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Investigating the Impact of Dynamic Vibrations on an Aluminium Plate with Cracks

Mohd Dawood[1], Dr.Prabhat Kumar Sinha[2]
[1]*Sam Higginbottom University of Agriculture, Technology and Sciences, Prayagraj (U.P).*
[2]*Sam Higginbottom University of Agriculture, Technology and Sciences, Prayagraj (U.P).*

ABSTRACT
This study delves into the analytical modeling of the impact of cracks on dynamic plates utilized in aerospace applications. The creation of a reduced order analytical model of the plate is the main focus. In this scenario, cracks with varying characteristics are subjected to force, which triggers vibrations. This situation is particularly perilous and may result in catastrophic consequences if left unchecked. The equation obtained is in the classical form, with coefficients containing information about the geometrical and mass properties of the plate, loading and boundary conditions, as well as the location, orientation, and geometry of the crack. The analytical solution obtained through the perturbation method of multiple scales is approximate. Furthermore, the study reveals that different boundary conditions can be applied to the plate, and the modal natural frequencies are affected by the crack's geometry. Various parameters were evaluated, and the research numerically studied the behavior of the cracked plate's divergence. The study also calculated the natural frequencies and corresponding Eigen functions as a function of the crack length. The results of this research illustrate that the Finite Element Method (FEM) is an efficient technique for dynamic plate analysis containing discontinuities. In conclusion, this study provides valuable insights into the behavior of dynamic plates containing cracks and emphasizes the importance of using analytical and numerical methods to study complex structures used in aerospace applications. The results obtained can be used to design safer and more efficient structures that can withstand the harsh conditions of aerospace environments.
Keywords: *vibration analysis, dynamic plate, FEM*

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I. INTRODUCTION

Plate structures have been a subject of great interest in both scientific and engineering communities for many years due to their numerous applications across various industries. The aircraft industry, in particular, has been highly interested in such structures, with early solutions motivated by this industry. Plate and beam structures are fundamental in engineering and are utilized in various structural applications, including aircraft wings, satellites, ships, steel bridges, sea platforms, helicopter rotor blades, spacecraft antennae, and subsystems of more complex structures. This dissertation focuses specifically on aircraft wing structures modeled as isotropic plates. The research explores an approximate analytical model of a cracked plate and examines the nonlinear vibrations of a plate with a crack at the center consisting of continuous line under transverse harmonic excitations with three sets of boundary conditions. Additionally, the bifurcately behaviors within this system are investigated for a nonlinear transition to chaos through the use of available software. Plate theory, and its behaviors in the dynamic and static domains, have been applied to reduce vibration and noise in structures since the end of the 19th century when German physicist Chladni et al. discovered various modes of free vibrations experimentally. Shen and Grady et al. studied the delamination effect of a composite plate using variation principles. However, the ‘constrained mode’ model failed to predict the opening in the mode shapes found in the experiments by Shen and Grady.To simulate the ‘open’ and ‘closed’ behaviors between the delaminated layers, Shu et al. presented an analytical solution to study a sandwich beam with double delamination. Their study drew attention towards the influence of the contact mode, ‘free’ and ‘constrained’, between the delaminated layers, and the local deformation at the delamination fronts. Della and Shu et al. further investigated the beam with double delamination by using ‘free mode’, ‘partially constrained mode,’ and ‘constrained mode’ models. Shu and Della presented ‘free mode’ and ‘constrained mode’ models to study composite beams with multiple delamination. Their study focused on the influence of a second short delamination on the bending frequencies and the corresponding mode shapes of the beam.Gim et al. developed a plate finite element to analyze a two-dimensional single delamination with multiple constraint conditions to find the strain energy release rate for the element. P.K Parhi et al. proposed an analytical model for arbitrarily located multiple delamination. Bangarubabu et al. presented an understanding of the effectiveness of the sandwich structures, the dynamics of a bare beam with free and constrained Viscoelastic layers were investigated.The Viscoelastic layer is bonded uniformly on the beam. The effect of distributed Viscoelastic layer treatment on the loss factors is studied. Baz et al. presented the vibration control of plates with active constrained layer damping. The discussion of bending vibration of flat plates is controlled using patches of active constrained layer damping (ACLD) treatments. Daraji et al. presented optimal placement of sensors and actuators for active vibration reduction of a flexible structure using a genetic algorithm based on modified H infinity.Galerkin's method is used to reduce the set of partial differential equations to a set of ordinary differential equations, and then it is possible to study linear, nonlinear, and chaotic behavior of the system. It uses the shape or characteristic functions which depend upon the boundary condition of the system. Galerkin's method can be divided into two main categories: Petrov-Galerkin method and Ritz-Galerkin method. In the Ritz-Galerkin method, the solution is expanded in terms of a series with unknown coefficients which depend on time and satisfy the given boundary conditions. In the Petrov-Galerkin method, the residual is orthogonal to each of the expansion functions. One may define a different set of test functions and require the residual to be orthogonal to each of these test functions.

