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# Synthesis of Quadrotor Control Law Using Modern Control Techniques

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***Abstract:*** *This paper applies the sliding mode control method combined with an extended state observer to synthesize a quadrotor control algorithm for tracking predefined flight trajectories. A mathematical model of the control object is constructed, from which a sliding mode controller for the quadrotor is designed. The controller's performance is validated and evaluated in cases with and without disturbances through simulations, showing that the controller responds well to the desired inputs.*
***Keywords:*** *Flying equipment, Sliding mode control, Extended state observer, Unmanned aerial vehicle, Trajectory.*

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## I. Introduction

Currently, unmanned aerial vehicles (UAVs) have been increasingly applied in various fields such as military, transportation, cargo delivery, agriculture, and search and rescue. Among them, the quadrotor is one of the most common configurations. This type of UAV uses four symmetrically arranged propeller motors, with a simple design, flexible maneuverability, and efficient operation in confined spaces. However, controlling such a system poses many technical challenges.

Quadrotor systems often face complex issues such as self-oscillations, external disturbances like wind or environmental changes, as well as model uncertainties and sensor errors. Operating in a tropical monsoon environment also causes the electronic components in the control system to degrade, reducing the system's reliability and efficiency. Therefore, it is necessary to design a control system with high reliability, good adaptability, and sufficient robustness to maintain stable quadrotor operation under various conditions.

Controllers for quadrotors typically require multiple sensors to compare reference signals with actual signals in order to generate appropriate control inputs. This increases the system's complexity and cost.Controllers based on Lyapunov function design ensure the stability of desired trajectories along the (X, Z) axes and tilt angles[1].PID and LQR controllers have been designed for altitude stabilization[2].While DC actuators with adaptive controllers have been employed for motion stabilization[3].However, under strong disturbances, these controllers become nonlinear, and the closed-loop stability of the system can only be achieved within a small region around the equilibrium point.Fuzzy logic control and neural networks have been used to ensure Euler angles converge to zero during motion[4].While sliding mode and higher-order sliding mode controllers[5]and[6]are applied to estimate unmeasured state variables and external disturbances such as wind or noise.Nevertheless, these controllers require heavy and complex computations.

From these considerations, this paper proposes a method for constructing and simulating a quadrotor control system using sliding mode control (SMC) combined with an extended state observer (ESO). SMC helps the system maintain stability under disturbances and uncertainties, while the ESO estimates unknown states and aggregated disturbances affecting the system, thus supporting the control process. This control structure is well-suited to the characteristics of quadrotor systems operating in real environments.

## II. Control Algorithm Synthesis

To build the quadrotor model, several simplified assumptions are made to describe the system as accurately as possible. In this paper, the dynamic model of the quadrotor is constructed based on the following assumptions[4]-[7]:

Assumption 1: The quadrotor frame and propellers are rigid and perfectly symmetrical.

Assumption 2: Rotor dynamics can be neglected.

Assumption 3: Ground effect is ignored.

Assumption 4: Moments around the axes and thrust forces are generated by the rotor speed, proportional to the square of the rotor angular velocity.

The functional block diagram of the control system is illustrated in Figure 1, which represents a structure with multiple interconnected control loops. Each loop is designed to control and adjust a specific flight parameter of the quadrotor. The outer loop focuses on position control, while the inner loop controls the rotational angles.

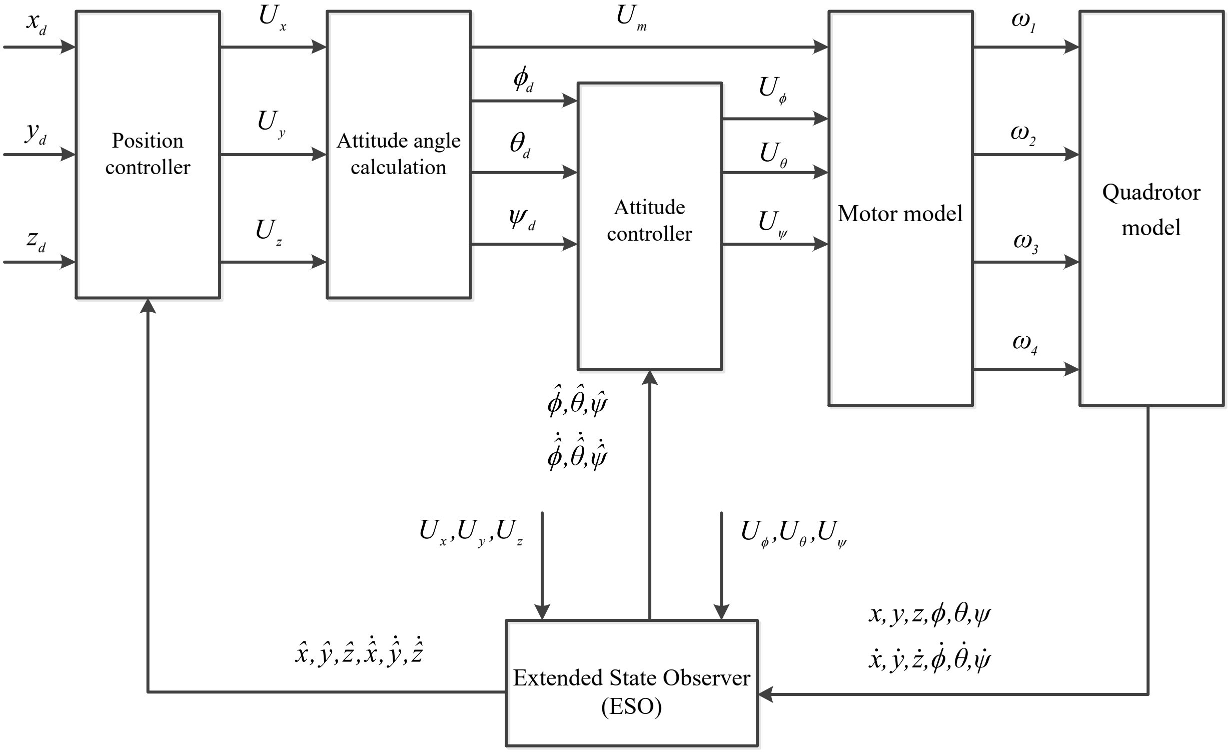



Figure 1. Quadrotor control system

### 2.1. Sliding mode control

The control technique applied to the quadrotor is SMC, which is well known for its strong robustness and good disturbance rejection. The basic principle of this method is to design a sliding surface and then construct a control law so that the system's state trajectory reaches and remains on this surface until it converges to the equilibrium point. The proposed approach replaces the discontinuous sign function with continuous approximations such as the saturation function.

