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Trajectory Design and Kinematic Analysis of Milling Robot

Ha Thanh Hai
Hanoi university of industry, No. 298 Cau Dien Street, Bac Tu Liem District, Hanoi, Vietnam

Abstract
The application of robots in machining such as drilling, milling, grinding, etc. is becoming increasingly popular. Multi-degree-of-freedom serial robots play the role of machining machines, bringing about many outstanding advantages in terms of flexibility and the ability to create complex shapes that can be machined on a variety of surfaces, from simple to complex, with sizes ranging from small to large, which traditional machining machines such as universal machines and CNC machines cannot do. However, the construction of machining technology programs for robots becomes difficult, first of all, the analysis of kinematics and the design of appropriate motion trajectories for the machining process to form the basis for calculating robot dynamics and control, because robots have a complex structure with many joints.
This paper presents a kinematic analysis model for milling robots and proposes a method for designing appropriate motion trajectories for robots in milling processes to create surface shapes of details. The survey and calculation results are illustrated by numerical simulations.
Keywords: *Robot, Kinematics, Trajectory Design, Milling*

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I. Introduction

Machining robots have many advantages due to their multi-degree-of-freedom structure, which allows them to perform complex cutting motions that are difficult for machine tools and CNC machines to achieve. In addition, robots also have the ability to be programmed flexibly, allowing for quick and easy changes to both the machining process and the workpiece. Thanks to these outstanding advantages, robots are increasingly being used in the field of machining [1-5].

The choice of robot with appropriate kinematic structure and degrees of freedom depends on the surface to be machined and the machining technique required. For milling and shaping of complex spatial surfaces of workpieces with configurations ranging from simple to complex, 6-degree-of-freedom serial industrial robots are commonly used. The use of 6-degree-of-freedom robots for machining ensures that the spatial shaping motion of the tool mounted on the operating unit reaches the desired position and orientation according to the requirements of the milling machining technology [5-8].

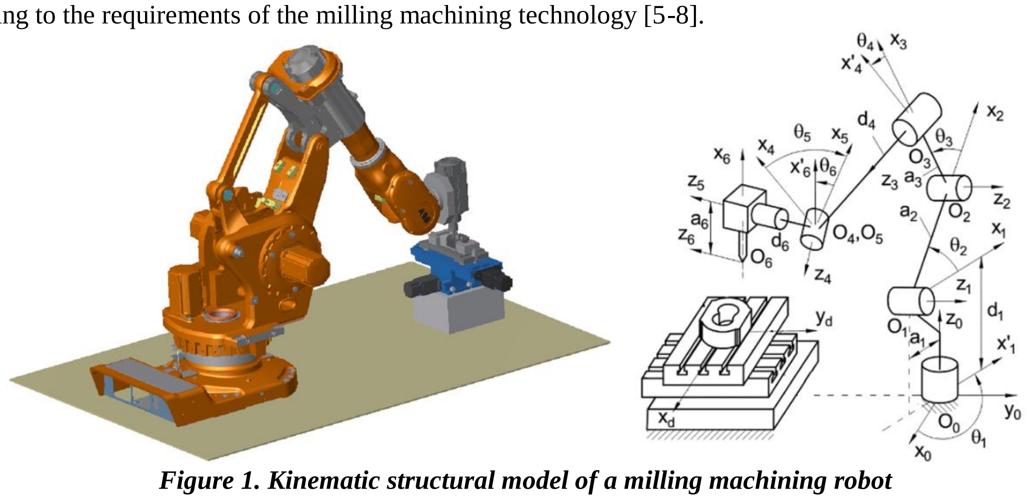


Figure 1. Kinematic structural model of a milling machining robot

Milling to create a shaped surface requires precise control of the position, orientation, velocity, and relative motion between the tool and the workpiece surface. The motion of the tool during milling is a combination of all the component motions of the links and joints, making kinematic analysis and trajectory design difficult and complex.

This paper presents a general method for establishing the equations of motion, performing kinematic analysis to determine the position, velocity, and acceleration of the joints according to the machining technology requirements, and designing motion trajectories for milling machining robots. To illustrate, the method is applied to a six-degree-of-freedom serial robot (modeled after an ABB robot) used in milling, with the tool attached to the operating link (the sixth link) and the workpiece fixed on the machine table.