II. Research Objectives

The research focused on developing an approximate analytical model for a cracked plate that took the form of a single-degree-of-freedom system. Here are some of the main points of the study:

- The crack was placed at the center of the plate in a continuous line, not in any specific location.
- Rotary inertial and through-thickness stresses were considered to be negligible by the model.
- In order to compare results, the method of multiple scales was suggested as an approximate solution technique, as well as direct integration and finite element analysis in ABAQUS.
- Finally, commercial software was used to conduct a dynamical systems analysis.

1. A Literature Review on the Use of Passive and Active Techniques for Vibration Minimization

In recent years, there has been a surge of interest in the development of passive and active techniques for minimizing vibration levels. Many researchers have focused on the dynamic analysis of plates subjected to vibration to identify different approaches to study the dynamic behaviour of simple structures. Despite some disadvantages, this literature survey provides a comprehensive summary of the research works done to date in this application-oriented field of smart material science engineering. One of the useful approaches employed for the study of vibration in cracked structures is the Finite Element Method (FEM). In 1979, Raju and Newman used a three-dimensional FEM model to analyse circular or penny-shaped cracks and an elliptical crack completely embedded in a finite thickness plate subjected to uniform tension. Isoparametric and singular elements were used to model elastic bodies with cracks. The calculated stress intensity factors for these crack configurations were compared with the exact solutions, and the validity of the FEM was verified. The stress intensity factors for embedded circular and elliptical cracks were generally about 0.4-1% below the exact solutions, while the calculated stress intensity factors for the elliptical crack in the sharpest curvature of the ellipse were about 3% higher than the exact solution.

Passive and active techniques have been widely studied in recent years to minimize vibration levels in various structures. The use of smart materials and engineering techniques has enabled researchers to develop effective methods for vibration control. The following are some of the notable approaches that have been employed in this field. In conclusion, the use of passive and active techniques, along with the Finite Element Method, has enabled researchers to develop effective methods for minimizing vibration levels in various structures. These methods have the potential to enhance the safety and performance of structures, especially those that are subjected to dynamic loads.

Nonlinear Plate Theory and its Applications: Plate structures are essential components in various engineering applications due to their versatility and durability. The dynamic and static behaviors of plate structures have been studied for decades, with a focus on reducing vibration, noise, and optimizing performance. This article aims to expand on the previous discussion of plate structures and their applications.

Nonlinear Vibrations of Cracked Plates: The nonlinear vibrations of a plate with a crack at the center have been explored in this dissertation. The approximate analytical model of a cracked plate has been studied under transverse harmonic excitations with three sets of boundary conditions. The research investigates the bifurcately behaviors within this system for a nonlinear transition to chaos, through the use of available software. The findings of this research can be applied to various industries, especially in the aircraft industry, to improve the performance of aircraft wing structures.

Delamination Effect of Composite Plates: The delamination effect of composite plates has been studied using variational principles. The 'constrained mode' model failed to predict the opening in the mode shapes found in the experiments. To simulate the 'open' and 'closed' behaviors between the delaminated layers, an analytical solution has been presented to study a sandwich beam with double delamination. The influence of the contact mode, 'free' and 'constrained,' between the delaminated layers, and the local deformation at the delamination fronts have been investigated.

Multiple Delamination Analysis: An analytical model has been proposed to study arbitrarily located multiple delamination. The study focused on the bending frequencies and the corresponding mode shapes of the beam. The effect of distributed Viscoelastic layer treatment on the loss factors has been studied, and an understanding of the effectiveness of the sandwich structures has been presented. The vibration control of plates with active constrained layer damping has been discussed, where patches of active constrained layer damping treatments are used to control the bending vibration of flat plates.

