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Some Applications of Minimal b-γ-Open Sets

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Abstract:–We characterize minimal b-γ-open sets in topological spaces. We show that any nonempty subset of a minimal b-γ-open set is pre b-γ-open. As an application of a theory of minimal b-γ-open sets, we obtain a sufficient condition for a b-γ-locally finite space to be a pre b-γ-Hausdorff space.

Keywords:–b-γ-open, minimal b-γ-open, pre b-γ-open, finite b-γ-open, b-γ-locally finite, pre b-γ-Hausdorff space

I. INTRODUCTION

Andrijevic [1] introduced and investigated the notions of b-open sets, and Kasahara [4] defined the concept of an operation on topological spaces. Ogata [5] introduced the concept of γ-open sets and investigated the related topological proporties of the associated topology τ_γ and τ, where τ_γ is the collection of all γ-open sets. In this paper, we study fundamental properties of minimal b-γ-open sets and apply them to obtain some results in topological spaces. In Section 3, we characterize minimal b-γ-open sets. In Section 4, we study minimal b-γ-open sets in b-γ-locally finite spaces. In Section 5, we apply the theory of minimal b-γ-open sets to study pre b-γ-open sets. Finally, we show that some conditions on minimal b-γ-open sets implies pre b-γ-Hausdorffness of a space.

II. PRELIMINARIES

The complement of a b-open set is said to be b-closed. The family of all b-open sets is denoted by BO(X, τ).

Definition 2.1. [4] Let (X, τ) be a topological space. An operation γ on the topology τ is a mapping from τ to power set P(X) of X such that V ⊆ γ(V) for each V ∈ τ, where γ(V) denotes the value of γ at V. It is denoted by γ : τ → P(X).

Definition 2.2. [5] A subset A of a topological spac (X, τ) is called γ-open set if for each x ∈ A there exists an open set U such that x ∈ U and γ(U) ⊆ A.

Definition 2.3. [2] Let γ be a mapping on BO(X) in to P(X) and γ : BO(X) → P (X) is called an operation on BO(X), such that V ⊆ γ(V) for each V ∈ BO(X).

Definition 2.4. [2] A subset A of a space X is called b-γ-open if for each x ∈ A, there exists a b-open set U such that x ∈ U and γ(U) ⊆ A.

Definition 2.5. [2] Let A be a subset of (X, τ), and γ : BO(X) → P (X) be an operation on BO(X). Then the b-γ-closure of A is denoted by τ_γ-bCl(A) and defined as τ_γ-bCl(A)= ∩{ F : F is b-γ-closed and A ⊆ F }.

Theorem 2.6. [2] For a point x ∈ X, x ∈ τ_γ-bCl(A) if and only if for every b-γ-open set V of X containing x, A ∩ V ≠ φ.

Definition 2.7. [2] An operation γ on BO(X) is said to be b-regular if for every b-open sets U and V of each x ∈ X, there exists a b-open set W of x such that γ(W) ⊆ γ(U) ∩ γ(V).

Proposition 2.8. [2] Let γ be a b-regular operation on BO(X). If A and B are b-γ-open sets in X, then A ∩ B is also a b-γ-open set.

Theorem 2.9. [2] Let A and B be subsets of a topological space (X, τ) and γ : BO(X) → P (X) an operation on BO(X, τ). Then we have the following properties:

(1) If A ⊆ B, then τ_γ-bCl(A) ⊆ τ_γ-bCl(B).

(2) If γ : BO(X) → P (X) is b-regular, then τ_γ-bCl(A ∪ B) = τ_γ-bCl(A) ∪ τ_γ-bCl(B) holds.

III. MINIMAL B-Γ-OPEN SETS

In view of the definition of minimal γ-open sets [3], we define minimal b-γ-open sets as:

Definition 3.1. Let X be a space and A ⊆ X a b-γ-open set. Then A is called a minimal b-γ-open set if φ and A are the only b-γ-open subsets of A.

The folloing examples shows that minimal b-γ-open sets and minimal γ-open sets are independent of each other.

