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Generation of Magic Squares from a Basic Magic Square

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***ABSTRACT**: Two simple methods, for generating different magic squares (MSs) from a basic MS, are presented. It is shown that no MS exists below a magic number (MN) 30. A lower limit of MN for a given basic MS is derived. Application of the MSs in generating the MS for a birthdate is given. Several examples are included.*

***KEYWORDS**: Magic Squares, Ramanujan birthday magic square, Super magic square, Quartet theory*

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I. INTRODUCTION

In this paper, we will restrict to the squares of order 4 with all distinct non-negative numbers (DNNs).

1.1 QUARTETS

Consider the general matrix of order 4

$$M_S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}. \quad (1)$$

There are 16 cells. We will group them into 4 sets, Q1,Q2,Q3,Q4. Each set will have four cells; each cell should be from a different row and a different column; only one of them must be on a diagonal. Let Q1 be one cell S_{14}. Then the other three cells cannot be in the first row and fourth column and the top right to bottom left diagonal. There are 6 possible cells S_{21}, S_{22}, S_{31}, S_{33}, S_{21}, S_{42}, S_{43} out of which 3 are to be chosen. Let us choose S_{22}. Now we can only choose out of S_{31}, S_{33}, S_{42}, S_{43}. Let us choose S_{31}. Only choice left is S_{43}. Thus, Q1 is identified as (S_{14},S_{22},S_{31},S_{43}). Similarly, the other quartets are identified as Q2(S_{41},S_{33},S_{24},S_{12}), Q3(S_{11},S_{23},S_{34}, S_{42}), Q4(S_{44},S_{32}, S_{21},S_{13}).

Similarly, choosing S33, instead of S22, in the above procedure, we get another set of quartets as Q5(S_{44}, S_{23},S_{12},S_{31}), Q6(S_{32},S_{11},S_{24},S_{43}), Q7(S_{21},S_{42},S_{33},S_{14}), Q8(S_{13},S_{34},S_{41},S_{22}). One can easily check that these are the only two sets possible.

Example: Out of many possible SMSs for S = 30, one is shown in Table 1 [1]. In the subscript, the numeral part stands for the magic number (MN) S. The quartets obtained are Q1(12,13,14,15), Q2(8,9,10,11), Q3(4,5,6,7), Q4(0,1,2,3), Q5(3,7,11,15), Q6(2,6,10,14), Q7(1,5,9,13), and Q8(0,4,8,12). The numbers are arranged in ascending order. These results are shown in Table 1. Quartets are marked in brackets. In the Q1-Q4 (Q5-Q8), the numbers are in arithmetic progression with a common difference 1 (4). The two MSs are the same, but the quartets identified are different. Therefore, one is identified as M_{30A} and the other M_{30B}.

Table 1. M_{30} with quartets marked in color

M_{30A} (Q1-Q4)	M_{30B} (Q5 − Q8)
$\begin{bmatrix} \text{(Q3) 06} & \text{(Q2) 11} & \text{(Q4) 00} & \text{(Q1) 13} \\ \text{(Q4) 01} & \text{(Q1) 12} & \text{(Q3) 07} & \text{(Q2) 10} \\ \text{(Q1) 15} & \text{(Q4) 02} & \text{(Q2) 09} & \text{(Q3) 04} \\ \text{(Q2) 08} & \text{(Q3) 05} & \text{(Q1) 14} & \text{(Q4) 03} \end{bmatrix}$	$\begin{bmatrix} \text{(Q6) 06} & \text{(Q5) 11} & \text{(Q8)00} & \text{(Q7) 13} \\ \text{(Q7) 01} & \text{(Q8)12} & \text{(Q5) 07} & \text{(Q6) 10} \\ \text{(Q5) 15} & \text{(Q6) 02} & \text{(Q7) 09} & \text{(Q8) 04} \\ \text{(Q8) 08} & \text{(Q7) 05} & \text{(Q6) 14} & \text{(Q5) 03} \end{bmatrix}$

II. Method

2.1 Theorem: If all the elements of any two quartets are increased by integers n_1 and n_2, then it becomes an MS of MN $S + n_1 + n_2$.