Optimal Placement of Sensors and Actuators: The optimal placement of sensors and actuators for active vibration reduction of a flexible structure has been presented using a genetic algorithm based on modified H infinity. Such findings can be applied to various industries to reduce the vibration and improve the performance of structures.

Galerkin's Method: Galerkin's method has been used to reduce the set of partial differential equations to a set of ordinary differential equations, which can be used to study linear, nonlinear, and chaotic behavior of the system. The method uses the shape or characteristic functions that depend upon the boundary condition of the system. The Ritz-Galerkin method expands the solution in terms of a series with unknown coefficients that depend on time and satisfy the given boundary conditions. On the other hand, in the Petrov-Galerkin method, the residual is orthogonal to each of the expansion functions. One may define a different set of test functions and require the residual to be orthogonal to each of these test functions. Such methods can be applied in various industries to optimize the performance of structures.

1.1 Adopted Methodology

The objective of this research paper is to present the equation of motion for a nonlinear vibration of an isotropic plate with a part-through crack located at the center. The paper emphasizes the importance of a tractable solution to the vibration problem by applying the equilibrium principle to derive the governing equation of motion, taking into account a continuous line. The rectangular plate's governing equation in its classical form is analyzed in detail, excluding rotary inertia and through-thickness shear forces. Based on this analysis, it can be concluded that:

$$D\left[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^4 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]=-\rho h\frac{\partial^2 w}{\partial t^2}+n_x\frac{\partial^2 w}{\partial x^2}+n_y\frac{\partial^2 w}{\partial y^2}+2n_{xy}\frac{\partial^2 w}{\partial x \partial y}+p_z \quad (1)$$

where w is the transverse deflection, $p_z$is the load per unit area acting at the surface, r is the density, h is the thickness of the plate and n_x, n_y,n_{xy} are the in-plane or membrane forces per unit length. D is the flexural rigidity and can be defined as$D = Eh^3 12(1-v^2)$; E is the modulus of elasticity, and v is the Poisson's ratio.

The equilibrium equations are obtained by resolving the forces in the zdirection and taking moments about the x and y-axes. The forces acting onthe plate element are shown in Figure 1. Summing the forces along the z-axis leads to,

$$\sum F_z = 0; -Q_x \partial y + \left(Q_x + \frac{\partial Q_x}{\partial y}dx\right)dy - Q_y dx + (Q_y + \frac{\partial Q_y}{\partial y})dx + P_z dxdy = \rho h \frac{\partial^2 w}{\partial t^2}dxdy \quad (2)$$

Where, the forces per unit length (projected along the z-axis) are represented by$Q_x$andQ_y, r is the density, h is the thickness and $P_z$is the load per unit area acting over the surface of the plate. To suit the experimental configuration, this is later substituted with a point load $P_z$utilizing the suitable delta function, which is a simple implementation in practice.

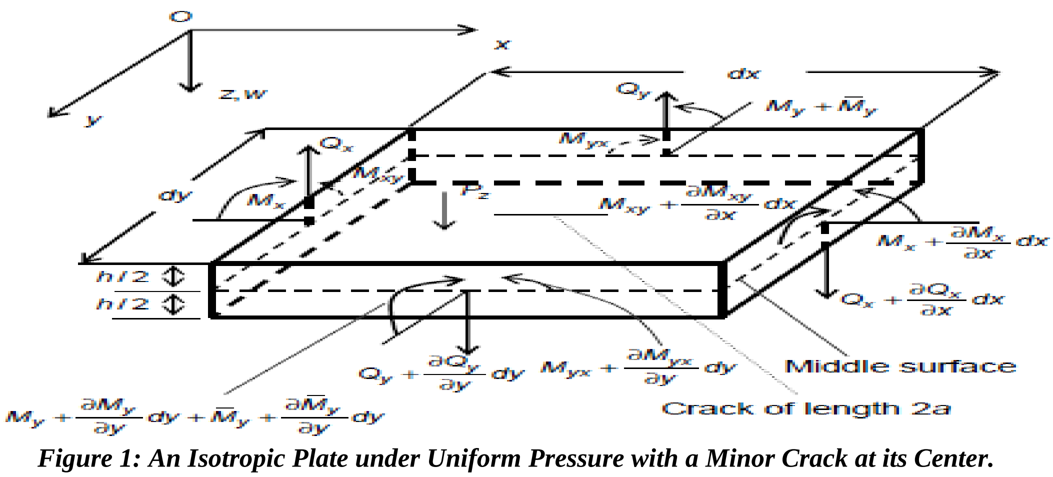


Figure 1: An Isotropic Plate under Uniform Pressure with a Minor Crack at its Center.