A nonlinear system of ordern with a single input can be represented as:

$$\boldsymbol{x}^{(n)}(t) = \boldsymbol{f}(x,t) + \boldsymbol{b}(x,t).\boldsymbol{u}(t) + \boldsymbol{d}(x,t) \quad (1)$$

Where: $x(t) = \left[ x(t)\ \dot{x}(t)\ \ddot{x}(t) \ldots x^{(n-1)}(t) \right]^T$ -is the state vector, $u(t)$ -is the control input, $d(\boldsymbol{x},t)$ -is bounded uncertainty and disturbance, $f(\boldsymbol{x},t)$ and $b(\boldsymbol{x},t)$ -are known nonlinear functions, $\boldsymbol{e}(t) \in R^{nx1}$ -is the error,be determined(2):

$$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_d(t) \quad (2)$$

Where: $\boldsymbol{x}_d(t)$ -is the desired trajectory.

For a second-order system $\boldsymbol{e}(t) = \left[ e(t)\ \dot{e}(t) \right]^T$ ,the sliding surface is defined as:

$$s(\boldsymbol{e}) = \dot{e}(t) + C_1 e(t) \quad (3)$$

Where: $C_1 > 0$ -determines the convergence speed of the error toward zero.

The system's stability is analyzed using Lyapunov stability theory. The target is to design a control law ensuring that the error converges asymptotically to zero. The Lyapunov function is selected as:

$$V(s) = \frac{1}{2} s^2(\mathbf{e}) \quad (4)$$

Satisfied $V(0) = 0,\ V(s) > 0\ \forall s(e) > 0$ .

The condition for system stability is $\dot{V}(s) < 0$ , accordingly[8],the control signal can be chosen as:

$$U = -b^{-1}[\hat{f}(\boldsymbol{x},t) - \ddot{x}_d(t) + C_1\dot{e}(t)] + k\,sign(s(\boldsymbol{e})) = U_{eq} + U_{dis} \quad (5)$$

Where, $\hat{f}(\boldsymbol{x},t)$ is the estimated nonlinear function $f(\boldsymbol{x},t)$, $k > 0$ determine the gain of the discontinuous component.

The sign(s) function is defined:

$$sign(s) = \begin{cases} -1 & if \quad s < 0 \\ 1 & if \quad s > 0 \end{cases} \quad (6)$$

The control signal in(5)consists of two components.The first component, $U_{eq}$ ,is continuous, based on the estimated parameters of the system to compensate for undesirable dynamic characteristics such as disturbances and model uncertainties.The second component, $U_{dis}$ is discontinuous, containing a sign function that forces the trajectory to converge to the sliding surface.

**2.2. Extended state observer**

Controllers for quadrotors typically require multiple sensors to compare reference and actual signals in order to generate appropriate control inputs. This increases the complexity and cost of the system. To overcome this, an ESO is proposed to estimate the system states in place of some sensors. When combined SMC, it ensures that the quadrotor can track the desired trajectory and maintain stability under small disturbances.

The ESO is used to estimate the system states as well as the total effects of external disturbances and uncertainties. The dynamic model of the quadrotor, when considering environmental disturbances, can be expressed as:

$$\ddot{\chi} = a + bu + d \quad (7)$$

Where: $\chi = \{x, y, z, \phi, \theta, \psi\}$, $a = \{a_x, a_y, a_z, a_\phi, a_\theta, a_\psi\}$, $b = \{b_x, b_y, b_z, b_\phi, b_\theta, b_\psi\}$, $u = \{U_x, U_y, U_z, U_\phi, U_\theta, U_\psi\}$, $d = \{d_x, d_y, d_z, d_\phi, d_\theta, d_\psi\}$.

To observe the states and velocities of the quadrotor, according to[9],the observer is designed as follows:

$$\begin{cases} \dot{\hat{\chi}}_1 = \hat{\chi}_2 + \dfrac{\alpha_1}{\varepsilon}(\chi_1 - \hat{\chi}_1) \\ \dot{\hat{\chi}}_2 = bu + \hat{d} + \dfrac{\alpha_2}{\varepsilon^2}(\chi_1 - \hat{\chi}_1) \\ \dot{\hat{d}} = \dfrac{\alpha_3}{\varepsilon^3}(\chi_1 - \hat{\chi}_1) \end{cases} \quad (8)$$

With $\chi_1 \in \{x, y, z, \phi, \theta, \psi\}$ ,the purpose of the observer is to ensure that the observed values converge to the actual values $\hat{\chi}_1 \to 1, \hat{\chi}_2 \to \hat{\chi}_2, \hat{d} \to d$ as $t \to \infty$ . Here, $\hat{\chi}_1, \hat{\chi}_2$ represents the estimated states of the observer, $\hat{d}$ denotes the extended states of the observer, $\varepsilon > 0$ , $\alpha_1, \alpha_2, \alpha_3 > 0$ and satisfies the Hurwitz stability condition of the polynomial $S^3 + \alpha_1 S^2 + \alpha_2 S + \alpha_3$ .

The observation errors are defined as::

$$\eta = [\eta_1 \;\; \eta_2 \;\; \eta_3]^T \quad (9)$$

Where: $\eta_1 = \dfrac{\chi_1 - \hat{\chi}_1}{\varepsilon^2}$ ; $\eta_2 = \dfrac{\chi_2 - \hat{\chi}_2}{\varepsilon}$ ; $\eta_3 = d - \hat{d}$

From (8) and(9),we obtain:

$$\varepsilon\dot{\eta}_1 = \frac{\dot{\chi}_1 - \dot{\hat{\chi}}_1}{\varepsilon} = \frac{1}{\varepsilon}\left(\chi_2 - \left(\hat{\chi}_2 + \frac{\alpha_1}{\varepsilon}(\chi_1 - \hat{\chi}_1)\right)\right) = -\frac{\alpha_1}{\varepsilon^2}(\chi_1 - \hat{\chi}_1) + \frac{1}{\varepsilon}(\chi_2 - \hat{\chi}_2) = -\alpha_1\eta_1 + \eta_2$$

$$\varepsilon\dot{\eta}_2 = \varepsilon\frac{\dot{\chi}_2 - \dot{\hat{\chi}}_2}{\varepsilon} = bu + d - \left(bu + \hat{d} + \frac{\alpha_2}{\varepsilon^2}(\chi_1 - \hat{\chi}_1)\right) = -\frac{\alpha_2}{\varepsilon^2}(\chi_1 - \hat{\chi}_1) + (d - \hat{d}) = -\alpha_2\eta_1 + \eta_3$$

$$\varepsilon\dot{\eta}_3 = \varepsilon(\dot{d} - \dot{\hat{d}}) = -\frac{\alpha_3}{\varepsilon^2}(\chi_1 - \hat{\chi}_1) + \varepsilon\dot{d} = -\alpha_3\eta_1 + \varepsilon\dot{d}$$

Thus, the system observation error can be expressed as:

$$\varepsilon\dot{\eta} = \boldsymbol{A}\eta + \varepsilon\boldsymbol{B}\dot{d} \quad (10)$$