Following the Introduction, the remaining sections of the paper include Section (2) kinematic analysis of a milling machining robot, Section (3) motion trajectory design for a milling machining robot, Section (4) numerical simulation of motion trajectory design for a milling machining robot, and Section (5) conclusions.

II. Kinematic Analysis of Milling Machine Robot

2.1. Establishment of the Kinematic Equation of Milling Machine Robot

Consider the milling machine robot model shown in Figure 1. To establish the motion equation for the robot, the Denavit-Hartenberg (DH) method is employed. The following coordinate systems are constructed for analysis:

Notation: $\Re_0 = O_0x_0y_0z_0$ - fixed base coordinate system.

$\mathscr{R}_1=O_1x_1y_1z_1$- coordinate system attached to link 1, described in coordinate system $\Re_0$ by the DH homogeneous transformation matrix $^0A_1$

$\mathscr{R}_2=O_2x_2y_2z_2$- coordinate system attached to link 2, described in coordinate system $\mathscr{R}_1$ by the DH homogeneous transformation matrix $^1A_2$, and in coordinate system $\Re_0$ by $^0A_2$

General coordinate system $\mathscr{R}_i=O_ix_iy_iz_i$(i = 1, ..., 6) - coordinate system attached to consecutive link i, described in coordinate system $\mathscr{R}_{i-1}$ by the DH homogeneous transformation matrix $^{i-1}A_i$, and in coordinate system $\Re_0$ by ^0A_i

Here, $\Re_6=O_6x_6y_6z_6$- coordinate system attached to the manipulator link, matrix $^0A_6$ describes the position and orientation of the manipulator link in coordinate system $\Re_0$ according to the robot's kinematic chain

$$^0A_6 = {}^0A_1\,{}^1A_2 ... {}^5A_6 \tag{1}$$

$\Re_E=O_Ex_Ey_Ez_E$- coordinate system representing the tool's cutting edge geometry (tool coordinate system) attached to the cutting point of the tool in contact with the workpiece surface, axis x_E is tangent to the cutting edge, axis z_E is normal to the cutting edge, axis yE forms the right-hand coordinate system OExEyEzE. Matrix ^6A_E describes the position and orientation of the tool relative to coordinate system $\Re_6$, synthesized from the homogeneous transformations, which are translations and rotations corresponding to the Cardan angles ^6x_E, ^6y_E, ^6z_E ,$^6\alpha_E$, $^6\beta_E$, $^6\eta$, respectively. Here, ^6x_E, ^6y_E, ^6z_E ,$^6\alpha_E$, $^6\beta_E$, $^6\eta$are constants dependent on the dimensions and shape of the manipulator link and tool

Matrix ^0A_E represents the position and orientation of the cutting tool in the base coordinate system $\Re_0$ according to the kinematic chain of the robot:

$$^0A_E = {}^0A_1\,{}^1A_2 ... {}^5A_6\,{}^6A_E \tag{2}$$

Joint coordinate vector notation:

$$q = \left[q_1,..,q_6\right]^T = \left[\theta_1,..,\theta_6\right]^T \tag{3}$$

Machine table coordinate system$\mathscr{R}_d=O_dx_dy_dz_d$: attached to the center of gravity above the machine table so that $x_d//x_0$, $y_d//y_0$, $z_d//z_0$, described in coordinate system $\mathscr{R}_0$by the homogeneous transformation matrix ^0A_d

According to the requirements of the surface shaping process, the position and orientation of the tool at each machining time are determined by the shape of the workpiece surface. The position and orientation of the tool relative to coordinate system Rd are synthesized from the homogeneous transformations, which are translations and rotations corresponding to the Cardan angles ^dx_E, ^dy_E, d^6z_E ,$^d\alpha_E$, $^d\beta_E$, $^d\eta_E$., respectively, denoted as:

$$p = \left[p_1, p_2, ..., p_6\right]^T = \left[{}^dx_E, {}^dy_E, {}^dz_E, {}^d\alpha_E, {}^d\beta_E, {}^d\eta_E\right]^T \tag{4}$$

