

# A State-Space-Based Model Predictive Control Method for a Twin Rotor MIMO System

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## Abstract

The Twin Rotor Multiple-Input Multiple-Output System (TRMS) is a laboratory-scale aerodynamic platform widely used for the development and validation of advanced control strategies. Due to its strong nonlinear characteristics, significant coupling effects, and multivariable dynamics, the TRMS represents a challenging benchmark for modern control applications. Although the system has been extensively investigated in the international research community, its utilization for advanced control studies remains limited in many laboratories because of the lack of accurate mathematical models and suitable control frameworks.

This paper presents the development and implementation of a Model Predictive Control (MPC) strategy for the TRMS based on a nonlinear state-space model. The nonlinear dynamics are linearized online at each sampling instant, resulting in a sequence of state-dependent linear models used for future state prediction. The MPC algorithm is formulated as a constrained optimization problem in which tracking performance and control effort are simultaneously considered. Simulation studies are carried out to evaluate the effectiveness of the proposed controller under square-wave and substep reference trajectories. The obtained results demonstrate accurate tracking performance for both pitch and yaw motions while effectively compensating for the cross-coupling interactions between the two rotational axes.

**Keywords:** Model Predictive Control, Twin Rotor MIMO System, Nonlinear Systems, State-Space Model, Multivariable Control, Predictive Optimization.

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Date of Submission: 13-06-2026

Date of acceptance: 27-06-2026

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## I. Introduction

Model Predictive Control (MPC) has become one of the most successful advanced control methodologies for industrial and engineering applications. Unlike conventional feedback controllers, MPC explicitly predicts future plant behavior over a finite prediction horizon and determines optimal control actions through online optimization. The ability to handle multivariable systems and operational constraints makes MPC particularly attractive for complex nonlinear processes.

The Twin Rotor Multiple-Input Multiple-Output System (TRMS) is a representative benchmark platform designed to emulate the dynamic characteristics of a helicopter. The system consists of two rotors positioned orthogonally, producing motion in both vertical and horizontal planes. Because of nonlinear aerodynamic forces, gyroscopic effects, and strong interactions between channels, the TRMS exhibits highly coupled dynamics that are difficult to control using traditional linear methods.

Several studies have investigated modeling and control techniques for TRMS, including PID control, fuzzy control, neural-network-based approaches, optimal control, and nonlinear state feedback methods. Among these techniques, MPC offers significant advantages because it can predict future system behavior while considering coupling effects and system constraints.

The primary objective of this work is to develop an MPC framework for the TRMS using a nonlinear state-space representation. The nonlinear model is linearized adaptively during operation, allowing the controller to capture system dynamics over a prediction horizon and generate optimal control actions in real time.

## II. Model Predictive Control Formulation

Consider a nonlinear discrete-time system described by

$$\begin{cases} x(k+1) = f(x(k), u(k)) \\ y(k) = h(x(k)) \end{cases} \quad (1)$$

Where  $x(k)$  is the state vector,  $u(k)$  is the input vector and  $y(k)$  is the output vector.

To facilitate predictive control design, the nonlinear model is linearized at each sampling instant around the current operating condition. The resulting local linear model can be expressed as

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k) \end{cases} \quad (2)$$

Where the matrices (A(k)), (B(k)), and (C(k)) depend on the current operating point.

For prediction purposes, a sequence of linearized models is generated throughout the prediction horizon. The predicted state vector can therefore be represented as a function of the current state and future control actions:

$$\hat{x}(k+i|k) = F_i x(k) + G_i U(k) \quad (3)$$

Where  $F_i$  and  $G_i$  are prediction matrices obtained from the state-transition dynamics. The corresponding predicted outputs are expressed as:

$$\hat{y}(k+i|k) = C_i \hat{x}(k+i|k) + d_i \quad (4)$$

Where  $d_i$  represents model uncertainties and disturbance effects.

The future outputs over the prediction horizon are assembled into the vector

$$Y(k) = M_c(k)M_A(k)x(k) + M_c(k)M_B(k)u(k) + M_c(k)M_U(k)\Delta U(k) + M_d(k) \quad (5)$$

which establishes the relationship between future outputs and future control inputs.

### III. Optimization Objective

The control objective is to minimize the tracking error while limiting excessive control activity. The MPC cost function is defined as

$$J(k) = \sum_{i=1}^{N_p} [r(k+i) - \hat{y}(k+i|k)]^T Q [r(k+i) - \hat{y}(k+i|k)] + \sum_{i=1}^{N_c} \Delta u(k+i-1)^T R \Delta u(k+i-1)$$

Where  $N_p$  is the prediction horizon,  $N_c$  is the control horizon, Q is the output weighting matrix, R is the control weighting matrix.

The optimization problem is solved subject to operational constraints on system outputs, control signals, and control increments:

$$y_{min} \leq y(k) \leq y_{max} \quad (6)$$

$$u_{min} \leq u(k) \leq u_{max} \quad (7)$$

$$\Delta u_{min} \leq \Delta u(k) \leq \Delta u_{max} \quad (8)$$

The resulting quadratic programming problem is solved online at each sampling instant. Only the first control action is applied to the plant, and the optimization procedure is repeated at the next sampling instant according to the receding horizon principle.