Where: $\boldsymbol{A} = \begin{bmatrix} -\alpha_1 & 1 & 0 \\ -\alpha_2 & 0 & 1 \\ -\alpha_3 & 0 & 0 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Based on Lyapunov stability theory[10], for any positive definite symmetric matrix $\mathbf{M} \in R^{3\times3}$ , there exists a unique positive definite symmetric matrix $\mathbf{N} \in R^{3\times3}$ that satisfies the Lyapunov equation as follows:

$$\boldsymbol{A^T N + NA + M = 0} \qquad (11)$$

The Lyapunov function is chosen as:

$$V_{ESO} = \varepsilon\eta^T \boldsymbol{N}\eta \qquad (12)$$

$$\begin{aligned}\dot{V}_{ESO} &= \varepsilon\dot{\eta}^T \mathbf{N}\eta + \varepsilon\eta^T \mathbf{N}\dot{\eta} = (\mathbf{A}\eta + \varepsilon\mathbf{B}\dot{d})^T \mathbf{N}\eta + \eta^T \mathbf{N}(\mathbf{A}\eta + \varepsilon\mathbf{B}\dot{d}) \\ &= \eta^T \mathbf{A}^T \mathbf{N}\eta + \varepsilon(\mathbf{B}\dot{d})^T + \eta^T \mathbf{N}\mathbf{A}\eta + \varepsilon\eta^T \mathbf{B}\dot{d} = \eta^T(\mathbf{A}^T\mathbf{N} + \mathbf{N}\mathbf{A})\eta + 2\varepsilon\eta^T \mathbf{N}\mathbf{B}\dot{d} \qquad (13) \\ &\le -\eta^T \mathbf{M}\eta + 2\varepsilon \|\mathbf{N}\mathbf{B}\|.\|\eta\|.\|\dot{d}\|\end{aligned}$$

Since $\left|\dot{d}\right| = L$ ,equation(13)can be rewritten as:

$$\dot{V}_{ESO} \le -\lambda_{\min}(\mathbf{M})\|\eta\|^2 + 2\varepsilon L\|\mathbf{N}\mathbf{B}\|.\|\eta\| \qquad (14)$$

Where: $\lambda_{min}(\mathbf{M})$ - the minimum eigenvalue ofM and with $\dot{V}_{ESO} \le 0$ , the observation error is bounded as follows:

$$\|\eta\| \le \frac{2\varepsilon L\|\mathbf{N}\mathbf{B}\|}{\lambda_{\min}(\mathbf{M})} \qquad (15)$$

From(15), it can be observed that the convergence of the observer error $\eta$ depends on $\varepsilon$ . If $\varepsilon$ is designed as a sufficiently small positive value, then the observer error $\eta$ will asymptotically converge.

### 2.3. Position control loop

The controller will drive the quadrotor to follow the desired trajectory $\{x_d, y_d, z_d\}$ , and the trajectory tracking errors are defined as:

$$e_x = x_d - x;\ e_y = y_d - y;\ e_z = z_d - z \qquad (16)$$

Where: $x_d, y_d, z_d$ -the desired trajectory of the quadrotor.

Since the design process of the position tracking controller is similar for each axis, in this paper the controller is synthesized for thex-axis. The sliding surface and the Lyapunov function are defined as follows:

$$s_x = \dot{e}_x + c_x e_x \qquad (17)$$

$$V_x = \frac{1}{2}s_x^2 \qquad (18)$$

Where: $c_x$ -positive definite control parameter.

Taking the derivative of(17), we obtain:

$$\dot{s}_x = c_x e_x + \ddot{e}_x = c_x\dot{e}_x + \ddot{x}_d - \frac{U_x}{m} - d_x \qquad (19)$$

Taking the derivative of(18),we obtain:

$$\dot{V}_x = s_x\dot{s}_x = s_x(c_x\dot{e}_x + \ddot{e}_x) = s_x(c_x\dot{e}_x + \ddot{x}_d - \frac{U_x}{m} - d_x) \qquad (20)$$

Based on the exponential law and to reduce the chattering phenomenon, the saturation functionsat() is applied instead of the sign function sign()[9]:

$$\dot{s}_x = -n_x sat(s_x) - k_x s_x \qquad (21)$$

Where$n_x$and $k_x$are positive definite control parameters, and the saturation function sat() is defined as:

$$sat(s) = \begin{cases} -1\ , & s < -1 \\ s\ , & |s| < 1 \\ 1\ , & s > 1 \end{cases} \qquad (22)$$

When using the state observer(8), the state variables of the system are estimated as follows:

$$\begin{cases} \hat{e}_x = x_d - x_1 \\ \dot{\hat{e}}_x = \dot{x}_d - \hat{x}_2 \\ \hat{s}_x = c_x\hat{e}_x - \dot{\hat{e}}_x \end{cases} \qquad (23)$$

The control signal is designed as follows:

$$U_x = m(-\hat{d} + \ddot{x}_d + c_x\dot{\hat{e}}_x + n_x sat(\hat{s}_x) + k_x\hat{s}_x) \qquad (24)$$

Where: $\hat{d}_x, \hat{e}_x, \hat{s}_x$ are respectively the estimated values of $d_x, e_x$ and $s_x$ .

Substituting(24)into(20),we obtain:

$$\begin{aligned}\dot{V}_x &= s_x[c_x\dot{e}_x + \ddot{x}_d - (-\hat{d}_x + \ddot{x}_d + c_x\dot{\hat{e}}_x + n_x sat(\hat{s}_x) + k_x\hat{s}_x) - d_x] \\ &= s_x[c_x(\dot{e}_x - \dot{\hat{e}}_x) - n_x sat(\hat{s}_x) - k_x\hat{s}_x + \hat{d}_x - d_x] \\ &= -(\hat{s}_x + s_x - \hat{s}_x)(n_x sat(\hat{s}_x) + k_x\hat{s}_x) + (\hat{s}_x + s_x - \hat{s}_x)c_x(\dot{e}_x - \dot{\hat{e}}_x) \\ &\quad + (\hat{s}_x + s_x - \hat{s}_x)(\hat{d}_x - d_x) \\ &\le -\hat{s}_x(n_x sat(\hat{s}_x) + k_x\hat{s}_x) + |s_x - \hat{s}_x|(n_x sat(\hat{s}_x) + k_x\hat{s}_x) \\ &\quad + (|\hat{d}_x - d_x| + c_x|\dot{e}_x - \dot{\hat{e}}_x|)|\hat{s}_x| + |s_x - \hat{s}_x|(|\hat{d}_x - d_x| + c_x|\dot{e}_x - \dot{\hat{e}}_x|) \\ &= -\hat{s}_x(n_x sat(\hat{s}_x) + k_x\hat{s}_x) + |s_x - \hat{s}_x|(n_x sat(\hat{s}_x) + k_x\hat{s}_x) \\ &\quad + (|\hat{d}_x - d_x| + c_x|\dot{e}_x - \dot{\hat{e}}_x|)|\hat{s}_x| + |s_x - \hat{s}_x|(|\hat{d}_x - d_x| + c_x|\dot{x} - \hat{x}_2|)\end{aligned} \quad (25)$$

Owing to the convergence of the extended state observer, $|s_x - \hat{s}_x|, |d_x - \hat{d}_x|, |\dot{x} - \hat{x}_2|$ remains bounded and sufficiently small, thus $\dot{V} \le 0$.