Matrix ^dA_E describes the position and orientation of the tool in coordinate system $\Re_d$ according to vector p:

$$^dA_E = {}^dA_E\left(p\right) \tag{5}$$

Matrix ^dA_E describes the position and orientation of the cutting tool in the base coordinate system $\Re_d$ according to the machining process requirements of the machine table kinematic chain:

$$^0A_E = {}^0A_d\,{}^dA_E \tag{6}$$

Subtracting side by side equations (2) and (5) gives the matrix equation (7)

$$^0A_d\,{}^dA_E = {}^0A_1\,{}^1A_2 ... {}^5A_6\,{}^6A_E \tag{7}$$

From matrix equation (7) and considering (2), the transformation is obtained (8), where (8) represents the position and orientation of the tool in the $\Re_0$ coordinate system according to coordinate q:

$$^dA_E = {}^0A_d^{-1}\,{}^0A_1\,{}^1A_2 ... {}^5A_6\,{}^6A_E = {}^dA_E(q) \tag{8}$$

From matrix equations (5) and (8), the kinematic equation is established in matrix form (9)

$$ {}^{d}A_{E}\left(q\right) = {}^{d}A_{E}\left(p\right) \tag{9}$$

The matrix form kinematic equation (9) allows us to derive a system of 6 independent kinematic equations that constrain the joint coordinates of the joint coordinate vector q to the workpiece coordinate vector p.

$$\begin{cases} f_1(q,p) = {}^{d}A_{E}(q)\left[1,4\right] - {}^{d}A_{E}(p)\left[1,4\right] = 0 \\ f_2(q,p) = {}^{d}A_{E}(q)\left[2,4\right] - {}^{d}A_{E}(p)\left[2,4\right] = 0 \\ f_3(q,p) = {}^{d}A_{E}(q)\left[3,4\right] - {}^{d}A_{E}(p)\left[3,4\right] = 0 \\ f_4(q,p) = {}^{d}A_{E}(q)\left[1,1\right] - {}^{d}A_{E}(p)\left[1,1\right] = 0 \\ f_5(q,p) = {}^{d}A_{E}(q)\left[2,2\right] - {}^{d}A_{E}(p)\left[2,2\right] = 0 \\ f_6(q,p) = {}^{d}A_{E}(q)\left[3,3\right] - {}^{d}A_{E}(p)\left[3,3\right] = 0 \end{cases} \tag{10}$$

2.2. Kinematic Problem of Milling Machine Robot

a. Forward Kinematics

The forward kinematics problem involves determining the tool's motion law when the motion laws of the links (joints) are known from sensors. Forward kinematics provides information for robot control so that the tool can approach the workpiece surface for machining.

At each machining time, the sensors determine the position q, angular velocity $\dot{q}$,, and angular acceleration $\ddot{q}$ of the joint by substituting them into the system of equations (10) and the first and second derivatives of (10) will determine the position-orientation p, velocity $\dot{p}$, and acceleration $\ddot{p}$ of the cutting tool.

b. Inverse Kinematics

The inverse kinematics problem involves determining the motion laws of the links (joints) when the motion law of the tool is known. Inverse kinematics plays a fundamental role in calculating the dynamics and control of robots.

The workpiece surface is defined by a coordinate system characteristic of the workpiece surface (detail coordinate system) $\Re_{fi}= O_{fi}x_{fi}y_{fi}z_{fi}$. Coordinate system Rfi is placed at point Ofi, the contact point between the workpiece surface and the cutting point of the tool, with the x_{fi}-axis tangent to the forming curve (tool path), the vertical z_{fi}-axis normal to the workpiece surface, and the y_{fi}-axis determined by the right-hand coordinate system rule. During surface shaping milling, the $\Re_{fi}$ coordinate system changes along the forming curve. Based on the knowledge of metal cutting in mechanical machining, the forming curve is determined based on two main factors: the workpiece surface and the chosen machining strategy. To form the workpiece surface, the tool coordinate system$\Re_{fi}$must coincide with the detail coordinate system$\Re_{fi}$.