Proof: Without any loss of generality, let us assume that n_1 and n_2 ($n_1 > n_2$) are added to Q1, and Q2, respectively, of M_S.

$$M_{S+n} = \begin{bmatrix} S_{11} & S_{12}+n_1 & S_{13} & S_{14}+n_2 \\ S_{21} & S_{22}+n_2 & S_{23} & S_{24}+n_1 \\ S_{31}+n_2 & S_{32} & S_{33}+n_1 & S_{34} \\ S_{41}+n_1 & S_{42} & S_{43}+n_2 & S_{44} \end{bmatrix}. \quad (2)$$

Since n_1 and n_2 appear in every diagonal, row and column, the MN is increased from S to $S + n_1 + n_2$.

Obviously, the theorem is also applicable to the M_{30B}.

2.2 QUARTETS Q1-Q4

2.2.1 Using only one quartet

Let $n_1 = n$ and $n_2 = 0$, i.e., the additions are made only in any one quartet.

(a) *Shifting technique*

Since the square is packed with all the integers 0-15, increase in Q1 by n will shift the numbers of Q1 above 15, say Qo (open space).

(b) *Jumping technique*

Q2 can be increased by $n \geq 8$. It jumps over Q1. Similarly, Q3 (Q4) can be increased by $n \geq 12$ (16), respectively.

Example 1: Applying the theorem to M_{30A}, M_{34} and M_{38} obtained by shifting technique are

$$M_{30+4=34} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{17} \\ 01 & \mathbf{16} & 07 & 10 \\ \mathbf{19} & 02 & 09 & 04 \\ 08 & 05 & \mathbf{18} & 03 \end{bmatrix}, M_{30+8=38} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{21} \\ 01 & \mathbf{20} & 07 & 10 \\ \mathbf{23} & 02 & 09 & 04 \\ 08 & 05 & \mathbf{22} & 03 \end{bmatrix}. \quad (3)$$

M_{38}, M_{46}, M_{101}, M_{257}, obtained by jumping technique applied to Q2, Q4, Q3 and Q2, respectively, are

$$M_{30+8=38} = \begin{bmatrix} 06 & \mathbf{19} & 00 & 13 \\ 01 & 12 & 07 & \mathbf{18} \\ 15 & 02 & \mathbf{17} & 04 \\ \mathbf{16} & 05 & 14 & 03 \end{bmatrix}, M_{30+16=46} = \begin{bmatrix} 06 & 11 & \mathbf{16} & 13 \\ \mathbf{17} & 12 & 07 & 10 \\ 15 & \mathbf{18} & 09 & 04 \\ 08 & 05 & 14 & \mathbf{19} \end{bmatrix} \quad (4)$$

$$M_{30+71=101} = \begin{bmatrix} \mathbf{77} & 11 & 00 & 13 \\ 01 & 12 & \mathbf{78} & 10 \\ 15 & 02 & 09 & \mathbf{75} \\ 08 & \mathbf{76} & 14 & 03 \end{bmatrix}, \quad M_{30+227=257} = \begin{bmatrix} 006 & \mathbf{238} & 000 & 013 \\ 001 & 012 & 007 & \mathbf{237} \\ 015 & 002 & \mathbf{236} & 004 \\ \mathbf{235} & 005 & 014 & 003 \end{bmatrix} \quad (5)$$

Unlike in the demonstrated video [2] for M_{257}, this theorem can handle numbers with any number of digits, including a 0.

Thus, infinite number of MSs of different S can be derived.

Example 2: Negative n

Any M_{30} contains all 0 to 15 numbers, such as M_{30A} and M_{30B} of Table 1. If we subtract n from Q1, Q2, Q3 there will be duplication, and from Q4 there will be negative numbers. As an example, if n = -1 (and below) is added to Q3, the numbers repeat.

$$M_{29} = \begin{bmatrix} 05 & 11 & 00 & 13 \\ 01 & 12 & 06 & 10 \\ 15 & 02 & 09 & \mathbf{03} \\ 08 & 04 & 14 & \mathbf{03} \end{bmatrix}. \quad (6)$$

Thus, MNs less than 30 cannot be obtained. If we subtract 4 from Q1 of (4), we get the same result shown in equation (3).