The control signals for the y- and z-positions can be designed using a similar approach:

$$U_y = m(-\hat{d} + \ddot{y}_d + c_y\dot{\hat{e}}_y + n_y sat(\hat{s}_y) + k_y\hat{s}_y) \quad (26)$$

$$U_z = m(-\hat{d} + \ddot{z}_d + c_z\dot{\hat{e}}_z + n_z sat(\hat{s}_z) + k_z\hat{s}_z) \quad (27)$$

Where $c_x, c_y, c_z$ denotes positive constants.

In the quadrotor control system, there are only four input control signals, while six outputs need to be controlled. Therefore, intermediate control signals $U_x, U_y, U_z$ are required to transform them into the total thrust and the desired tilt angles $\theta_d, \phi_d$ .Based on the lift force equations of the four motors in the body-fixed reference frame[1]-[2],these values are determined as follows:

$$\begin{cases} U_m = \sqrt{U_x^2 + U_y^2 + U_z^2} \\ \theta_d = \arctan\left(\dfrac{U_x \cos\psi_d + U_y \sin\psi_d}{U_z}\right) \\ \phi_d = \arctan\left(\dfrac{U_x \sin\psi_d - U_y \cos\psi_d}{U_z} \cos\theta_d\right) \end{cases} \quad (28)$$

**2.4. Attitude control loop**

With the desired yaw angle set either by the operator or by a predefined program, the roll and pitch angles are determined from(28).The attitude sliding mode controllers are then designed in a similar manner to the controllers in the position control loop:

$$\begin{cases} U_\phi = I_{xx}(-\hat{d}_\phi + \ddot{\phi}_d + c_\phi\dot{\hat{e}}_\phi + n_\phi sat(\hat{s}_\phi) + k_\phi\hat{s}_\phi) \\ U_\theta = I_{yy}(-\hat{d}_\theta + \ddot{\theta}_d + c_\theta\dot{\hat{e}}_\theta + n_\theta sat(\hat{s}_\theta) + k_\theta\hat{s}_\theta) \\ U_\psi = I_{zz}(-\hat{d}_\psi + \ddot{\psi}_d + c_\psi\dot{\hat{e}}_\psi + n_\psi sat(\hat{s}_\psi) + k_\psi\hat{s}_\psi) \end{cases} \quad (29)$$

Where: $c_\phi$, $c_\theta$, $c_\psi$, $k_\Phi$, $k_\theta$, $k_\psi$, $n_\phi$, $n_\theta$, $n_\psi$ are positive constants.

## III. Simulation and performance evaluation

To evaluate the sliding mode controller combined with the extended state observer, a simulation model is constructed in the MATLAB-Simulink environment.

Quadrotor parameters:
Mass: m =1,776 (kg)
Gravity: g = 9,81(m/s$^2$)
Thrust coefficient: $k_T$= 0.0087 (N.s$^2$)
Drag coefficient: $k_d$= 55.10$^{-6}$ (N.m.s$^2$)
Length: L = 0,225 (m)
Moment of inertia about x-axis: $I_{xx}$= 0,0035 (kg.m$^2$)
Moment of inertia abouty-axis: $I_{yy}$= 0,0035 (kg.m$^2$)
Moment of inertia aboutz-axis: $I_{zz}$= 0,0055 (kg.m$^2$)
Propeller inertia: J = 2,8.10$^{-6}$ (kg.m$^2$)
Drag coefficients: $k_{cx} = k_{cy} = k_{cz}$= 5,567.10$^{-4}$ (N.s/m); $k_{d\Phi}$=$k_{d\theta}$=$k_{d\psi}$= 5,567.10$^{-4}$ (N.m.s$^2$)

The parameters of the controller: $c_x = c_y = c_z = 1{,}55$; $c_\phi = c_\theta = c_\psi = 10$; $k_x = k_y = k_z = 0{,}75$; $k_\phi = k_\theta = k_\psi = 100$; $n_x = n_y = n_z = 1$; $n_\phi = n_\theta = n_\psi = 1$.

To simplify the process of analysis and controller performance evaluation, the reference trajectory is chosen in the form of a spiral:

$x_d$ = 5sin(0.25t) (m);

$y_d$ = 5cos(0.25t) (m);

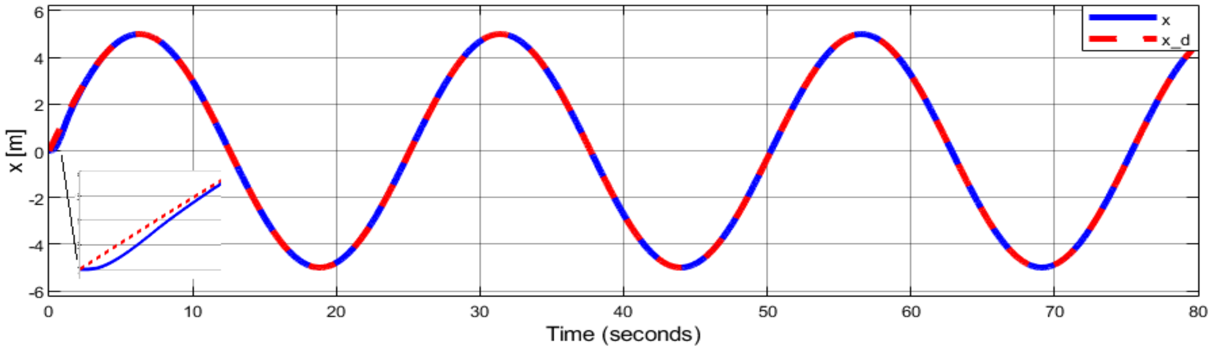
$z_d$ = 0.25t (m);

The psi angle is predefined as:

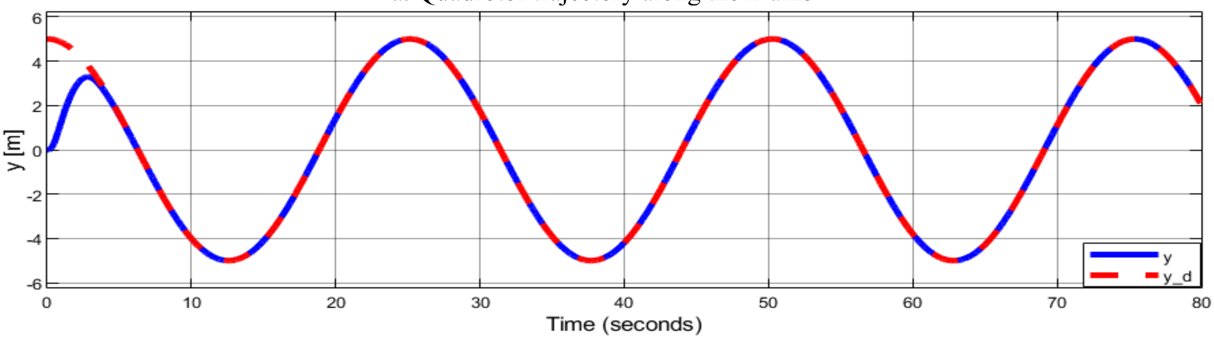
$\psi_d = \pi/8$ (rad).