Example 3.2. Consider X = {a, b, c} with the topology τ = {φ, X}. Define an operation γ : BO(X) → P (X) by γ(A) = A. The b-γ-open sets are φ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c} and X. Here {a} is minimal b-γ-open but not minimal γ-open. Also we consider X = {a, b, c} with the topology τ = {φ, {a, b}, X}. Define γ : BO(X) → P (X) as γ(A) = A, the set {a, b} is minimal γ-open but not minimal b-γ-open.
Proposition 3.3. Let X be a space. Then:
(1) Let A be a minimal b-γ-open set and B a b-γ-open set. Then A ∩ B = φ or A ⊆ B, where γ is b-regular.
(2) Let B and C be minimal b-γ-open sets. Then B ∩ C = φ or B = C, where γ is b-regular.
Proof. (1) Let B be a b-γ-open set such that A ∩ B ≠ φ. Since A is a minimal b-γ-open set and A ∩ B ⊆ A, we have A ∩ B = A. Therefore A ⊆ B.
(2) If B ∩ C ≠ φ, then we see that B ⊆ C and C ⊆ B by (1). Therefore B = C.
Proposition 3.4. Let A be a minimal b-γ-open set. If x is an element of A, then A ⊆ B for any b-γ-open neighborhood B of x, where γ is b-regular.
Proof. Let B be a b-γ-open neighborhood of x such that A ⊄ B. Since γ is b-regular operation, then A ∩ B is b-γ-open set such that A ∩ B ⊆ A and A ∩ B ≠ φ. This contradicts our assumption that A is a minimal b-γ-open set.

The following example shows that the condition that γ is b-regular is necessary for the above proposition.
Example 3.5. Consider X = {a, b, c} with the topology τ = {φ, {a}, {b}, {a, b}, {a, c}, X}. Define an operation γ on BO(X) by γ(A) = A if b ∈ A and γ(A) = Cl(A) if b∉ A. Then calculations show that the operation γ is not b-regular. Clearly A = {a, c} is a minimal b-γ-open set. Thus for a ∈ A, there exist a b-γ-open set B = {a, b} of a such that A ⊄ B.
Proposition 3.6. Let A be a minimal b-γ-open set. Then for any element x of A, A= ∩{ B : B is b-γ-open neighborhood of x}, where γ is b-regular.
Proof. By Proposition 3.4 and the fact that A is b-γ-open neighborhood of x, we have A ⊆ ∩{ B : B is b-γ-open neighborhood of x } ⊆ A. Therefore we have the result.
Proposition 3.7. Let A be a minimal b-γ-open set in X and x ∈ X such that x ∉ A. Then for any b-γ-open neighborhood C of x, C ∩ A = φ or A ⊆ C, where γ is b-regular.
Proof. Since C is a b-γ-open set, we have the result by Proposition 3.3.
Corollary 3.8. Let A be a minimal b-γ-open set in X and x ∈ X such that x ∉ A. Define A_x = ∩{ B: B is b-γ-open neighborhood of x}. Then A_x ∩ A = φ or A ⊆ A_x, where γ is b-regular.
Proof. If A ⊆ B for any b-γ-open neighborhood B of x, then A ⊆ ∩{ B: B is b-γ-open neighborhood of x}. Therefore A ⊆ A_x. Otherwise there exists a b-γ-open neighborhood B of x such that B ∩ A = φ. Then we have A_x ∩ A = φ.
Corollary 3.9. If A is a nonempty minimal b-γ-open set of X, then for a nonempty subset C of A, A ⊆ τ_γ-bCl(C), where γ is b-regular.
Proof. Let C be any nonempty subset of A. Let y ∈ A and B be any b-γ-open neighborhood of y. By Proposition 3.4, we have A ⊆ B and C = A ∩ C ⊆ B ∩ C. Thus we have B ∩ C ≠ φ and hence y ∈ τ_γ-bCl(C). This implies that A ⊆ τ_γ-bCl(C). This copmletes the proof.
Proposition 3.10. Let A be a nonempty b-γ-open subset of a space X. If A ⊆ τ_γ-bCl(C), then τ_γ-bCl(A) = τ_γ-bCl(C), for any nonempty subset C of A.
Proof. For any nonempty subset C of A, we have τ_γ-bCl(C) ⊆ τ_γ-bCl(A). On the other hand, by supposition we see τ_γ-bCl(A) ⊆ τ_γ-bCl(τ_γ-bCl(C)) = τ_γ-bCl(C) implies τ_γ-bCl(A) ⊆ τ_γ-bCl(C). Therefore we have τ_γ-bCl(A) = τ_γ-bCl(C) for any nonempty subset C of A.
Proposition 3.11. Let A be a nonempty b-γ-open subset of a space X. If τ_γ-bCl(A) = τ_γ-bCl(C), for any nonempty subset C of A, then A is a minimal b-γ-open set.
Proof. Suppose that A is not a minimal b-γ-open set. Then there exists a nonempty b-γ-open set B such that B ⊆ A and hence there exists an element x ∈ A such that x ∉ B.Then we have τ_γ-bCl({ x }) ⊆ (X \ B) implies that τ_γ -bCl({x}) ≠ τ_γ-bCl(A). This contradiction proves the proposition.