Therefore,

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \rho h \frac{\partial^2 w}{\partial t^2} - P_z \quad (3)$$

By taking the moment equilibrium about the y-axis, we get,

$$\sum M_y = 0; -M_x\, dy + \left(M_x + \frac{\partial M_x}{\partial x} dx\right) dy - M_{xy} dx + \left(M_{xy} + \frac{\partial M_{xy}}{dy} dy\right) dx - \left(Q_x + \frac{\partial Q_x}{dx} dx\right) \frac{dx}{2} dy - Q_x dy \frac{dx}{2} = 0 \quad (4)$$

After simplification, the term containing $\frac{1}{2}\frac{\partial Q_y}{\partial x}(dx)^2 dy$ is neglected,Therefore,

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{dy} = Q_x \quad (5)$$

And so,

$$\frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} = -\frac{\partial Q_y}{\partial y} \quad (6)$$

Where M_y= bending moment per unit length.

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (7)$$

Where $M_x, M_y\ and\ M_{xy}$ =the bending moment per unit length. In both the x and y directions
It is as follows:

$$M_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x\, zdz \quad (8)$$

$$M_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y\, zdz \quad (9)$$

$$M_{xy} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \tau_{xy}\, zdz \quad (10)$$

$$\sigma_x = -\frac{EZ}{1-v^2}\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \quad (11)$$

$$\sigma_y = -\frac{EZ}{1-v^2}\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) \quad (12)$$

$$\tau_{xy} = \frac{EZ}{1-v^2}(1-v)\left(\frac{\partial^2 w}{\partial x \partial y}\right) \quad (13)$$

From equation (11), (12) and (13) we get the following $M_x, M_y and\ M_{xy}$

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \quad (14)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) \quad (15)$$

$$M_{xy} = -M_{xy} = -D(1-v)\left(\frac{\partial^2 w}{\partial x \partial y}\right) \quad (16)$$

Where D = flexural rigidity and can be express as D=$Eh^3/12(1-v^2)$; E is the modulus of elasticity and v is the poisson's ratio. Equations (14),(15) and (16)leads to the following result,

$$D\left(\frac{\partial^2 w}{\partial x^2} + 2\frac{\partial^2 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h \frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 M_y}{\partial y^2} + P_Z \quad (17)$$

Taking into account the equilibrium of the dxdy element shown in Figure 2,and given that it is subjected to in-plane forcesn_x ,n_y,$n_{xy} = n_{yx}$.Given that there are no body forces, and the plate has a crack at its center per unit length, the projection of the membrane forces on the x-axis is as follows:

$$-n_x dy + \left(n_x + \frac{\partial n_c}{\partial x} dx\right) dy - n_{yx} dx + \left(n_{yx} + \frac{\partial n_{yx}}{\partial y} dy\right) dx = 0 \quad (18)$$

Therefore,

$$\frac{\partial n_x}{\partial x} + \frac{\partial n_{yx}}{\partial y} = 0 \qquad (19)$$