### 3.1. Simulation case without external disturbances

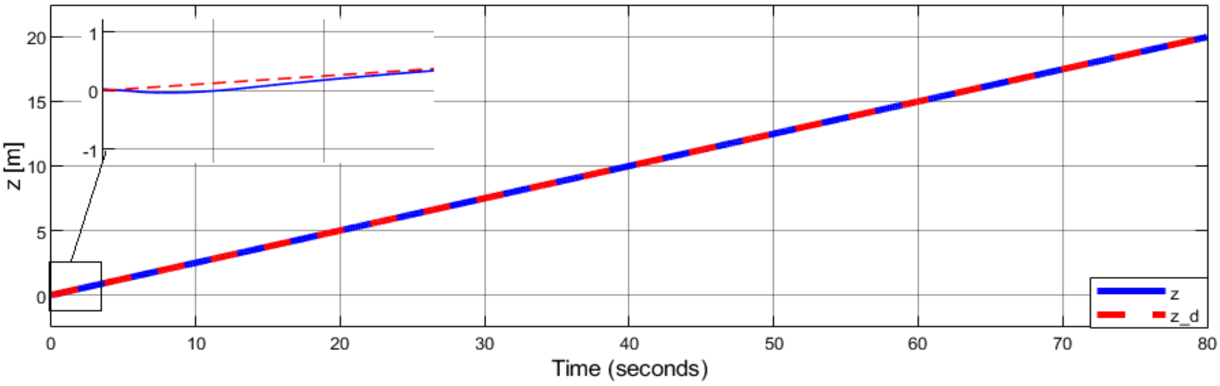
In figure 2, the quadrotor's position along the x-axis, controlled by SMC, converges to the desired position within less than 1 second. The actual trajectory closely follows the reference trajectory throughout the entire process. The position along the y-axis is also effectively controlled by SMC, with the system reaching the reference trajectory within approximately 3,5 seconds. Although an initial error appears, the controller successfully adjusts the quadrotor to track the reference trajectory closely for the remainder of the time. The position along the z-axis approaches the desired value within less than 1 second under the effect of the SMC controller. The actual trajectory remains stable, exhibits no overshoot, and continuously follows the reference trajectory.




a. Quadrotor trajectory along the x-axis



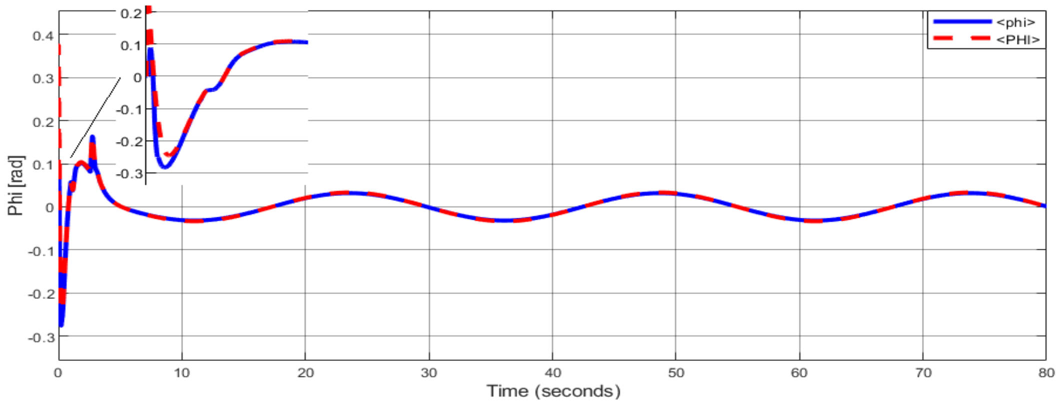

b.Quadrotor trajectory along the y-axis



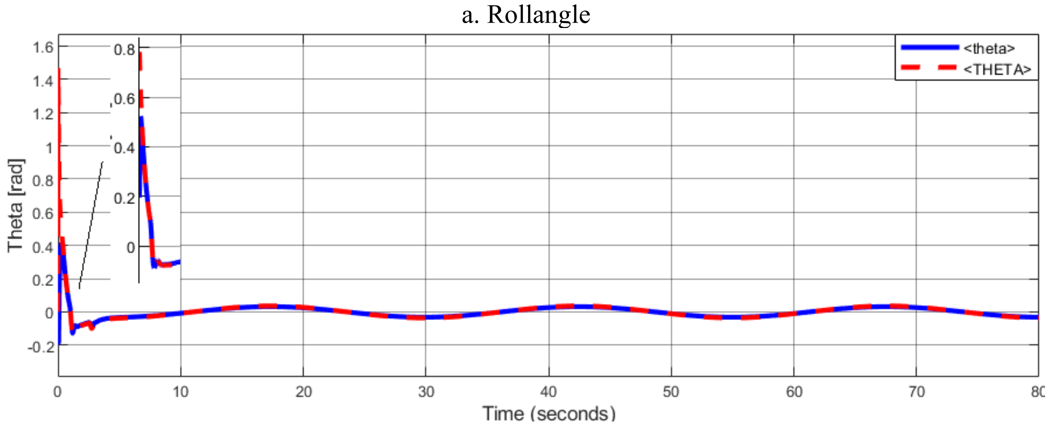

c.Quadrotor trajectory along the z-axis

Figure 2. Quadrotor trajectory along the coordinate axes

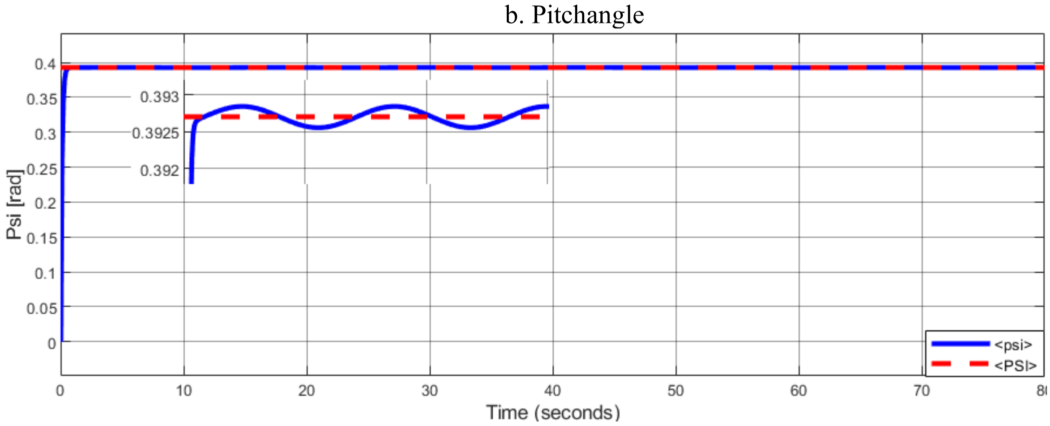
In Figure 3, the reference attitude angles and the actual measured angles are shown throughout the quadrotor's operation. For roll and pitch tracking, although initial errors occur, the SMC enables the quadrotor to follow the reference angles within 0,4 seconds. In yaw tracking, the SMC demonstrates effectiveness as the yaw angle oscillations are very small and no overshoot is observed.




a. Rollangle




b. Pitchangle




c. Yawangle

Figure 3. Quadrotor attitude angles

From the results in figures 2 and 3, it can be seen that the controller designed using the SMC method effectively ensures that the quadrotor tracks the reference trajectory. The spiral reference trajectory and the actual trajectory of the quadrotor in space are illustrated in figure 4.