Combining Corollary 3.9 and Propositions 3.10 and 3.11, we have:
Theorem 3.12. Let A be a nonempty b-γ-open subset of space X. Then the following are equivalent:
(1) A is minimal b-γ-open set, where γ is b-regular.
(2) For any nonempty subset C of A, A ⊆ τ_γ-bCl(C).
(3) For any nonempty subset C of A, τ_γ-bCl(A) = τ_γ-bCl(C).
Definition 3.13. Let A be a subset of (X, τ), and γ : BO(X) → P (X) be an operation on BO(X). Then the b-γ-interior of A is denoted by τ_γ-bInt(A) and defined as τ_γ-bInt(A)= ∪{U : U is b-γ-open and U ⊆ A}.
Definition 3.14. A subset A of a space X is called a pre b-γ-open set if A ⊆ τ_γ-bInt(τγ -bCl(A)). The family of all pre b-γ-open sets of X will be denoted by PBOγ(X).
Definition 3.15. A space X is called pre b-γ-Hausdorff if for each x, y ∈ X, x ≠ y there exist subsets U and V of PBOγ(X) such that x ∈ U, y ∈ V, and U ∩ V = φ.

Theorem 3.16. Let A be a minimal b-γ-open set. Then any nonempty subset C of A is a pre b-γ-open set, where γ is b-regular.
Proof. By Corollary 3.9, we have $A \subseteq \tau_\gamma$-bCl(C) implies τ_γ-bInt(A) $\subseteq \tau_\gamma$-bInt(τ_γ-bCl(C)). Since A is a b-γ-open set, we have $C \subseteq A = \tau_\gamma$-bInt(A) $\subseteq \tau_\gamma$-bInt(τ_γ-bCl(C)) or $C \subseteq \tau_\gamma$-bInt(τ_γ-bCl(C)), that is C pre b-γ-open. Hence the proof.

Theorem 3.17. Let A be a minimal b-γ-open set and B be a nonempty subset of X. If there exists a b-γ-open set C containing B such that $C \subseteq \tau_\gamma$-bCl($B \cup A$), then $B \cup D$ is a pre b-γ-open set for any nonempty subset D of A, where γ is b-regular.

Proof. By Theorem 3.12 (3), we have τ_γ-bCl($B \cup D$) = τ_γ-bCl(B) $\cup$ τ_γ-bCl(D) = τ_γ-bCl(B) $\cup$ τ_γ-bCl(A) = τ_γ-bCl($B \cup A$). By supposition $C \subseteq \tau_\gamma$-bCl($B \cup A$) = τ_γ-bCl($B \cup D$) implies τ_γ-bInt(C) $\subseteq \tau_\gamma$-bInt(τ_γ-bCl($B \cup D$)). Since C is a b-γ-open neighborhood of B, namely C is a b-γ-open such that $B \subseteq C$, we have $B \subseteq C = \tau_\gamma$-bInt(C) $\subseteq \tau_\gamma$-bInt(τ_γ-bCl($B \cup D$)). Moreover we have τ_γ-bInt(A) $\subseteq \tau_\gamma$-bInt(τ_γ-bCl($B \cup A$)), for τ_γ-bInt(A) = $A \subseteq \tau_\gamma$-bCl(A) $\subseteq \tau_\gamma$-bCl(B) $\cup$ τ_γ-bCl(A) = τ_γ-bCl($B \cup A$). Since A is a b-γ-open set, we have $D \subseteq A = \tau_\gamma$-bInt(A) $\subseteq \tau_\gamma$-bInt(τ_γ-bCl($B \cup A$)) = τ_γ-bInt(τ_γ-bCl($B \cup D$)). Therefore $B \cup D \subseteq \tau_\gamma$-bInt($\tau_\gamma$-bCl($B \cup D$)) implies $B \cup D$ is a pre b-γ-open set.

Corollary 3.18. Let A be a minimal b-γ-open set and B a nonempty subset of X. If there exists a b-γ-open set C containing B such that $C \subseteq \tau_\gamma$-bCl(A), then $B \cup D$ is a pre b-γ-open set for any nonempty subset D of A, where γ is b-regular.
Proof. By assumption, we have $C \subseteq \tau_\gamma$-bCl(B) $\cup$ τ_γ-bCl(A) = τ_γ-bCl($B \cup A$). By Theorem 3.17, we see that $B \cup D$ is a pre b-γ-open set.