To generate an MS of different *S*, at least one MS should be available. However, one can use the M_{30A} which can easily be memorized.

2.2.2 Limitations

(a) If the elements of a particular quartet are greater than all other quartets, then any positive integer value of *n* can be added to this quartet.

(b) If any of the quartet has number less than the other quartets, then

$$n_{min} = \{\text{Quartet } Q_{j\ max}\ (j \neq i) + 1 - (\text{quartet } Q_{i\ min}\}. \qquad (7)$$

Table 2 gives the values of n_{min} for various quartets.

Table 2: values of n_{min} for various quartets

Quartet	n_{min}	Quartet	n_{min}	n*
Q1	00	Q5	12	4,8,12
Q2	08	Q6	14	4,8,12
Q3	12	Q7	15	4,8,12
Q4	16	Q8	16	4,8,12

*Values of *n* permitted for M_{30B}.

Q1 gives MSs for *n* = 0 and above, Q2, for 8 and above, Q3, for 12 and above, and Q4, for 16 and above. Thus, the only MSs possible for M_{31} to M_{34} are

$$M_{30+1=31} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{14} \\ 01 & \mathbf{13} & 07 & 10 \\ \mathbf{16} & 02 & 09 & 04 \\ 08 & 05 & \mathbf{15} & 03 \end{bmatrix}, \ M_{30+2=32} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{15} \\ 01 & \mathbf{14} & 07 & 10 \\ \mathbf{17} & 02 & 09 & 04 \\ 08 & 05 & \mathbf{16} & 03 \end{bmatrix}, \qquad (8)$$

$$M_{30+3=33} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{16} \\ 01 & \mathbf{15} & 07 & 10 \\ \mathbf{18} & 02 & 09 & 04 \\ 08 & 05 & \mathbf{17} & 03 \end{bmatrix}, \ M_{30+4=34} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{17} \\ 01 & \mathbf{16} & 07 & 10 \\ \mathbf{19} & 02 & 09 & 04 \\ 08 & 05 & \mathbf{18} & 03 \end{bmatrix}, \qquad (9)$$

1) In M_{31} number 12, in M_{32}, 12, 13, in M_{33}, 12,13,14 are missing. This trend continues for higher MSs.
2) Any one particular M_{30} gives only one M_{31}.

The only MS possible for M_{38} is

$$M_{30+8=38} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{21} \\ 01 & \mathbf{20} & 07 & 18 \\ \mathbf{23} & 02 & 19 & 04 \\ 08 & 05 & \mathbf{22} & 03 \end{bmatrix}. \qquad (10)$$

2.3 QUARTETS Q5-Q8

Q5 gives MSs for *n* = 12 and above, Q6, 14 and above, Q7, 15 and above, and Q8, 16 and above. In addition to these, Q5-Q8 give MSs for discrete values of *n* = 4,8,12. We are skipping the details to conserve the space, but for the benefit of the readers, we are giving some typical MSs. The MSs obtained by adding 4, 8, 12 and 16 to Q5,Q6,Q7 and Q8, respectively.

$$M_{30+4=34} = \begin{bmatrix} 06 & \mathbf{15} & 00 & 13 \\ 01 & 12 & \mathbf{11} & 10 \\ \mathbf{19} & 02 & 09 & 04 \\ 08 & 05 & 14 & \mathbf{07} \end{bmatrix}, M_{30+8=38} = \begin{bmatrix} \mathbf{14} & 11 & 00 & 13 \\ 01 & 12 & 07 & \mathbf{18} \\ 15 & \mathbf{10} & 09 & 04 \\ 08 & 05 & \mathbf{22} & 03 \end{bmatrix}, \qquad (11)$$

$$M_{30+12=42} = \begin{bmatrix} 14 & 15 & 00 & \mathbf{13} \\ \mathbf{01} & 12 & 11 & 18 \\ 19 & 10 & \mathbf{09} & 04 \\ 08 & \mathbf{05} & 22 & 07 \end{bmatrix}, \quad M_{30+16=46} = \begin{bmatrix} 06 & 11 & \mathbf{16} & 13 \\ 01 & \mathbf{28} & 07 & 10 \\ 15 & 02 & 09 & \mathbf{20} \\ \mathbf{24} & 05 & 14 & 03 \end{bmatrix}, \tag{12}$$

2.4 USING MORE THAN ONE QUARTETS

(a) If all the elements of Q1 are increase by n, then there will be a vacancy of n numbers created. This vacancy can be filled in by increasing Q2 by n_2, Q3 by n_3 and Q4 by n_4 such that

$$n_1 + n_2 + n_3 + n_4 = n \tag{13}$$

where $n_i \geq n_{i+1}$, i = 1 to 3.