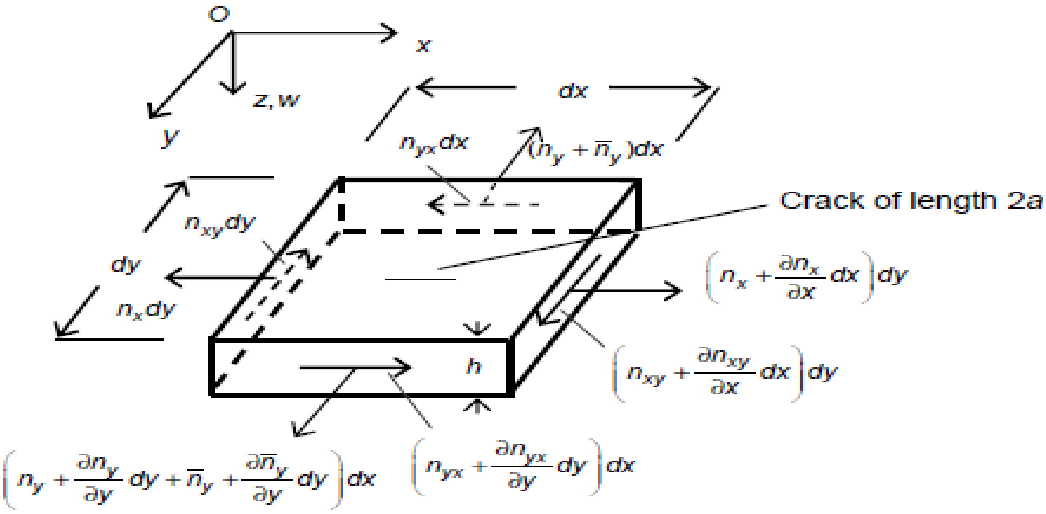


Figure2: In-Plane Forces and Central Crack Length (2a) in Plate Element.

Similarly,

Along the y-axis we find that,

$$(-n_y)dx + \left(n_y + \frac{\partial n_y}{\partial y}dy\right)dx - n_{xy}dy + \left(n_{xy} + \frac{\partial n_{xy}}{\partial x}dx\right)dy = 0 \qquad (20)$$

leading to,

$$\frac{\partial n_y}{\partial y} + \frac{\partial n_x}{\partial x} + \frac{\partial n_{xy}}{\partial x} = 0 \qquad (21)$$

Next, we will examine the equilibrium of the dxdy element in the z-direction. For the purpose of analysis, we will assume that the left-hand and rear edges of the plate element are fixed and situated in the xy plane, as illustrated in Figure 3.

$$\sum F_z(x,y) = \left(n_x + \frac{\partial n_x}{\partial x}dx\right)dy\frac{\partial^2 w}{\partial x^2}dx + \left(n_y + \frac{\partial n_y}{\partial y}dy\right)dx\frac{\partial^2 w}{\partial y^2}dy + \left(n_{xy} + \frac{\partial n_{xy}}{\partial x}dx\right)dy\frac{\partial^2 w}{\partial x \partial y}dx + \left(n_{yx} + \frac{\partial n_{yx}}{\partial y}\right)dx\frac{\partial^2 w}{\partial x \partial y}dy \quad (22)$$

So,we obtain

$$\sum F_z(x,y) = n_x\frac{\partial^2 w}{\partial x^2} + n_y\frac{\partial^2 w}{\partial y^2} + 2n_{xy}\frac{\partial^2 w}{\partial x \partial y} (23)$$

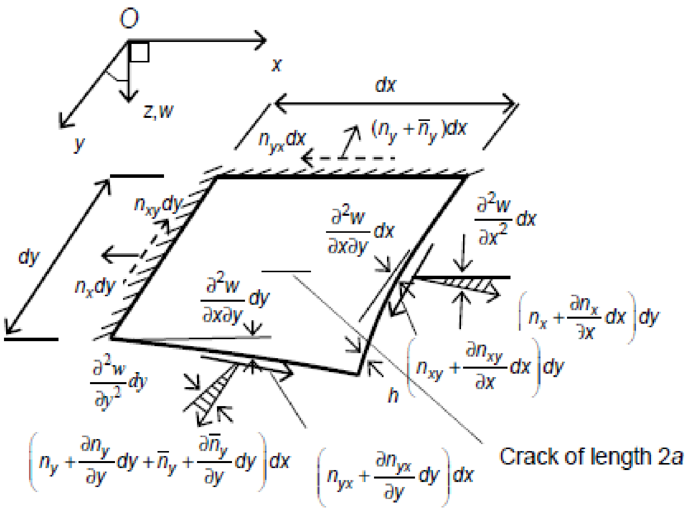


Figure 3: Plate Deformation with a Part-Through Crack at the Center: The Role of Two-Sided Constraints

By utilizing equation (23), it can be inferred that the impact of membrane forces on deflection is equal to a presumed lateral force (referred to as "X"). By including this force in the governing equation, specifically in the x-direction, the terms in the y and xy directions can be disregarded. Ultimately, this results in the final version of equation (17), which is as follows:

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = -\rho h \frac{\partial^2 w}{\partial t^2} + n_x \frac{\partial^2 w}{\partial x^2} + P_z (24)$$

The cracked vibrating plate's equation of motion is represented by equation (24), which applies to free vibration. Table 1 reveals the computed natural frequencies with and without a crack, for various aspect ratios and boundary conditions. These calculations indicate that the first mode's natural frequency is noticeably affected by the presence of a crack at the plate's centre, across all three cases.