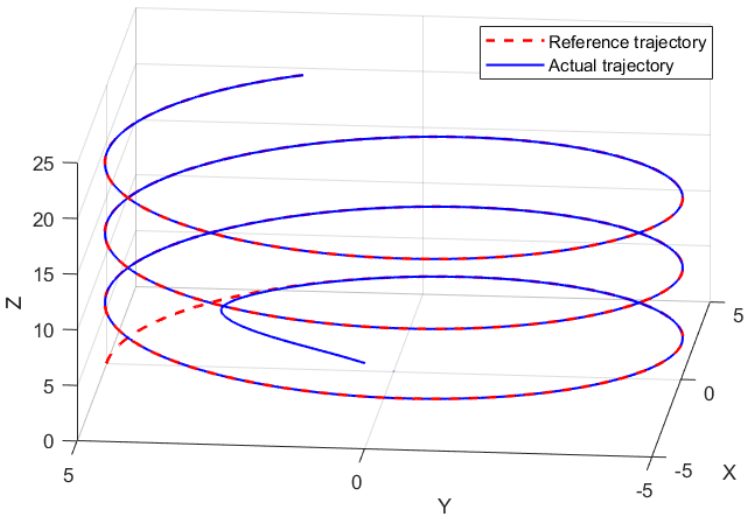



Figure 4. Quadrotor trajectory in space

### 3.2. Simulation case under external disturbances

The disturbances acting on the translational and rotational motions of the quadrotor are assumed to have the following form:

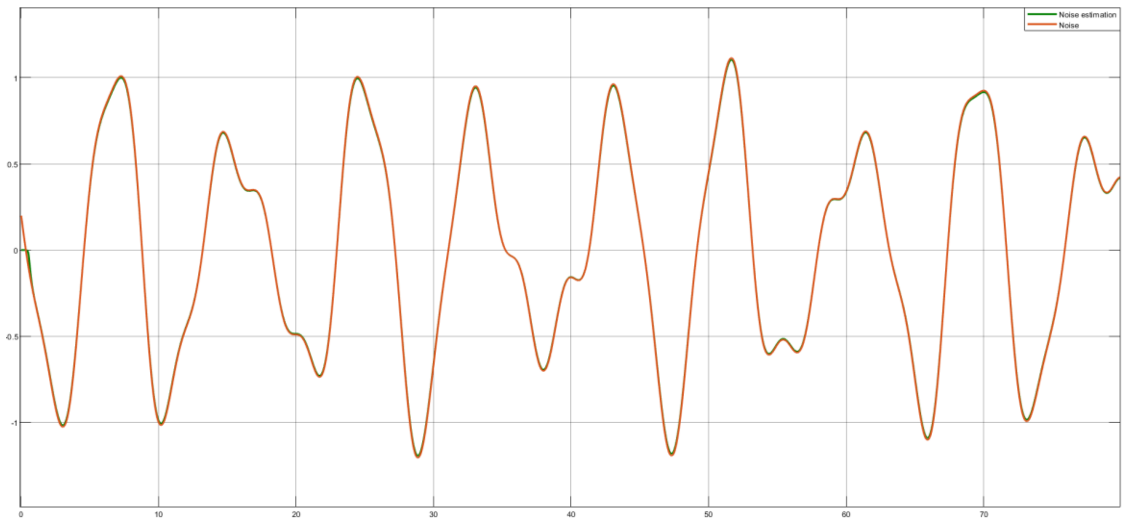
$D(t) = 0.8\sin(0.7t) + 0.3\sin(t - 2) + 0.15\sin(2t + 1) + 0.055\sin(\pi/2t + 5))$

$D_x = D_y = D_z = -2D(t)$; $D_\Phi = D_\theta = D_\psi = 0.2D(t)$

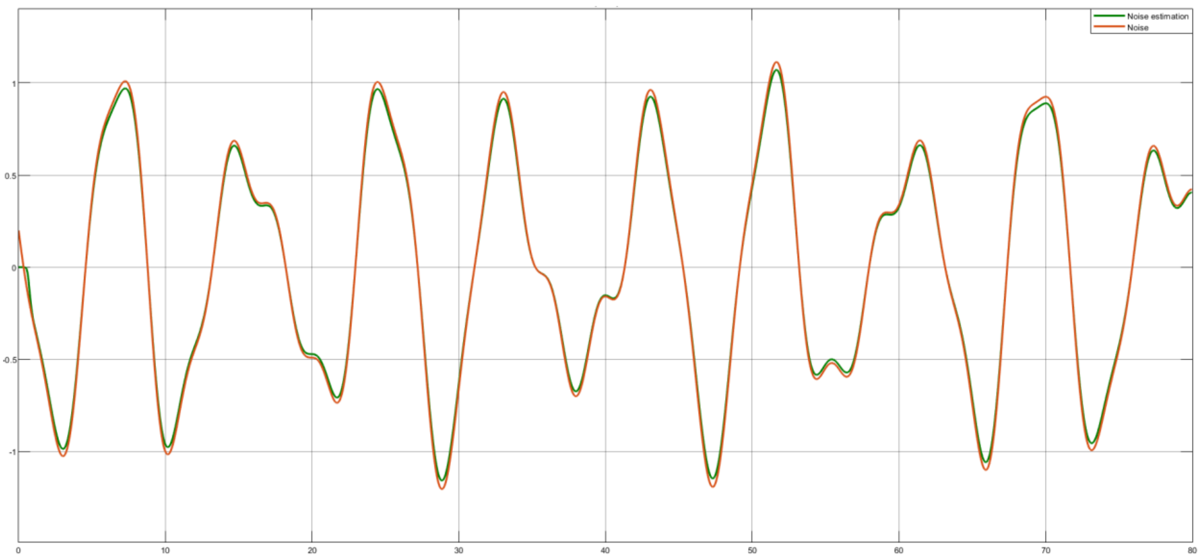
The parameters of the extended state observer:

$\alpha_{1x} = 3$, $\alpha_{1y} = 2$, $\alpha_{1z} = 2$, $\alpha_{1\phi} = 3$, $\alpha_{1\theta} = 2$, $\alpha_{1\psi} = 2$; $\alpha_{2x} = 3$, $\alpha_{2y} = 2$, $\alpha_{2z} = 2$, $\alpha_{2\phi} = 3$ $\alpha_{2\theta} = 2$, $\alpha_{2\psi} = 2$; $\alpha_{3x} = 1$, $\alpha_{3y} = 0,5$, $\alpha_{3z} = 0,5$, $\alpha_{3\phi} = 1$, $\alpha_{3\theta} = 0,5$, $\alpha_{3\psi} = 0,5$.