At each machining time according to the technological requirements, the forming motion of the tool with the workpiece surface is determined by the position-orientation vectors p, velocity $\dot{p}$, and acceleration $\ddot{p}$ as (11), (12), (13). Substituting vectors p, $\dot{p}$, $\ddot{p}$into the system of equations (10) and the first and second derivatives of (10) will determine the position vectors q, angular velocity $\dot{q}$, and angular acceleration $\ddot{q}$ of the robot joints as (14), (15), (16).

$$p = \left[p_1, p_2, ..., p_6\right]^T = \left[{}^{d}x_E, {}^{d}y_E, {}^{d}z_E, {}^{d}\alpha_E, {}^{d}\beta_E, {}^{d}\eta_E\right]^T \tag{11}$$

$$\dot{p} = \left[\dot{p}_1, \dot{p}_2, ..., \dot{p}_6\right]^T = \left[{}^{d}\dot{x}_E, {}^{d}\dot{y}_E, {}^{d}\dot{z}_E, {}^{d}\dot{\alpha}_E, {}^{d}\dot{\beta}_E, {}^{d}\dot{\eta}_E\right]^T \tag{12}$$

$$\ddot{p} = \left[\ddot{p}_1, \ddot{p}_2, ..., \ddot{p}_6\right]^T = \left[{}^{d}\ddot{x}_E, {}^{d}\ddot{y}_E, {}^{d}\ddot{z}_E, {}^{d}\ddot{\alpha}_E, {}^{d}\ddot{\beta}_E, {}^{d}\ddot{\eta}_E\right]^T \tag{13}$$

$$q = \left[q_1, .., q_6\right]^T \tag{14}$$

$$\dot{q} = \left[\dot{q}_1, \dot{q}_2, ..., \dot{q}_8\right]^T \tag{15}$$

$$\ddot{q} = \left[\ddot{q}_1, \ddot{q}_2, ..., \ddot{q}_8\right]^T \tag{16}$$

III. Design of Motion Trajectory for Milling Machine Robot

To utilize the milling robot for shaping workpiece surfaces according to technological requirements, it is necessary to design the geometric and kinematic trajectories for the robot so that during machining, the tool achieves the appropriate cutting speed, feed rate, and cutting depth for the material and workpiece surface. During machining, the tool must move in contact with the workpiece surface along the forming curve at the cutting point on the cutting edge. At the same time, the tool must ensure high accuracy of position, orientation, and speed and acceleration relative to the workpiece surface according to the requirements of the technological machining operation.

a. Geometric Trajectory Design:

The design of the geometric trajectory for the milling robot includes two main parts: the design of the geometric trajectory for the tool and the design of the geometric trajectory for the robot.

Design of the Forming Curve (Tool Path) on the Workpiece Surface:The forming curve is the path along which the tool moves on the workpiece surface to create the desired shape. The forming curve is defined based on various factors, including the type of cutting tool, machining method, shape and dimensions of the workpiece, and machining requirements. The tool path can be described in analytical form:

The forming curve can be represented as the coordinates of points as a function of time t in the machine table coordinate system $\Re_d$:

$$\begin{cases} x_{fi} = x_{fi}(t) \\ y_{fi} = y_{fi}(t) \\ z_{fi} = z_{fi}(t) \end{cases} \tag{17}$$

The forming curve can be represented as a function of the coordinates x_{fi}, y_{fi}, z_{fi}in the machine table coordinate system $\Re_d$:

$$f_j(x_{fi}, y_{fi}, z_{fi}) = 0, \quad j = 1,2 \tag{18}$$

The forming curve can be described numerically by a set of points i with x_{fi}, y_{fi}, z_{fi}in the machine table coordinate system Rd. The points i are tabulated. Table 1 describes the order of the points i(x_{fi}, y_{fi}, z_{fi}) on the forming curve i=1,...,n.

