDUST_{COST_t}$	1.300.000	1.300.000	1.300.000	1.300.000	1.300.000	1.300.000	1.300.000	1.300.000	1.300.000	1.300.000
$PRICE_t$:	222.5	222.5	222.5	222.5	222.5	222.5	222.5	222.5	222.5	222.5

IV. RESULTS

By substituting the data into the equations of the methodology, the following results are obtained.
To investigate the performance of the industrial investment project, in case of simultaneous change of two or more variables, stochastic analysis or otherwise risk analysis is used. The simultaneous change of two or more variables is done by randomly sampling their values from specific distributions using the established Monte Carlo method. Initially, the distributions from which the values for the variables will be drawn by random sampling are defined. (Table 2)

The results of the Monte Carlo simulation for 1000 iterations give the distribution of the NPV and the IRR of the investment for all three evaluation perspectives (national economy, investment scheme, shareholders

Table 2: Risk Analysis

Elements of risk analysis parameters					
Variable	**Name**	**Distribution**	**min/mean**	**likely**	**s.d./max**
Production cost cost	cost_product	Normal	12000000		500000
Capacity of Production	capacity_factory	Triangular	90000	100000	110000
Special Consumption Tax	consum_tax	Normal	1250		50
Price	price	Triangular	210	222,5	240

The results include plots of the probability density distributions for NPV and IRR for all three evaluation approaches. (Fig. 2)