Example 3: Let n = 4. Then the possible values are $\{n_1, n_2, n_3, n_4\}$ ={4,0,0,0}, {3,1,0,0}, {2,2,0,0}, {2,1,1,0}, {1,1,1,1}. There are 5 possible M_{34} from M_{30A}.

Example 4: Let n be 10. One possible choice is $\{n_1, n_2, n_3, n_4\}$ = {4,3,2,1}. The resulting MS is

$$M_{30+1+2+3+4=40} = \begin{bmatrix} 08 & 14 & 01 & 17 \\ 02 & 16 & 09 & 13 \\ 19 & 03 & 12 & 06 \\ 11 & 07 & 18 & 04 \end{bmatrix}. \tag{14}$$

(b) *n* is a multiple of 4

In this case we can have

$$n_1 = n_2 = n_3 = n_4 = n/4 \tag{15}$$

Since all the quartets have increased by the same number, the resulting square will be an MS M_{S+n}.

$$M_{S+n} = \begin{bmatrix} S_{11}+n/4 & S_{12}+n/4 & S_{13}+n/4 & S_{14}+n/4 \\ S_{21}+n/4 & S_{22}+n/4 & S_{23}+n/4 & S_{24}+n/4 \\ S_{31}+n/4 & S_{32}+n/4 & S_{33}+n/4 & S_{34}+n/4 \\ S_{41}+n/4 & S_{42}+n/4 & S_{43}+n/4 & S_{44}+n/4 \end{bmatrix} \tag{16}$$

Example 5: From M_{30},

$$M_{34} = \begin{bmatrix} 07 & 12 & 01 & 14 \\ 02 & 13 & 08 & 11 \\ 16 & 03 & 10 & 05 \\ 09 & 06 & 15 & 04 \end{bmatrix} \tag{17}$$

is obtained by adding 1 to all the quartets.

(c) n_1 and n_2 are added into 2 quartets such that

$$n_1 + n_2 = n. \tag{18}$$

where are 10 possible MSs as shown in equations (17)-(26)

$$M_{S+n} = \begin{bmatrix} S_{11}+n_1 & S_{12} & S_{13} & S_{14}+n_2 \\ S_{21} & S_{22}+n_2 & S_{23}+n_1 & S_{24} \\ S_{31}+n_2 & S_{32} & S_{33} & S_{34}+n_1 \\ S_{41} & S_{42}+n_1 & S_{43}+n_2 & S_{44} \end{bmatrix} \tag{19}$$

$$M_{S+n} = \begin{bmatrix} S_{11}+n_1 & S_{12} & S_{13} & S_{14}+n_2 \\ S_{21} & S_{22}+n_2 & S_{23}+n_1 & S_{24} \\ S_{31}+n_2 & S_{32} & S_{33} & S_{34}+n_1 \\ S_{41} & S_{42}+n_1 & S_{43}+n_2 & S_{44} \end{bmatrix} \tag{20}$$

$$M_{S+n} = \begin{bmatrix} S_{11} & S_{12}+n_1 & S_{13}+n_2 & S_{14} \\ S_{21}+n_2 & S_{22} & S_{23} & S_{24}+n_1 \\ S_{31} & S_{32}+n_2 & S_{33}+n_1 & S_{34} \\ S_{41}+n_1 & S_{42} & S_{43} & S_{44}+n_2 \end{bmatrix} \tag{21}$$

$$M_{S+n} = \begin{bmatrix} S_{11} & S_{12}+n_1 & S_{13} & S_{14}+n_2 \\ S_{21} & S_{22}+n_2 & S_{23} & S_{24}+n_1 \\ S_{31}+n_2 & S_{32} & S_{33}+n_1 & S_{34} \\ S_{41}+n_1 & S_{42} & S_{43}+n_2 & S_{44} \end{bmatrix} \tag{22}$$