Table 1: Tool path parameter table

Point	x_{fi}	y_{fi}	z_{fi}
1	x_{f1}	y_{f1}	z_{f1}
2	x_{f2}	y_{f2}	z_{f2}
…	…	…	..
n	x_{fn}	y_{fn}	z_{fn}

At each machining instant in the machine table coordinate system Rd, the position of the cutting point of the tool is determined by:

$$^{d}x_E = x_{fi}, \quad ^{d}y_E = y_{fi}, \quad ^{d}z_E = z_{fi} \tag{19}$$

To shape the workpiece surface, the tool coordinate system $\Re_E$ must coincide with the detail coordinate system $\Re_{fi}$.. Therefore, at each machining instant corresponding to the cutting points on the forming curve, the orientation of the tool is completely determined by the angles $^{d}\alpha_E = \alpha_{fi}$,$^{d}\beta_E = \beta_{fi}$, $^{d}\eta_E = \beta_{fi}$in the machine table coordinate system $\Re_d$. Thus, the geometric trajectory of the tool (position and orientation of the tool in the machine table coordinate system $\Re_d$ during surface milling) is determined by the geometric characteristics of the forming curve and the workpiece surface as:

$$p = \left[^{d}x_E, ^{d}y_E, ^{d}z_E, ^{d}\alpha_E, ^{d}\beta_E, ^{d}\eta_E \right]^T = \left[x_{fi}, y_{fi}, z_{fi}, \alpha_{fi}, \beta_{fi}, \eta_{fi} \right]^T \tag{20}$$

The matrix A_E describing the position and orientation of the tool in the coordinate system $\Re_d$ in formula (5) is determined for each point on the forming curve represented by:

$$^{d}A_E = {}^{d}A_E(p) \tag{21}$$

Substituting the results calculated in (20), (21) into the system of equations (10) and solving will determine the geometric trajectory of the robot during surface milling of the workpiece (the inverse position kinematics problem is solved).

b. Kinematic Trajectory Design:

The design of the kinematic trajectory for the robot for product shaping plays a crucial role and demands meticulous selection of cutting speed and feed rate to ensure productivity, surface quality, and machining tool life. Cutting speed depends on the relative motion velocity of the cutting tool relative to the workpiece surface along a path that is always tangent to the forming curve. Cutting speed depends on the material type, cutting tool, cutting depth, etc. Cutting speed is determined from the technical requirements of the machining process.

At each machining instant, the cutting speed and cutting acceleration of the tool on the forming curve are determined using analytical or numerical methods. Thus, the kinematic trajectory (cutting speed, cutting acceleration) of the tool is generally calculated by differentiating the position of the tool with respect to time, as expressed in (22), (23).

$$ {}^{d}\dot{x}_{E} = \dot{x}_{fi}(t), \quad {}^{d}\dot{y}_{E} = {}^{d}\dot{y}_{fi}(t), \quad {}^{d}\dot{z}_{E} = {}^{d}\dot{z}_{fi}(t) \tag{22} $$

$$ {}^{d}\ddot{x}_{E} = \ddot{x}_{fi}(t), \quad {}^{d}\ddot{y}_{E} = \ddot{y}_{fi}(t), \quad {}^{d}\ddot{z}_{E} = \ddot{z}_{fi}(t) \tag{23} $$

Substituting (17), (18), (19), (22), (23) into the system of equations (10) and taking the first and second derivatives of (10) and solving will determine the kinematic trajectory (motion law of the links) of the robot over time.

IV. Numerical Simulation for Design of Motion Trajectory for Milling Machine Robot

Design the trajectory for a milling machine robot to process a part made of Ti_6Al_4V material, with the machined surface and dimensions as shown in Figure 2.