### IV. Mathematical Model of the TRMS

The Twin Rotor MIMO System consists of two independently driven rotors mounted on a beam structure, allowing motion in both vertical (pitch) and horizontal (yaw) directions. The system dynamics are highly nonlinear due to aerodynamic forces, gyroscopic effects, gravitational torques, and coupling interactions between the two axes.

The state vector is defined as:  $x = [\omega_h, \alpha_h, S_h, \omega_v, \alpha_v, S_v]$  (9)

Where  $\omega_h$  and  $\omega_v$  denote rotor angular velocities,  $\alpha_h$  and  $\alpha_v$  represent yaw and pitch angles.

The nonlinear model can be expressed in state-space form as:  $\dot{x} = f(x, u)$  and  $y = [\alpha_h, \alpha_v]$  (10)

Where  $u = [U_h, U_v]$  is the input voltage vector applied to the tail and main motors.

The nonlinear continuous state space equations of the TRMS are expressed in [1]

$$\frac{d}{dt} \begin{pmatrix} \omega_h \\ S_h \\ \alpha_h \\ \omega_v \\ S_v \\ \alpha_v \end{pmatrix} = \begin{pmatrix} -\frac{(k_{ah}\varphi_h)^2}{J_{tr}R_{ah}}\omega_h - \frac{B_{tr}}{J_{tr}}\omega_h - \frac{f_1(\omega_h)}{J_{tr}} + \frac{k_{ah}\varphi_h}{J_{tr}R_{ah}}f_6(U_h) \\ \frac{l_t\gamma_t f_2(\omega_h)\cos\alpha_v - f_7(\Omega_h) - f_3(\alpha_h)}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ S_h + \frac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ -\frac{(k_{av}\varphi_v)^2}{J_{mr}R_{mr}}\omega_v - \frac{B_{mr}}{J_{mr}}\omega_v - \frac{f_4(\omega_v)}{J_{mr}} + \frac{k_{av}\varphi_v}{J_{mr}R_{av}}f_8(U_v) \\ \frac{f_5(\omega_v)(l_m\gamma_m + k_g\Omega_h\cos\alpha_v) - f_9(\Omega_v) + g[(A-B)\cos\alpha_v - C\sin\alpha_v] - 0.5\Omega_h^2 H \sin 2\alpha_v}{J_v} \\ S_v + \frac{k_t}{J_v}\omega_h \end{pmatrix}$$

(11)

### V. Simulation Results

Figure 1 shows the block diagram of the MPC approach for TRMS

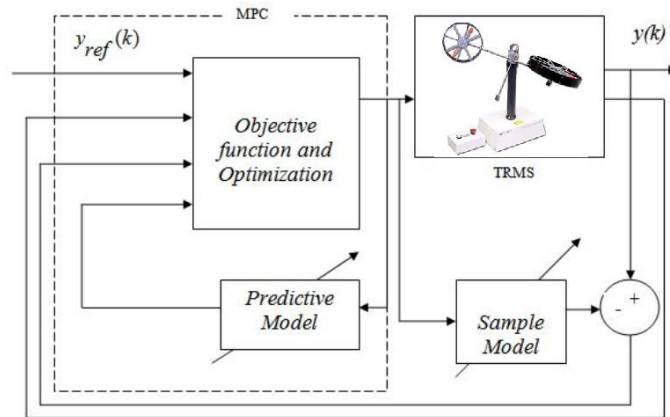
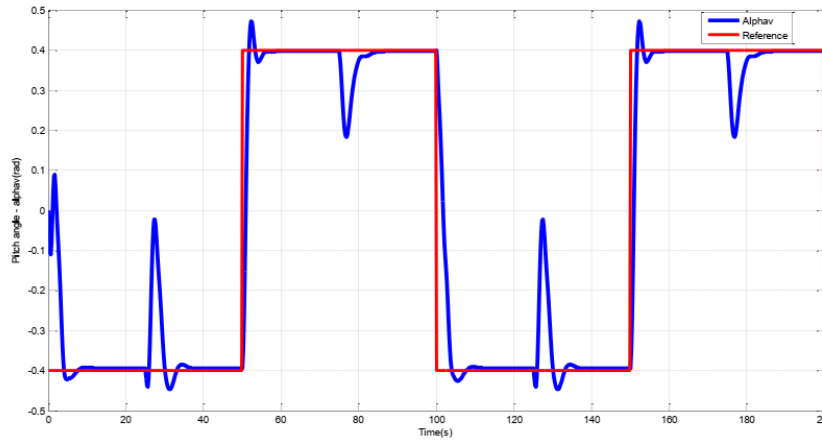
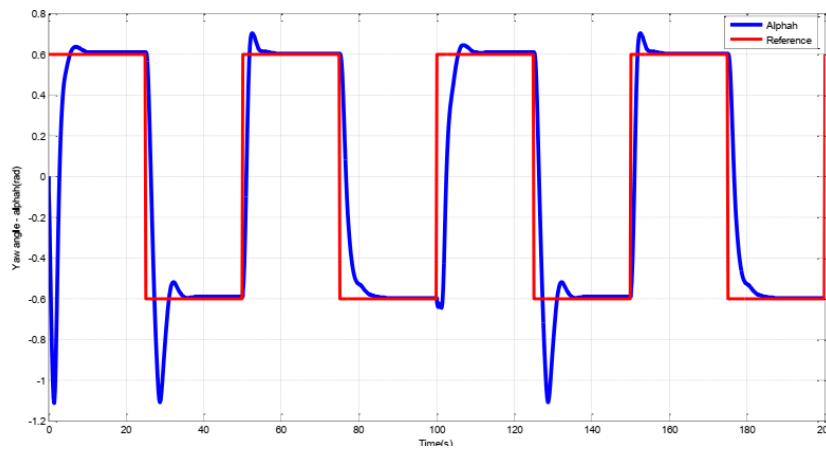