$$M_{S+n} = \begin{bmatrix} S_{11}+n_1 & S_{12}+n_2 & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23}+n_1 & S_{24}+n_2 \\ S_{31}+n_1 & S_{32}+n_2 & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43}+n_1 & S_{44}+n_2 \end{bmatrix} \tag{23}$$

$$M_{S+n} = \begin{bmatrix} S_{11}+n_1 & S_{12}+n_2 & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23}+n_1 & S_{24}+n_2 \\ S_{31}+n_1 & S_{32}+n_2 & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43}+n_1 & S_{44}+n_2 \end{bmatrix} \tag{24}$$

$$M_{S+n} = \begin{bmatrix} S_{11} & S_{12}+n_1 & S_{13}+n_2 & S_{14} \\ S_{21} & S_{22}+n_2 & S_{23}+n_1 & S_{24} \\ S_{31}+n_2 & S_3 & S_{33} & S_{34}+n_1 \\ S_{41}+n_1 & S_{42} & S_{43} & S_{44}+n_2 \end{bmatrix} \tag{25}$$

$$M_{S+n} = \begin{bmatrix} S_{11} & S_{12} & S_{13}+n_1 & S_{14}+n_2 \\ S_{21}+n_1 & S_{22}+n_2 & S_{23} & S_{24} \\ S_{31}+n_2 & S_{32}+n_1 & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43}+n_2 & S_{44}+n_1 \end{bmatrix} \tag{26}$$

$$M_{S+n} = \begin{bmatrix} S_{11}+n_1 & S_{12} & S_{13}+n_2 & S_{14} \\ S_{21}+n_2 & S_{22} & S_{23}+n_1 & S_{24} \\ S_{31} & S_{32}+n_2 & S_{33} & S_{34}+n_1 \\ S_{41} & S_{42}+n_1 & S_{43} & S_{44}+n_2 \end{bmatrix} \tag{27}$$

$$M_{S+n} = \begin{bmatrix} S_{11}+n_1 & S_{12} & S_{13} & S_{14}+n_2 \\ S_{21}+n_2 & S_{22} & S_{23} & S_{24}+n_1 \\ S_{31} & S_3+n_2 & S_{33}+n_1 & S_{34} \\ S_{41} & S_{42}+n_1 & S_{43}+n_2 & S_{44} \end{bmatrix}. \tag{28}$$

$$M_{S+n} = \begin{bmatrix} S_{11} & S_{12}+n_1 & S_{13}+n_2 & S_{14} \\ S_{21} & S_{22}+n_2 & S_{23}+n_1 & S_{24} \\ S_{31}+n_2 & S_3 & S_{33} & S_{34}+n_1 \\ S_{41}+n_1 & S_{42} & S_{43} & S_{44}+n_2 \end{bmatrix}. \tag{29}$$

This is obvious as every row, column and diagonal is increased by $S = n_1 + n_2$, except in equations (25) and (26). However, it is valid for all the 10 cases when $n_1 = n_2 = n/2$. The n_1 and n_2 are interchangeable. Although these are 10 cases, only one will be applicable depending upon the choice of M_{30}. For M_{30A} only equation (19) will be applicable.

From Table 1, if $n_{min} \geq 16$ (or $S \geq 30 + 16$), then all equations (17)-(26) can be used. However, if n_{min} is below 16, and if we choose quartet Q1, then the other possible quartet may be Q2, Q3 or Q4. However, Q3 and Q4 give repetition; while Q2 does not, because adding to Q1 makes space for Q2 to increase. The possible MS for M_{38} can be obtained by choosing $\{n_1, n_2\} = \{7,1\}, (6,2\}, \{5,3\}, \{4,4\}$. For $\{5,3\}$, it is

$$M_{38} = \begin{bmatrix} 06 & \mathbf{14} & 00 & \mathbf{18} \\ 01 & \mathbf{17} & 07 & \mathbf{13} \\ \mathbf{20} & 02 & \mathbf{12} & 04 \\ \mathbf{11} & 05 & \mathbf{19} & 03 \end{bmatrix} \tag{30}$$