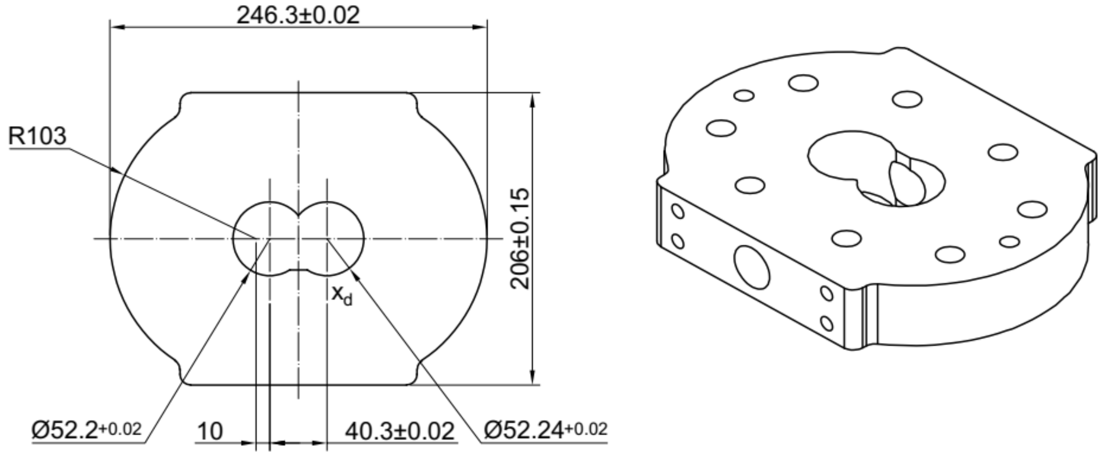
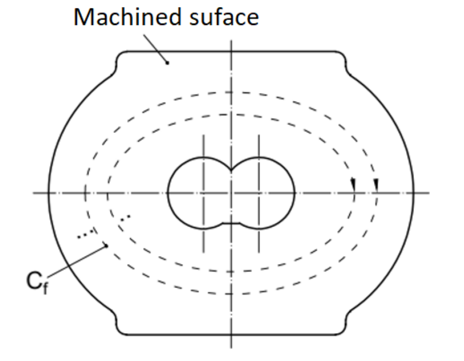


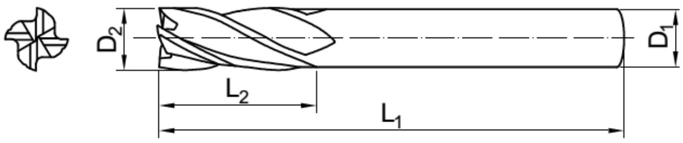
***Figure 2**. Machining details*

Applying the Denavit-Hartenberg rule, the Ri coordinate systems (i=0,6) mentioned above have been established, from which the DH kinematic parameters for the robot have been determined and described in Table 2.

Table 1: Kinematic parameters of the robot

Khâu-Khớp	θ_i (độ)	d_i (mm)	a_i (mm)	α_i (rad)
1	θ_1	d_1 = 514,5	a_1 = 300	$\pi/2$
2	θ_2	0	a_2 = 700	0
3	θ_3	0	a_3 = 280	$-\pi/2$
4	θ_4	d_4 = 1060,24	0	$\pi/2$
5	θ_5	0	0	$-\pi/2$
6	θ_6	d_6 = 377	a_6 = -256	0





a. The tool paths The end mill cutter

Figure 3. The tool paths and the cutter

The machined surface is a numerical mesh surface. Figure 3a shows the forming curve Cf designed on the numerical machined part surface and saved as a data file.

Table 3 describes the kinematic parameter values of the tool and machine table in the coordinate systems A6 and A0.

Table 3. Kinematic parameter values of tool and machine table

^6x_E (mm)	^6y_E (mm)	^6z_E (mm)	$^6\alpha_E$ (rad)	$^6\beta_E$ (rad)	$^6\eta_E$ (rad)	^0x_d (mm)	^0y_d (mm)	^0z_d (mm)	$^0\alpha_d$ (rad)	$^0\beta_d$ (rad)	$^0\eta_d$ (rad)
0	0	0	π	$\pi/2$	0	0	-1361	196,6	0	0	0

A ball-end end mill (Figure 3b) is the cutting tool used for milling and shaping the part surfaces. The technical parameters of the end mill and cutting mode when milling with the robot are presented in detail in Table 4. Table 4 shows the number of teeth of the end mill z, the spindle speed n, the feed per tooth S_z, and the cutting depth h0.