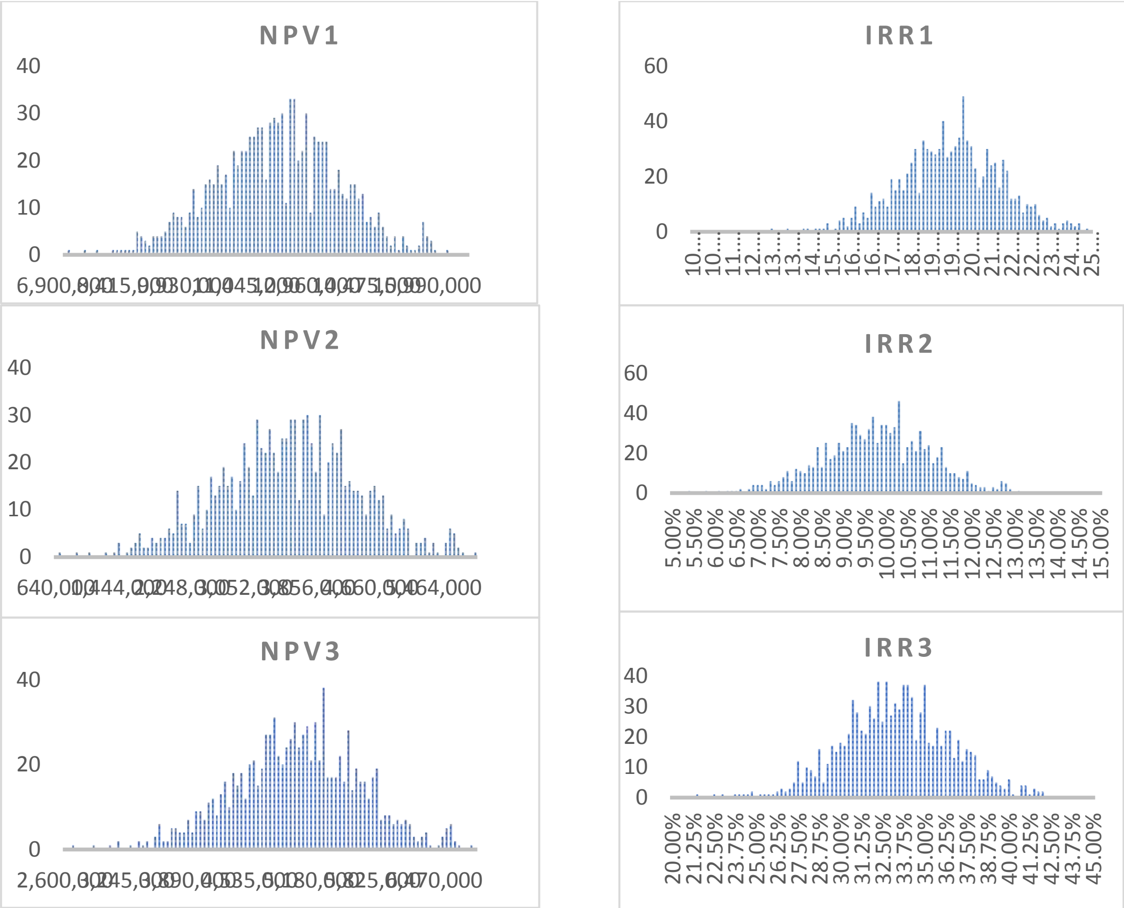

Figure 2: Distribution of base case NPV1, NPV2, NPV3, IRR1, IRR2, IRR3

Table 3 shows the corresponding cumulative distribution showing above which value α% of the observations of the financial indicators $NPV_1, NPV_2, NPV_3, IRR_1, IRR_2, IRR_3$

Table 3: Cumulative distribution table

	NPV1	IRR1	NPV2	IRR2	NPV3	IRR3
100%	6.977.254	13,0%	648.921	5,4%	2.675.543	21,5%
90%	10.124.786	17,3%	2.448.107	8,1%	4.144.417	29,2%
80%	10.844.941	18,1%	2.840.753	8,7%	4.452.583	30,6%
70%	11.394.500	18,7%	3.162.897	9,1%	4.686.296	31,6%
60%	11.792.889	19,2%	3.376.036	9,4%	4.851.050	32,4%
50%	12.195.286	19,7%	3.608.331	9,8%	5.020.542	33,2%
40%	12.586.606	20,1%	3.823.840	10,1%	5.188.600	33,9%
30%	13.016.111	20,6%	4.073.080	10,4%	5.352.000	34,8%
20%	13.460.792	21,2%	4.313.387	10,8%	5.567.000	35,9%
10%	14.158.533	22,0%	4.706.453	11,3%	5.838.579	37,4%
0%	16.495.000	24,9%	6.000.000	13,1%	6.857.000	42,0%