IV. FINITE B-Γ-OPEN SETS

In this section, we study some properties of minimal b-γ-open sets in finite b-γ-open sets and b-γ-locally finite spaces.
Proposition 4.1. Let X be a space and $\varphi \neq B$ a finite b-γ-open set in X. Then there exists at least one (finite) minimal b-γ-open set A such that $A \subseteq B$.
Proof. Suppose that B is a finite b-γ-open set in X. Then we have the following two possibilities:
(1) B is a minimal b-γ-open set.
(2) B is not a minimal b-γ-open set.
In case (1), if we choose B = A, then the proposition is proved. If the case (2) is true, then there exists a nonempty (finite) b-γ-open set B_1 which is properly contained in B. If B_1 is minimal b-γ-open, we take $A = B_1$. If B_1 is not a minimal b-γ-open set, then there exists a nonempty (finite) b-γ-open set B_2 such that $B_2 \subseteq B_1 \subseteq B$. We continue this process and have a sequence of b-γ-open sets $... \subseteq B_m \subseteq ... \subseteq B_2 \subseteq B_1 \subseteq B$. Since B is a finite, this process will end in a finite number of steps. That is, for some natural number k, we have a minimal b-γ-open set Bk such that $B_k = A$. This completes the proof.
Definition 4.2. A space X is said to be a b-γ-locally finite space, if for each $x \in X$ there exists a finite b-γ-open set A in X such that $x \in A$.
Corollary 4.3. Let X be a b-γ-locally finite space and B a nonempty b-γ-open set. Then there exists at least one (finite) minimal b-γ-open set A such that $A \subseteq B$, where γ is b-regular.
Proof. Since B is a nonempty set, there exists an element x of B. Since X is a b-γ-locally finite space, we have a finite b-γ-open set Bx such that $x \in$ Bx . Since $B \cap B_x$ is a finite b-γ-open set, we get a minimal b-γ-open set A such that $A \subseteq B \cap B_x \subseteq B$ by Proposition 4.1.
Proposition 4.4. Let X be a space and for any $\alpha \in I$, B_α a b-γ-open set and $\varphi \neq A$ a finite b-γ-open set. Then $A \cap (\cap_{\alpha \in I} B_\alpha)$ is a finite b-γ-open set, where γ is b-regular.
Proof. We see that there exists an integer n such that $A \cap (\cap_{\alpha \in I} B_\alpha) = A \cap (\cap^n_{i=1} B_{\alpha i})$ and hence we have the result.
Using Proposition 4.4, we can prove the following:
Theorem 4.5. Let X be a space and for any $\alpha \in I$, B_α a b-γ-open set and for any $\beta \in J$, A_β a nonempty finite b-γ-open set. Then $(\cup_{\beta \in J} A_\beta) \cap (\cap_{\alpha \in I} B_\alpha)$ is a b-γ-open set, where γ is b-regular.

V. APPLICATIONS

Let A be a nonempty finite b-γ-open set. It is clear, by Proposition 3.3 and Proposition 4.1, that if γ is b-regular, then there exists a natural number m such that $\{A_1, A_2, ..., A_m\}$ is the class of all minimal b-γ-open sets in A satisfying the following two conditions:
(1) For any l, n with $1 \leq l, n \leq m$ and $l \neq n$, $A_l \cap A_n = \varphi$.
(2) If C is a minimal b-γ-open set in A, then there exists l with $1 \leq l \leq m$ such that $C = A_l$.
Theorem 5.1. Let X be a space and $\varphi \neq A$ a finite b-γ-open set such that A is not a minimal b-γ-open set. Let $\{A_1, A_2, ..., A_m\}$ be a class of all minimal b-γ-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup ... \cup A_m)$. Define $A_y = \cap\{$ B:

B is a b-γ-open neighborhood of y}. Then there exists a natural number k ∈ { 1, 2, ..., m} such that A_k is contained in A_y, where γ is b-regular.
Proof. Suppose on the contrary that for any natural number k ∈ { 1, 2, ..., m }, A_k is not contained in A_y. By Corollary 3.8, for any minimal b-γ-open set A_k in A, $A_k \cap A_y = \varphi$. By Proposition 4.4, $\varphi \neq A_y$ is a finite b-γ-open set. Therefore by Proposition 4.1, there exists a minimal b-γ-open set C such that $C \subseteq A_y$. Since $C \subseteq A_y \subseteq A$, we have C is a minimal b-γ-open set in A. By supposition, for any minimal b-γ-open set A_k, we have $A_k \cap C \subseteq A_k \cap Ay = \varphi$. Therefore for any natural number k ∈ { 1, 2, ..., m} , $C \neq A_k$. This contradicts our assumption. Hence the proof.
Proposition 5.2. Let X be a space and $\varphi \neq A$ be a finite b-γ-open set which is not a minimal b-γ-open set. Let $\{A_1, A_2, ..., A_m\}$ be a class of all minimal b-γ-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup ... \cup A_m)$. Then there exists a natural number k ∈ {1, 2, ..., m} such that for any b-γ-open neighborhood B_y of y, A_k is contained in B_y, where γ is b-regular.
Proof. This follows from Theorem 5.1, as $\cap\{ B : B$ is a b-γ-open of y $\} \subseteq B_y$. Hence the proof.
Theorem 5.3. Let X be a space and $\varphi \neq A$ be a finite b-γ-open set which is not a minimal b-γ-open set. Let $\{A_1, A_2, ..., A_m\}$ be the class of all minimal b-γ-open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup ... \cup A_m)$. Then there exists a natural number k ∈ {1, 2, ..., m} such that $y \in \tau_\gamma$-bCl(A_k), where γ is b-regular.
Proof. It follows from Proposition 5.2, that there exists a natural number k ∈ {1, 2, ..., m} such that $A_k \subseteq B$ for any b-γ-open neighborhood B of y. Therefore $\varphi \neq A_k \cap A_k \subseteq A_k \cap B$ implies $y \in \tau_\gamma$-bCl(A_k). This completes the proof.
Proposition 5.4. Let $\varphi \neq A$ be a finite b-γ-open set in a space X and for each k ∈ {1, 2, ..., m}, A_k is a minimal b-γ-open sets in A. If the class $\{A_1, A_2, ..., A_m\}$ contains all minimal b-γ-open sets in A, then for any $\varphi \neq B_k \subseteq A_k$, $A \subseteq \tau_\gamma$-bCl($B_1 \cup B_2 \cup ... \cup B_m$), where γ is b-regular.
Proof. If A is a minimal b-γ-open set, then this is the result of Theorem 3.12 (2). Otherwise A is not a minimal b-γ-open set. If x is any element of $A \setminus (A_1 \cup A_2 \cup ... \cup A_m)$, we have $x \in \tau_\gamma$-bCl(A1) ∪ τ_γ-bCl(A_2) ∪ ... ∪ τ_γ-bCl(A_m) by Theorem 5.3. Therefore $A \subseteq \tau_\gamma$-bCl(A_1) ∪ τ_γ-bCl(A_2) ∪ ... ∪ τ_γ-bCl(A_m) = τ_γ-bCl(B_1) ∪ τ_γ-bCl(B_2) ∪ ... ∪ τ_γ-bCl(B_m) = τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$) by Theorem 3.12 (3).
Proposition 5.5. Let $\varphi \neq A$ be a finite b-γ-open set and A_k is a minimal b-γ-open set in A, for each k ∈ {1, 2, ..., m}. If for any $\varphi \neq B_k \subseteq A_k$, $A \subseteq \tau_\gamma$-bCl($B_1 \cup B_2 \cup ... \cup B_m$) then τ_γ-bCl(A) = τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$).
Proof. For any $\varphi \neq B_k \subseteq A_k$ with k ∈ {1, 2, ..., m}, we have τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$) ⊆ τ_γ-bCl(A). Also, we have τ_γ-bCl(A) ⊆ τ_γ-bCl(τ_γ-bCl($B_1 \cup B_2 \cup ... \cup Bm$)) = τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$). Therefore we have τ_γ-bCl(A) = τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$) for any nonempty subset B_k of A_k with k ∈{1, 2, ..., m}.
Proposition 5.6. Let $\varphi \neq A$ be a finite b-γ-open set and for each k ∈ {1, 2, ..., m}, A_k is a minimal b-γ-open set in A. If for any $\varphi \neq B_k \subseteq A_k$, τ_γ-bCl(A) = τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$), then the class $\{A_1, A_2, ..., A_m\}$ contains all minimal b-γ-open sets in A.
Proof. Suppose that C is a minimal b-γ-open set in A and $C \neq A_k$ for k ∈ {1, 2, ..., m}. Then we have $C \cap \tau_\gamma$-bCl(A_k) = φ for each k ∈ {1, 2, ..., m}. It follows that any element of C is not contained in τ_γ-bCl($A_1 \cup A_2 \cup ... \cup A_m$). This is a contradiction to the fact that $C \subseteq A \subseteq \tau_\gamma$-bCl(A) = τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$). This completes the proof.