Figure 1. Block diagram of the MPC

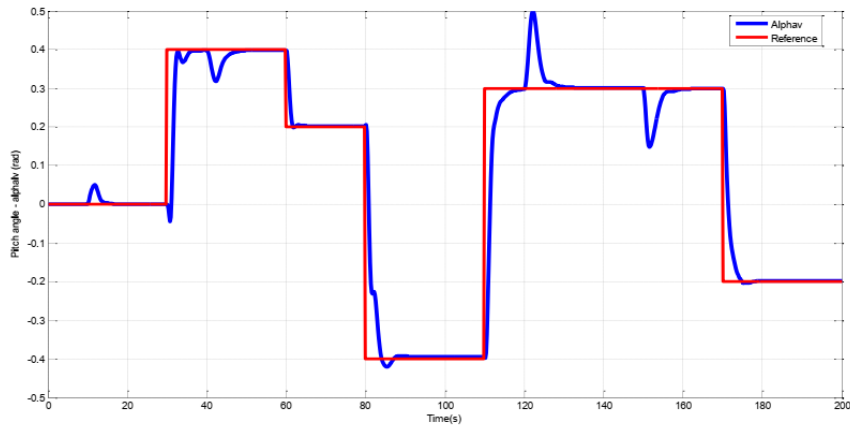
The simulation results with square and substep waves are represented in the following figures. Figure 2 is the response of the pitch angle in which the reference is a square wave. Figure 3 is the response of the Yaw angle in which the reference is a square wave. Figure 4 is the response of the pitch angle in which the reference is a substep. Figure 5 is the response of the Yaw angle in which the reference is a substep. Based on the simulation results in 200 seconds when applying Model Predictive Control for TRMS, the output responses of Yaw angle and pitch angle track the reference in predictive window. Especially, the cross-coupling channels between Yaw angle and pitch angle is best known. As soon as  $\alpha_h$  varies,  $\alpha_v$  changes and vice versa.



*Figure 2. The response of the pitch angle control loop with respect to a square*



*Figure 3. The response of the Yaw angle control loop with respect to a square*



*Figure 4. The response of the pitch angle control loop with respect to a substep*

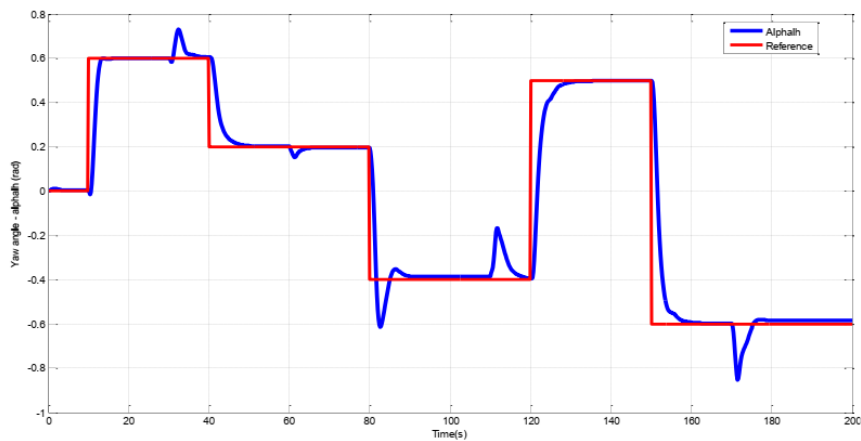


Figure 5. The response of the Yaw angle control loop with respect to a substep

## VI. Conclusion

This paper presented a Model Predictive Control approach for a Twin Rotor MIMO System based on a nonlinear state-space model. The controller employs online linearization and finite-horizon optimization to predict future system behavior and generate optimal control actions.

Simulation studies show that the proposed method provides accurate trajectory tracking for both pitch and yaw motions while effectively handling channel interactions. The obtained results confirm the capability of MPC to improve control performance for nonlinear multivariable systems.

Future work will focus on experimental validation and the investigation of disturbance rejection capabilities under external perturbations.

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