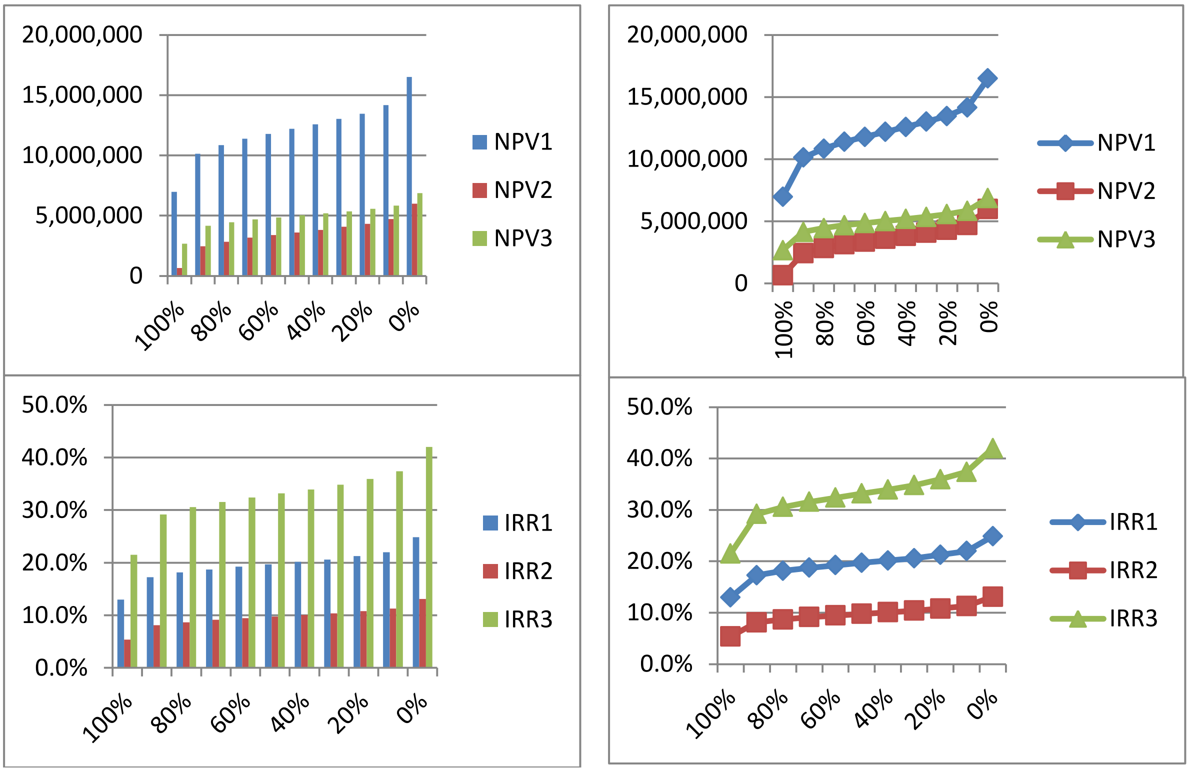

Figure 3: Histograms - polygons of cumulative distributions $\mathrm{NPV}_1, \mathrm{NPV}_2, \mathrm{NPV}_3$ IRR1, IRR2, IRR3

Then the average values, standard deviations, and coefficients of variability of the indicators $NPV_1, NPV_2, NPV_3, IRR_1, IRR_2, IRR_3$ are calculated (Table 4). It is observed that CV<1, (CV∈[0,10-0, 25]) which means reduced risk.

Table 4: Tables of mean values, standard deviations, and coefficients of variability of indicators

AVERAGE VALUES OF INDICATORS $NPV_1, NPV_2, NPV_3, IRR_1, IRR_2, IRR_3$					
$\overline{NPV_1}$	$\overline{IRR_1}$	$\overline{NPV_2}$	$\overline{IRR_2}$	$\overline{NPV_3}$	$\overline{IRR_3}$
12.178.723	19,65%	3.592.555	9,73%	5.010.899	33,18%

STANDARD DEVIATIONS OF INDICATORS $NPV_1, NPV_2, NPV_3, IRR_1, IRR_2, IRR_3$					
S_{NPV1}	S_{IRR1}	S_{NPV2}	S_{IRR2}	S_{NPV3}	S_{IRR3}
1.558.513,282	0,0186232	873.595,691	0,012448	663.574,9702	0,03221969

COEFFICIENTS OF VARIABILITY OF THE INDICATORS $NPV_1, NPV_2, NPV_3, IRR_1, IRR_2, IRR_3$					
CV_{NPV1}	CV_{IRR1}	CV_{NPV2}	CV_{IRR2}	CV_{NPV3}	CV_{IRR3}
0,127970171	0,0947685	0,24316837	0,127947	0,13242633	0,09711069

Finally, the probability (uncertainty) of the financial indicator $NPV_1, NPV_2, NPV_3, IRR_1, IRR_2, IRR_3$ is calculated.

	NPV1	IRR1	NPV2	IRR2	NPV3	IRR3
lower interval limit	6.500.000	13%	500.000	5%	2.500.000	21%
upper interval limit	9.000.000	16%	2.000.000	7%	3.500.000	26%
PROBABILITY	2,1%	2,2%	3,4%	1,4%	1,1%	1,2%

	NPV1	IRR1	NPV2	IRR2	NPV3	IRR3
lower interval limit	9.000.000	16%	2.000.000	7%	3.500.000	26%

upper interval limit	12.500.000	19%	3.500.000	9%	4.500.000	31%
PROBABILITY	53,6%	30,5%	40,5%	23,3%	20,0%	21,1%

	NPV1	IRR1	NPV2	IRR2	NPV3	IRR3
lower interval limit	12.500.000	19%	3.500.000	9%	4.500.000	31%
upper interval limit	14.500.000	21%	5.000.000	11%	5.500.000	35%
PROBABILITY	37,7%	42,0%	50,8%	57,9%	54,5%	46,9%

	NPV1	IRR1	NPV2	IRR2	NPV3	IRR3
lower interval limit	14.500.000	21%	5.000.000	11%	5.500.000	35%
upper interval limit	16.500.000	25%	6.000.000	13%	7.000.000	42%
PROBABILITY	6,6%	25,3%	5,2%	17,3%	24,4%	30,6%

V. CONCLUSIONS

In the Monte Carlo simulation, the change in NPV and IRR from a simultaneous change in the uncertain parameters was studied. A probability distribution (normal and triangular) was chosen for each of the uncertain variables. This was followed by the MC simulation with the results of the probability distributions of the NPV and IRR of the Investment Plan. All the above was studied for three scenarios (basic, optimistic, pessimistic) from three perspectives: the national economy, investors, and shareholders.

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