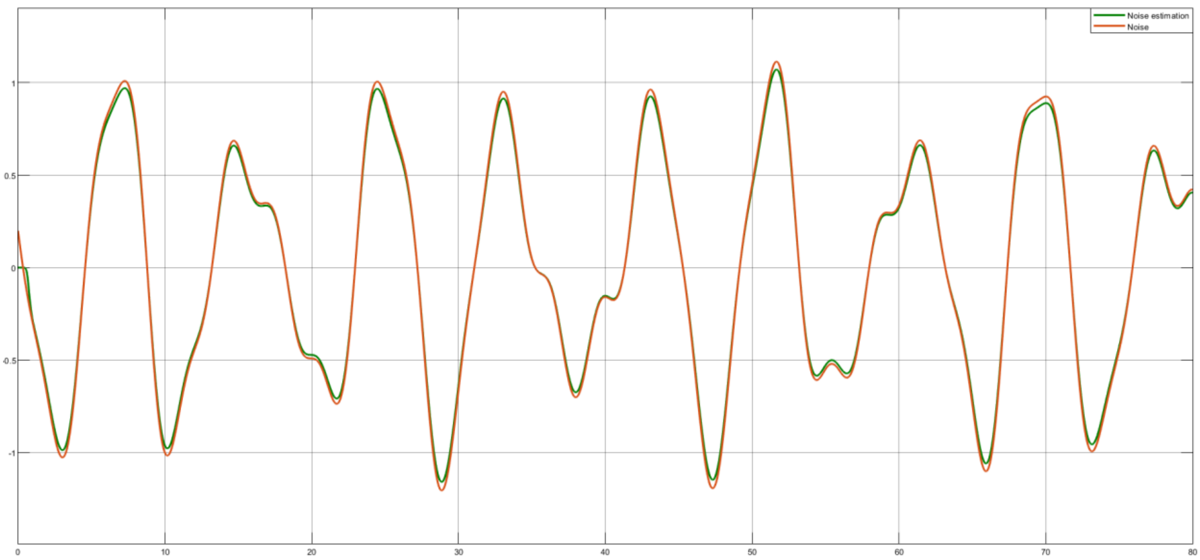
Figure 5 illustrates the external disturbances acting along the axes and the disturbances estimated by the extended state observer. The ESO estimates the disturbances quite accurately with errors nearly equal to zero, where the estimation errors along the x- and y-axes are smaller than 0,003 m/s², while the error along the z-axis is approximately 0,04 m/s².




a.Disturbance and estimated disturbance along the x-axis



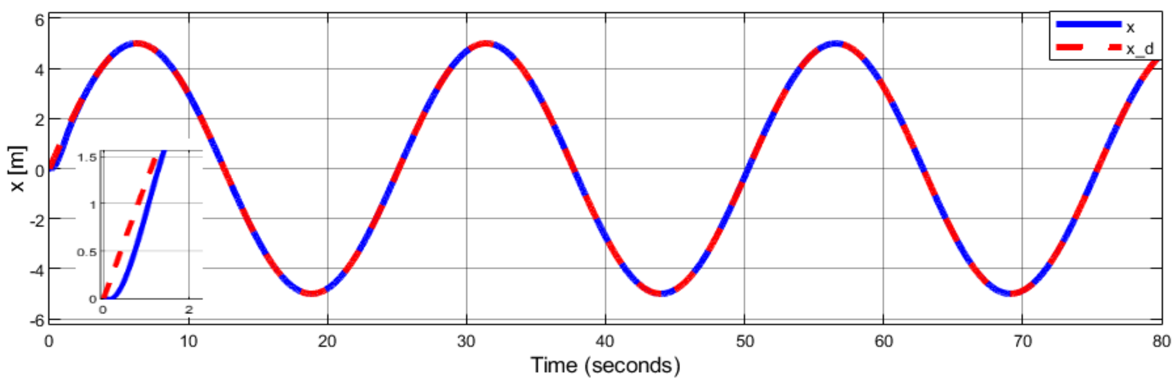

b. Disturbance and estimated disturbance along the y-axis



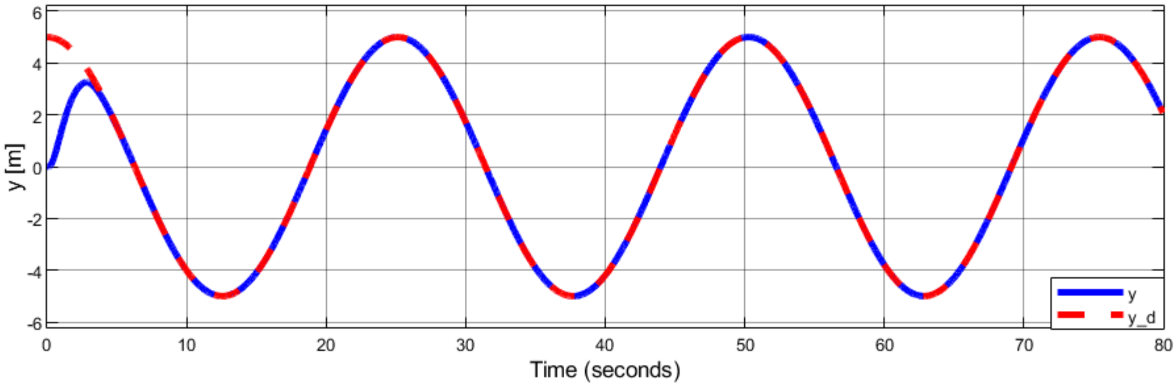

c.Disturbance and estimated disturbance along the z-axis

Figure 5. Disturbances and estimated disturbances acting on translational motion

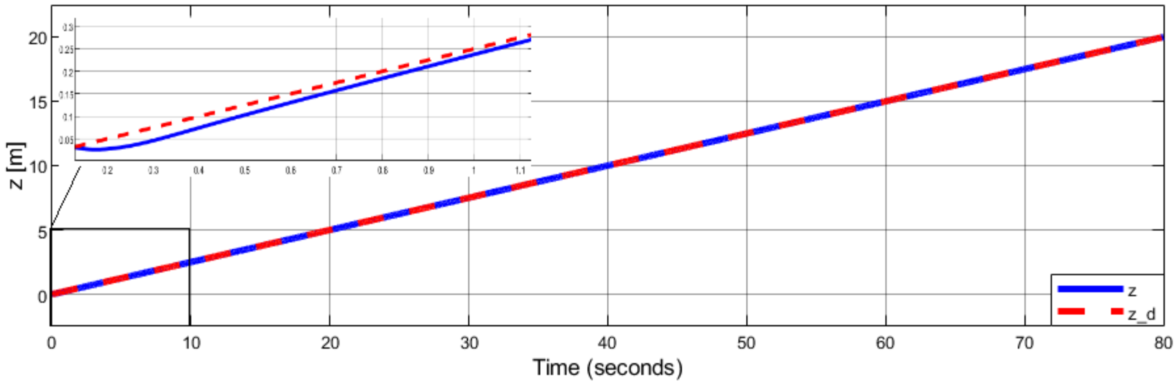
Figure 6 illustrates the process of tracking the predefined trajectory along the coordinate axes. In figure 6a, the controller drives the quadrotor to follow the desired position along the x-axis within 1,8 seconds. In figure 6b, the position along the y-axis reaches the reference value in less than 3 seconds. Along the z-axis, the position converges to the desired value within 0,5 seconds.




