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# On The Simultaneous Equations $x+y=2a^2, 2x+y=5a^2+b^2, x+2y=c^3$

A. Vijayasankar[1], Sharadha Kumar[2], M.A. Gopalan[3]

[1]*Assistant Professor, Department of Mathematics, National College, Affiliated to Bharathidasan University, Trichy-620 001, Tamil Nadu, India.*
*e[2]Research Scholar, Department of Mathematics, National College, Affiliated to Bharathidasan University ,Trichy-620 001, Tamil Nadu, India.*
[3]*Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.*
*Corresponding Author: Sharadha Kumar*

***ABSTRACT:*** *An attempt is made to obtain non-zero distinct integer quintuples $(x,y,a,b,c)$ satisfying the system of three equations $x+y=2a^2, 2x+y=5a^2+b^2, x+2y=c^3$. Different sets of integer solutions are presented.*

***KEYWORDS:*** *system of triple equations, triple equations with five unknowns, integer solutions.*

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## I. Introduction

In [1], an attempt has been made to obtain pairs of non-zero distinct integers $x, y$ such that, in each pair

i. $x+y=a^2, 2x+y=b^2, x+2y=a^3$

ii. $x+y=a^2, 2x+y=b^2, x+2y=c^3$

[2] illustrates the analysis of obtaining different sets of distinct integer solutions to the two systems of triple equations with five unknowns given by

i. $x+y=a^2, 2x+y=b^2, x+2y=3c^2$

ii. $x+y=a^2, 2x+y=b^2, x+2y=2c^2$ respectively.

In [3], the system of three equations $x+y=a^2, 2x+y=b^2, x+2y=a^2-c^2$ has been studied for its non-zero distinct integer solutions.

This communication exhibits different sets of non-zero distinct integer solutions for the system of triple equations with five unknowns given by $x+y=2a^2, 2x+y=5a^2+b^2, x+2y=c^3$.

## II. Method Of Analysis

Let $x, y, a, b$ and $c$ be five non-zero distinct integers such that

$$x+y=2a^2 \tag{1}$$

$$2x+y=5a^2+b^2 \tag{2}$$

$$x+2y=c^3 \tag{3}$$

Eliminating $x$ and $y$ between (1) to (3), the resulting equation is

$$a^2-b^2=c^3 \tag{4}$$

Solving (4) through different methods, one obtains different sets of solutions to the system (1) to (3).

**Method 1:**

It is observed that (4) is satisfied by

$$a=m(m^2-n^2), b=n(m^2-n^2), c=(m^2-n^2) \tag{5}$$

where $m \neq n$ and $n \neq 1$. Eliminating $y$ between (1) and (2), the values of $x$ is given by

$$x=3a^2+b^2=(m^2-n^2)^2(3m^2-n^2) \tag{6}$$

From (1), $$y=2a^2-x=-(m^2-n^2)^2(m^2+n^2) \tag{7}$$

Note that, (5) to (7) satisfy (1) to (3). A few numerical examples are given in Table 1 below:

**Table 1: Numerical Examples**

| *m* | *n* | *a* | *b* | *c* | *x* | *y* |
|---|---|---|---|---|---|---|
| 2 | 3 | -10 | -15 | -5 | 525 | -325 |
| 5 | -7 | -120 | 168 | -24 | 71424 | -42624 |
| 11 | 9 | 440 | 360 | 40 | 710400 | -323200 |
| 9 | 2 | 693 | 154 | 77 | 1464463 | -503965 |

**Method 2:**
After performing numerical calculations, it is seen that (4) is satisfied by

$$a = t_{3,k+1}, b = t_{3,k}, c = (k+1) \quad (8)$$

where $t_{3,k}$ is the triangular number of rank $k$.
The corresponding values of $x$ and $y$ satisfying (1) to (3) are represented by

$$x = 3(t_{3,k+1})^2 + (t_{3,k})^2 = (k+1)^4 + (k+1)^3 + (k+1)^2$$

$$y = -(t_{3,k+1})^2 - (t_{3,k})^2 = -\frac{1}{2}\left[(k+1)^4 + (k+1)^2\right]$$

A few numerical examples are given in Table 2 below:

**Table 2: Numerical Examples**

| *k* | *a* | *b* | *c* | *x* | *y* |
|---|---|---|---|---|---|
| 2 | 6 | 3 | 3 | 117 | -45 |
| 3 | 10 | 6 | 4 | 336 | -136 |
| 4 | 15 | 10 | 5 | 775 | -325 |
| 5 | 21 | 15 | 6 | 1548 | -666 |

**Method 3:**

Observe that (4) is satisfied by

$$a = \frac{c^3+1}{2}, b = \frac{c^3-1}{2}$$

Since our interest is on finding integer solutions,
take

$$c = 2k+1$$

and we have

$$a = 4k^3 + 6k^2 + 3k + 1$$

$$b = 4k^3 + 6k^2 + 3k$$

For this choice, the values of $x$ and $y$ satisfying (1) to (3) are given by

$$x = 4(4k^3+6k^2+3k)^2 + 6(4k^3+6k^2+3k) + 3$$

$$y = -2(4k^3+6k^2+3k)^2 - 2(4k^3+6k^2+3k) - 1$$

A few numerical examples are presented in Table 3 below:

**Table 3: Numerical Examples**

| *k* | *a* | *b* | *c* | *x* | *y* |
|---|---|---|---|---|---|
| 2 | 63 | 62 | 5 | 15751 | -7813 |
| 3 | 172 | 171 | 7 | 117993 | -58825 |
| 4 | 365 | 364 | 9 | 532171 | -265721 |
| 5 | 666 | 665 | 11 | 1772893 | -885781 |

**Method 4:**
Introducing the transformations

$$a = u+v, b = u-v, c = 2\alpha \quad (9)$$

in (4), we have

$$uv = 2\alpha^3 \quad (10)$$

One may choose $u$ and $v$ suitably in (10) and using (9) the corresponding values of $x$ and $y$ satisfying the system of equations (1) to (3) are obtained.

Choice 1:

$u=\alpha^3, v=2$

$\therefore a=\alpha^3+2, b=\alpha^3-2$

Thus, $x=3(\alpha^3+2)^2+(\alpha^3-2)^2=4\alpha^6+8\alpha^3+16$

$y=-(\alpha^3+2)^2-(\alpha^3-2)^2=-(2\alpha^6+8)$

Choice 2:

$u=2\alpha^3, v=1$

$\therefore a=2\alpha^3+1, b=2\alpha^3-1$

Thus, $x=3(2\alpha^3+1)^2+(2\alpha^3-1)^2=16\alpha^6+8\alpha^3+4$

$y=-(2\alpha^3+1)^2-(2\alpha^3-1)^2=-(8\alpha^6+2)$

Choice 3:

$u=2\alpha^2, v=\alpha$

$\therefore a=2\alpha^2+\alpha, b=2\alpha^2-\alpha$

Thus, $x=3(2\alpha^2+\alpha)^2+(2\alpha^2-\alpha)^2=16\alpha^4+8\alpha^3+4\alpha^2$

$y=-(2\alpha^2+\alpha)^2-(2\alpha^2-\alpha)^2=-(8\alpha^4+2\alpha^2)$

Choice 4:

$u=\alpha^2, v=2\alpha$

$\therefore a=\alpha^2+2\alpha, b=\alpha^2-2\alpha$

Thus, $x=3(\alpha^2+2\alpha)^2+(\alpha^2-2\alpha)^2=4\alpha^4+8\alpha^3+16\alpha^2$

$y=-(\alpha^2+2\alpha)^2-(\alpha^2-2\alpha)^2=-(2\alpha^4+8\alpha^2)$

**Method 5:**

The introductions of the transformations

$$a=u+2k^3v, b=u-2k^3v, c=2k\alpha \tag{11}$$

in (4), leads to

$$uv=\alpha^3 \tag{12}$$

One may choose $u$ and $v$ suitably in (11) and using (12) the corresponding values of $x$ and $y$ satisfying the system of equations (1) to (3) are obtained.

Choice 5:

$u=\alpha^3, v=1$

$\therefore a=\alpha^3+2k^3, b=\alpha^3-2k^3$

Thus, $x=3(\alpha^3+2k^3)^2+(\alpha^3-2k^3)^2=4\alpha^6+8k^3\alpha^3+16k^6$

$y=2(\alpha^3+2k^3)^2-(4\alpha^6+8k^3\alpha^3+16k^6)=-(2\alpha^6+8k^6)$

Choice 6:

$u=1, v=\alpha^3$

$\therefore a=1+2k^3\alpha^3, b=1-2k^3\alpha^3$

Thus, $x=3(1+2k^3\alpha^3)^2+(1-2k^3\alpha^3)^2=16k^6\alpha^6+8k^3\alpha^3+4$

$y=2(1+2k^3\alpha^3)^2-(16k^6\alpha^6+8k^3\alpha^3+4)=-(8k^6\alpha^6+2)$

Choice 7:

$u=\alpha^2, v=\alpha$

$\therefore a=\alpha^2+2k^3\alpha, b=\alpha^2-2k^3\alpha$

Thus, $x=3(\alpha^2+2k^3\alpha)^2+(\alpha^2-2k^3\alpha)^2=4\alpha^4+16k^6\alpha^2+8k^3\alpha^3$

$$y=2(\alpha^2+2k^3\alpha)^2-(4\alpha^4+16k^6\alpha^2+8k^3\alpha^3)=-(8k^6\alpha^2+2\alpha^4)$$

Choice 8:

$$u=\alpha, v=\alpha^2$$

$$\therefore a=\alpha+2k^3\alpha^2, b=\alpha-2k^3\alpha^2$$

Thus, $x=3(\alpha+2k^3\alpha^2)^2+(\alpha-2k^3\alpha^2)^2=4\alpha^2+8k^3\alpha^3+16k^6\alpha^4$

$$y=2(\alpha+2k^3\alpha^2)^2-(4\alpha^2+8k^3\alpha^3+16k^6\alpha^4)=-(2\alpha^2+8k^6\alpha^4)$$

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