Combining Proposition 5.4, 5.5 and 5.6,we have the following theorem:
Theorem 5.7. Let A be a nonempty finite b-γ-open set and Ak a mini- mal b-γ-open set in A for each k ∈ {1, 2, ..., m}. Then the following three conditions are equivalent:
(1) The class $\{A_1, A_2, ..., A_m\}$ contains all minimal b-γ-open sets in A.
(2) For any $\varphi \neq B_k \subseteq A_k$, $A \subseteq \tau_\gamma$-bCl($B_1 \cup B_2 \cup ... \cup B_m$).
(3) For any $\varphi \neq B_k \subseteq A_k$, τ_γ-bCl(A) = τ_γ-bCl($B_1 \cup B_2 \cup ... \cup B_m$), where γ is b-regular.

Suppose that $\varphi \neq A$ is a finite b-γ-open set and $\{A_1, A_2, ..., A_m\}$ is a lass of all minimal b-γ-open sets in A such that for each k ∈ {1, 2, ..., m}, $y_k \in A_k$. Then by Theorem 5.7, it is clear that $\{y_1, y_2, ..., y_m\}$ is a pre b-γ-open set.

Theorem 5.8. Let A be a nonempty finite b-γ-open set and $\{A_1, A_2, ..., A_m\}$ is a class of all minimal b-γ-open sets in A. Let B be any subset of $A \setminus (A_1 \cup A_2 \cup ... \cup A_m)$ and B_k be any nonempty subset of A_k for each k ∈ {1, 2, ..., m}. Then $B \cup B_1 \cup B_2 \cup ... \cup B_m$ is a pre b-γ-open set.
Proof. By Theorem 5.7, we have $A \subseteq \tau_\gamma$-bCl($B_1 \cup B_2 \cup ... \cup B_m$) ⊆ τ_γ-bCl($B \cup B_1 \cup B_2 \cup ... \cup B_m$). Since A is a b-γ-open set, then we have $B \cup B_1 \cup B_2 \cup ... \cup B_m \subseteq A = \tau_\gamma$-bInt(A) ⊆ τ_γ-bInt(τ_γ-bCl($B \cup B_1 \cup B_2 \cup ... \cup B_m$)). Then we have the result.
Theorem 5.9. Let X be a b-γ-locally finite space. If a minimal b-γ-open set $A \subseteq X$ has more than one element, then X is a pre b-γ-Hausdorff space, where γ is b-regular.

Proof. Let x, y ∈ X such that x ≠ y. Since X is a b-γ-locally finite space, there exists finite b-γ-open sets U and V such that x ∈ U and y ∈ V. By Proposition 4.1, there exists the class $\{U_1, U_2, ..., U_n\}$ of all minimal b-γ-open sets in U and the class $\{V_1, V_2, ..., V_m\}$ of all minimal b-γ-open sets in V. We consider three possibilities:

(1) If there exists i of {1, 2, ..., n} and j of {1, 2, ..., m} such that $x \in U_i$ and $y \in V_j$, then by Theorem 3.16 , {x} and {y} are disjoint pre b-γ-open sets which contains x and y, respectively.
(2) If there exists i of {1, 2, ..., n} such that $x \in U_i$ and $y \notin V_j$ for any j of {1, 2, ..., m}, then we find an element y_j of V_j for each j such that {x} and $\{y, y_1, y_2, ..., y_m\}$ are pre b-γ-open sets and $\{x\} \cap \{y, y_1, y_2, ..., y_n\} = \varphi$ by Theorems 3.16, 5.8 and the assumption.
(3) If $x \notin U_i$ for any i of {1, 2, ..., n} and $y \notin Vj$ for any j of {1, 2, ..., m}, then we find elements xi of U_i and y_j of V_j for each i, j such that $\{x, x_1, x_2, ..., x_n\}$ and $\{y, y_1, y_2, ..., y_m\}$ are pre b-γ-open sets and $\{x, x_1, x_2, ..., x_n\} \cap \{y, y_1, y_2, ..., y_m\} = \varphi$ by Theorem 5.8 and the assumption. Hence X is a pre b-γ-Hausdorff space.

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