Table 4. Parameter values of ball-end end mill and cutting mode when milling with robot

Cutter's meterial	D_1 (mm)	D_2 (mm)	L_1 (mm)	L_2 (mm)	Z (tooth)	n (vg/ph)	S_z (mm/tooth)	h_0 (mm)	Cooling liquor
Carbide	20	20	125	50	4	1200	0,1	0,5	Emunxi

With the selected tool values, the parameters in Tables 3 and 4, together with the tool forming motion law that has been calculated and saved as a data file mentioned above, are substituted into (11) to calculate the motion law of the robot performing the milling processes.

Substitute the values in Tables 2, 3, 4 and the numerical value of the tool path into the system of equations (10) and the first and second derivatives of the system of equations (10) to find the motion law of the robot when machining and shaping the part surface according to the requirements of the milling engineering.

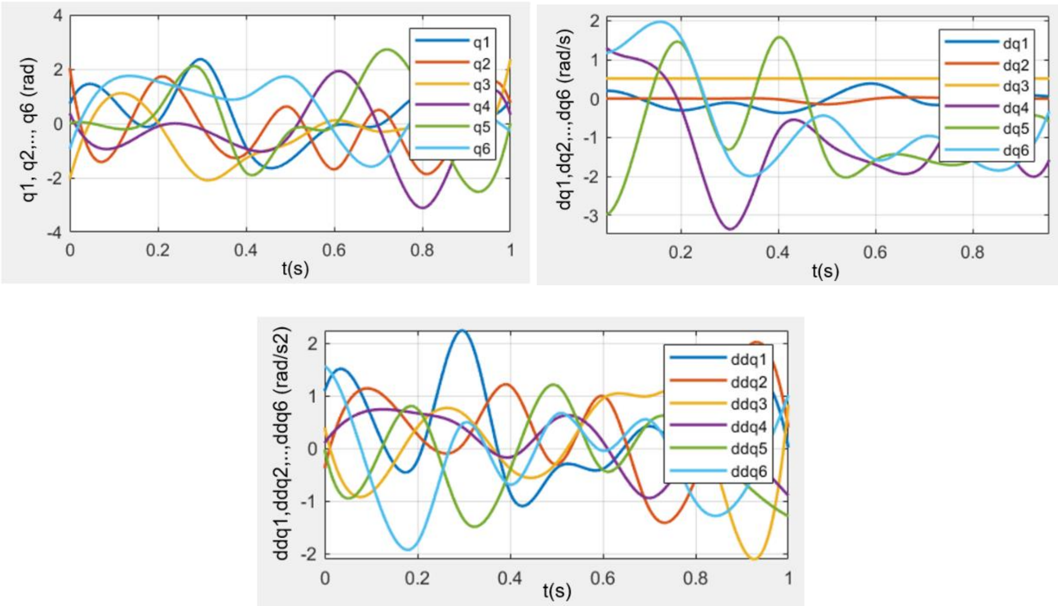


Figure 4. Graph describing the motion law
(Position, velocity, acceleration) of the robot's links - joints

Figure 4 shows the motion law of the links - joints when machining the part surface. Figure 4a shows the position of the joints from q1,..,q6 of the robot; Figure 4b shows the velocity of the joints from $\dot{q}_1,\ldots,\dot{q}_6$ of the robot; Figure 4c shows the angular acceleration $\ddot{q}_1,..,\ddot{q}_6$of the robot.

V. Conclusion

The paper presents a method for designing motion trajectories for milling robots to shape part surfaces, offering several benefits such as improved visibility, more efficient selection of technological operations, and enhanced machining quality and efficiency. The method involves establishing the forming curve, motion law for the robot and tool so that during machining the tool moves relative to the workpiece surface along the forming curve in a way that the tool is always within the serviceable area. The process involves setting up and solving the robot's dynamic equations with the constraints of position, orientation, and velocity of the coordinate system representing the tool placed at the cutting point coinciding with the set of coordinate systems representing the workpiece surface placed at the machining point on the forming curve. With the established dynamic equation and the represented motion law of the tool on the forming curve, the design of the robot's motion trajectory is effectively performed. The computational and simulation results demonstrate the accuracy of the presented method, providing a basis for its application to real-world machining of complex surfaces using robots.

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