From equation (27), M_{34} can be obtained by subtracting (2+2), (3+1), (1+3) as

$$M_{38\text{-}2\text{-}2=34} = \begin{bmatrix} 06 & \mathbf{12} & 00 & \mathbf{16} \\ 01 & \mathbf{15} & 07 & \mathbf{11} \\ \mathbf{18} & 02 & \mathbf{10} & 04 \\ \mathbf{09} & 05 & \mathbf{17} & 03 \end{bmatrix} \quad M_{38\text{-}3\text{-}1=34} = \begin{bmatrix} 06 & \mathbf{11} & 00 & \mathbf{17} \\ 01 & \mathbf{16} & 07 & \mathbf{10} \\ \mathbf{19} & 02 & \mathbf{09} & 04 \\ \mathbf{08} & 05 & \mathbf{18} & 03 \end{bmatrix} \tag{31}$$

$$M_{38\text{-}1\text{-}3=34} = \begin{bmatrix} 06 & \mathbf{13} & 00 & \mathbf{15} \\ 01 & \mathbf{14} & 07 & \mathbf{12} \\ \mathbf{17} & 02 & \mathbf{11} & 04 \\ \mathbf{10} & 05 & \mathbf{16} & 03 \end{bmatrix}. \tag{32}$$

Example 6: M_{48} obtained from M_{38} of equation (27) by adding 6 and 4 in Q1 and Q2, respectively, is

$$M_{38+6+4=48} = \begin{bmatrix} 06 & \mathbf{18} & 00 & \mathbf{24} \\ 01 & \mathbf{23} & 07 & \mathbf{17} \\ \mathbf{26} & 02 & \mathbf{16} & 04 \\ \mathbf{15} & 05 & \mathbf{25} & 03 \end{bmatrix}. \tag{33}$$

Example 7: Number 73 = 30 + 20 + 13 + 8 + 2. Using Q1, Q2, Q3, Q4, we get

$$M_{30+20+13+8+2=73} = \begin{bmatrix} 14 & 24 & 02 & 33 \\ 03 & 32 & 15 & 23 \\ 35 & 04 & 22 & 12 \\ 21 & 13 & 34 & 05 \end{bmatrix}. \tag{34}$$

(d) If MSs M and N are multiplied by m and n, respectively, and then added, will result into another MS $M(m\times M+n\times N)$. It can easily be proved using the properties of matrix algebra.

Example 8:

$$2M_{30} + 3M_{34} = \begin{bmatrix} 33 & 58 & 03 & 68 \\ 08 & 63 & 38 & 53 \\ 78 & 13 & 48 & 23 \\ 43 & 28 & 73 & 18 \end{bmatrix} = M_{162}. \tag{35}$$

Similar theory can be developed for Q5-Q8. We are skipping to conserve space, but typical MSs are given for the benefit of the readers.

Example 9: Applying the theorem to $M_{30\mathrm{B}}$, M_{38} obtained by shifting technique is

$$M_{30+8=38} = \begin{bmatrix} 06 & 19 & 00 & 13 \\ 01 & 12 & 15 & 10 \\ 23 & 02 & 09 & 04 \\ 08 & 05 & 14 & 11 \end{bmatrix} \tag{36}$$

Unlike in the demonstrated video [2] for M_{257}, this theorem can handle numbers with any number of digits, including a 0.

2.5 SIMPLE PROCEDURE

A stepwise simple procedure for obtaining guaranteed a valid MS of desired $S = Sp$ is given below.

1. Use the basic M_{30A} which has $n_{min} = 0$, such as given by Q1.
2. Find $n = S_p - 30$.
3. Use the methods given in Section 2, if n is a multiple of 4, even, and/or use the theorem.

Example 10: Find MSs for $S = 39$. Here $S_p = 39$. Therefore $n = 39 - 30 = 9$. Using the theorem, we get

$$M_{30+9=39} = \begin{bmatrix} 06 & 11 & 00 & \mathbf{22} \\ 01 & \mathbf{21} & 07 & 10 \\ \mathbf{24} & 02 & 09 & 04 \\ 08 & 05 & \mathbf{23} & 03 \end{bmatrix}. \tag{37}$$

Since the maximum possible value among all the quartets is 15, and the minimum possible value of 00, n_{min} cannot be greater than 16. (Actual value will depend upon the quartets). Therefore, $S \geq 46$ is guaranteed from any basic M_{30}.