a. Quadrotor trajectory along the x-axis



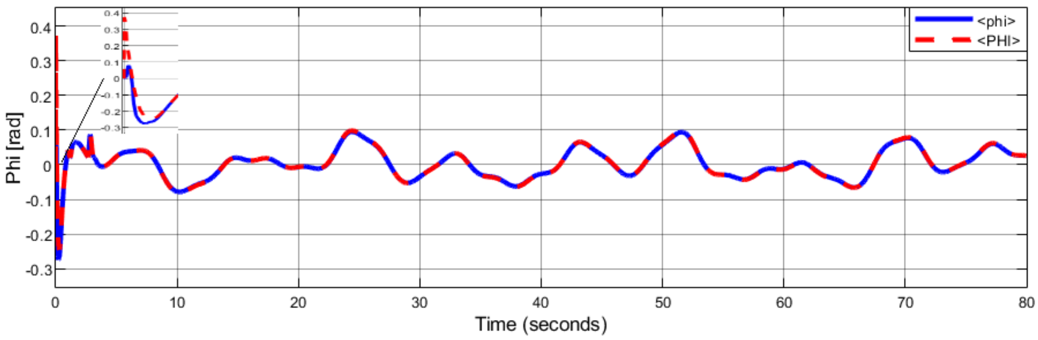

b. Quadrotor trajectory along the y-axis



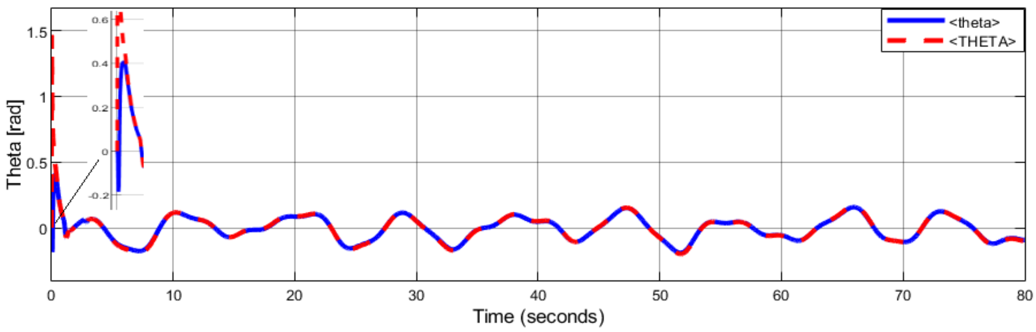

c. Quadrotor trajectory along the z-axis

Figure 6. Quadrotor trajectory along the coordinate axes under disturbances

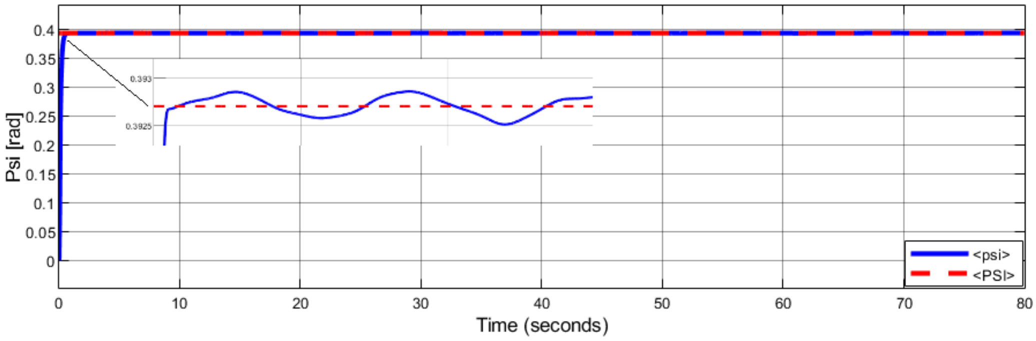
During the attitude tracking process, the SMC controlled the roll angle to follow the desired value within 0,45 seconds. For pitch angle tracking, the SMC drove the angle to the reference value in the shortest time, within less than 0,6 seconds. Regarding the yaw angle (rotation around the vertical axis), the SMC enabled the angle to converge to the reference value within 1 second. The tracking of the reference attitude angles is illustrated in figure 7.




a. Rollangle




b. Pitchangle




c. Yawangle

Figure 7. Quadrotor attitude angles under disturbances

The quadrotor's trajectory in space when tracking the predefined spiral reference trajectory is illustrated in figure 8. This trajectory demonstrates that the observer operates effectively in combination with the SMC to mitigate the influence of disturbances.

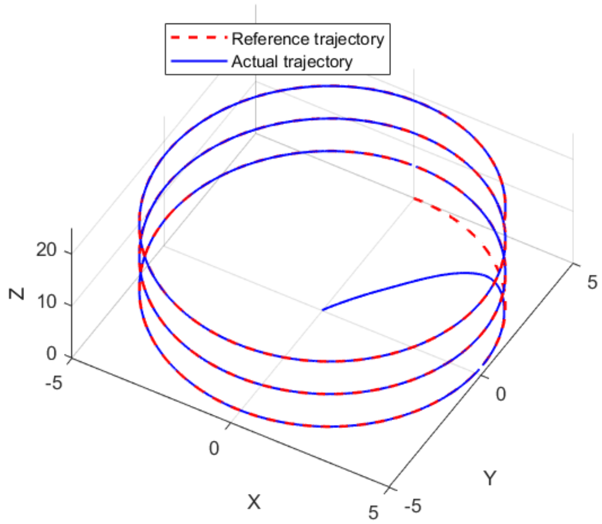



Figure 8. Quadrotor trajectory under disturbances

## IV. Conclusion

This paper proposed a sliding mode controller based on Lyapunov theory to stabilize a quadrotor. The controller achieved desired positions with minimal Euler angle deviations, ensuring smooth UAV motion without oscillation and accurate response to inputs.

Simulation results demonstrated that SMC ensures robustness and trajectory tracking. When combined with ESO, the controller can estimate and reject disturbances and uncertainties, making it highly practical for real-world quadrotor applications requiring stability and robustness.

This research provides meaningful contributions to designing and operating quadrotor systems for transportation and data collection, enhancing reliability and efficiency.

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