Table No 1: Natural Frequencies of the Cracked Plate Model for Different Aspect Ratios and Boundary Conditions

First Mode Natural Frequency for half cracked length, A=0.01 and B=0.025 m										
the Length of the Plate's Sides		Two edges are clamped together while the other two remain free (CCFF)			CCSS: Two Adjacent Edges Clamped, Two Others Simply Supported			Edge Supports: All Simply Supported (SSSS)		
$l_1\,(m)$	$l_2\,(m)$	Un-cracked	cracked		Un-cracked	cracked		Un-cracked	cracked	
			0.01 m	0.025 m		0.01 m	0.025 m		0.01 m	0.025 m
1	1	80.50	77.40	74.12	445.69	432.52	418.50	77.56	75.50	73.35
0.5	1	231.05	229.90	228.81	1160.74	11154.28	1146.50	193.94	192.50	191.07
1	0.5	231.05	213.92	194.69	1160.74	1089.96	1011.02	193.92	183.16	171.40
0.5	0.5	321.90	310.52	296.38	1730.05	1730.02	1674.28	310.30	302.12	293.50

III. APPROXIMATE ANALYTICAL TECHNIQUES

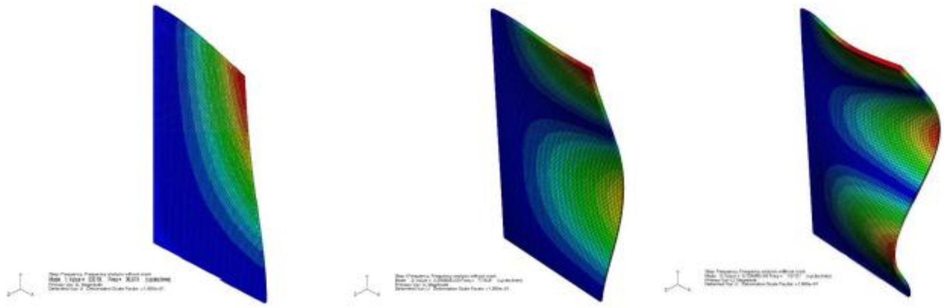
Linear vibration problems have been long studied, and many exact solutions are available in the literature. In contrast, nonlinear problems present a challenge for analysis due to their inherent complexity. As a result, numerical approximate techniques are commonly used to solve them. In this study, we employed the perturbation method of multiple scales to obtain a closed-form solution. We also conducted direct integration within Mathematica™ and a finite element analysis in ABAQUS to compare numerical solutions for the cracked plate model. Although several perturbation methods exist, each has its own limitations and advantages. They work by dividing the coordinate into successively smaller parts, expressed as a power series in terms of a small parameter, typically denoted as e. In our case, the coordinate of interest is y, and its expansion follows this form.

$$\varphi = \varphi_0 + \varepsilon\varphi_1 + \varepsilon^2\varphi_2 + \ \dots\dots\dots\dots + \varepsilon^n\varphi_n \qquad (25)$$

Each term in the series is known as a perturbation.

3.1 FE analysis

Figures 4,5and 6 display the ABAQUS/CAE results of the nonlinear FE analysis of a plate with a part-through crack at its center. The aspect ratio of the plate is 0.5/1, and the boundary condition used is CCFF. Other boundary conditions, such as CCSS and SSSS, can be employed to obtain results.