Example 11: Find MSs for $S = 50$. They are 8 in numbers. Two of them (using Q1 and Q7) are

$$M_{30+20=50} = \begin{bmatrix} 06 & 11 & 00 & 33 \\ 01 & 32 & 07 & 10 \\ 35 & 02 & 09 & 04 \\ 08 & 05 & 34 & 03 \end{bmatrix} = \begin{bmatrix} 26 & 11 & 00 & 13 \\ 01 & 12 & 07 & 30 \\ 15 & 22 & 09 & 04 \\ 08 & 05 & 34 & 03 \end{bmatrix}. \quad (38)$$

III. APPLICATIONS

3.1 MS of a desired birthdate

Let the birthdate be 29.10.1943 ($S = 101$). In M_{30A}, add 23 to Q1, -1 to Q2, 19 to Q3 and 30 to Q4. We have shown the final result in equation (38).

$$M_{30+23-1+19+30=101} = \begin{bmatrix} 29 & 10 & 19 & 43 \\ 20 & 42 & 30 & 09 \\ 45 & 21 & 08 & 27 \\ 07 & 28 & 44 & 22 \end{bmatrix}. \quad (39)$$

Example 12: For Ramanujan's birthdate (22.12.1887), the MS is shown to the left side of equation (40). Of course, it does not satisfy all the sum properties of original Ramanujan's birthday MS shown to the right side. [2].

$$M_{30+16+1+18+74=139} \begin{bmatrix} 22 & 12 & 18 & 87 \\ 19 & 86 & 23 & 11 \\ 89 & 20 & 10 & 20 \\ 09 & 21 & 88 & 21 \end{bmatrix}, \text{Ramanujan's birthdate MS} \begin{bmatrix} 22 & 12 & 18 & 87 \\ 88 & 17 & 09 & 25 \\ 10 & 24 & 89 & 16 \\ 19 & 86 & 23 & 11 \end{bmatrix}. \quad (40)$$

If the change in the day only is required, use Q4. Similarly, for month, century and year use corresponding quartets. One has to choose the proper MS so that the conditions for n_{min} are satisfied for change in the day, month, century and year.

Example 13: Let the date of birth be 28.02.1975 ($S = 124$). Then we get M_{124} as

$$M_{30+22-9+19+62=124} = \begin{bmatrix} 28 & 02 & 19 & 75 \\ 20 & 74 & 29 & 01 \\ 79 & 23 & -02 & 24 \\ -03 & 25 & 78 & 24 \end{bmatrix}. \quad (41)$$

Observe the repeat of 24 and the negative numbers -02 and -03. This is due to the restriction of n_{min}. Therefore, one should choose the proper basic MS. Let us choose the basic M_{30} given by left side of equation (41). Now add 17 to Q3, 00 to Q2, 06 to Q4 and 71 to Q1, respectively. The resulting MS is shown on the right side.

$$M_{30} = \begin{bmatrix} 11 & 02 & 13 & 04 \\ 05 & 12 & 03 & 10 \\ 06 & 15 & 00 & 09 \\ 08 & 01 & 14 & 07 \end{bmatrix} \rightarrow M_{30+17+0+6+71=124} = \begin{bmatrix} 28 & 02 & 19 & 75 \\ 11 & 83 & 20 & 10 \\ 77 & 21 & 00 & 26 \\ 08 & 18 & 85 & 13 \end{bmatrix} \quad (42)$$

Here we cannot use $n/4$ or $n/2$ techniques discussed in Section 2 as they require change in numbers more than 1 in the first row.

3.2 Only *S* is mentioned: Split S into the sum of 4 DNNs. All the properties above can be applied now. For example, if S is 101, we can split in to 29+10+19+43. The MS will be the same as in equation (38).

IV. CONCLUSION

Two simple methods for obtaining different MNs from a basic MS are presented. There exists no MS for MN below 30. For the remaining cases, MSs are possible. A lower limit of MN for a given MS is derived. Some practical applications of the method are illustrated.

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