(a) Mode – I (b) Mode – II (c) Mode – III

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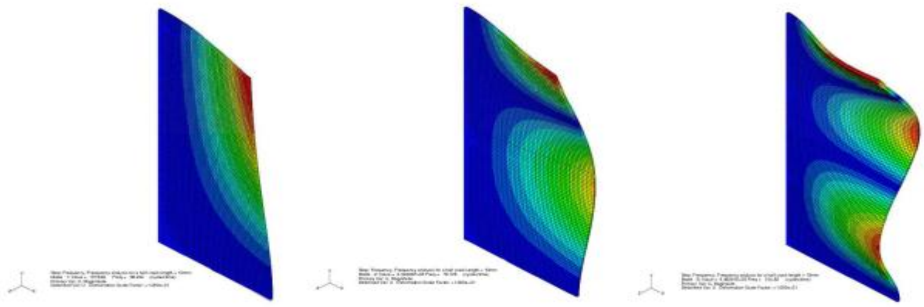
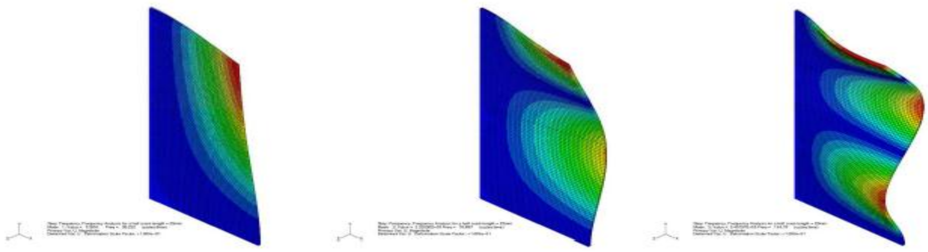
Figure 4: Three Vibration Modes of an Aluminium Plate in the Absence of Cracks.



(a) Mode – I (b) Mode – II (c) Mode – III

Figure 5: Vibration Modes of an Aluminium Plate with a 0.01m Half Crack Length and Sides Measuring 0.5m x 1m.



(a) Mode – I (b) Mode – II (c) Mode – III

Figure 6: Vibration Modes of an Aluminium Plate with Dimensions of 0.5 x 1m and a Half Crack Length of 0.025m

3.2Amplitude Analysis

Amplitude responses of an un-cracked and cracked plate model were obtained through implicit dynamic nonlinear analysis in ABAQUS/CAE. As predicted, the amplitudes showed an increase after a small crack was introduced to the aluminium plate. Refer to Table 2 for the amplitude values, and to compare theoretical models to FEA results.

Table No 2: Finite Element Analysis Results

		Theortical results		FEA Results			
		Frequency (Hz)	Amplitude (mm)	Frequency (Hz)			Amplitude (mm)
				M-1	M-2	M-3	M-1 only
Un-cracked Plate		36.79	1.1910	36.820	77.86	157.08	1.0162
Cracked	10 mm	36.62	1.1970	36.242	76.70	154.80	1.228
plates	25 mm	36.38	1.2050	36.238	76.69	154.79	1.385

Table 3: Comparing Experimental Results of Vibrational Modes in Cracked and Uncracked Aluminium Plate

Aluminum plates	Frequency of excitation (Hz)	Measurement of Amplitude (mm) at the Free End of the Aluminum Plate
Uncracked	25	0.91890
Cracked	24.5	0.93694

IV. RESULT

In an experimental setup, the vibration of a cracked aluminium plate was analyzed at its center. The findings indicated a consistent decrease in frequencies and an increase in amplitudes, which matched theoretical calculations based on approximate techniques. However, the measured values were significantly lower than expected. This discrepancy was likely due to the presence of microscopic flaws or cracks in materials under normal conditions. Subsequent experiments showed a loss of local stability in plates with cracks under periodic loading.

V. CONCLUSIONS

In order to study the response behavior of a cracked plate model, three distinct methods were implemented. These methods displayed comparable trends, showing the impact of decreasing natural frequencies and increasing amplitude values. Moreover, dynamical system studies indicate that the presence of chaos becomes more pronounced as the system becomes increasingly nonlinear due to excitation acceleration. These experimental findings attest to the practical significance of the theoretical approach. Further analysis of the experimental results revealed some interesting findings. Here are some key takeaways:

1. The decreasing natural frequencies observed in all three methods indicate that the structure becomes more flexible and less stable as the crack grows. This is expected behavior for a cracked plate model, as cracks can significantly alter the stiffness and damping properties of the structure.

4. The increasing amplitude values also demonstrate the impact of crack growth on the response behavior of the structure. As the crack grows, it creates new modes of vibration that can lead to higher amplitudes and even structural failure.

5. The presence of chaos in the system is a concern for engineers because it can make it difficult to predict the response behavior of the structure. However, by understanding the underlying dynamics of the system, it is possible to design control strategies that can mitigate the effects of chaos.

Overall, the experimental findings provide valuable insights into the behavior of cracked plate models and demonstrate the practical significance of the theoretical approach. As researchers continue to study these structures, they will gain a deeper understanding of their complex dynamics and be better equipped to design safe and reliable structures.

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