

Optimal Synthesis of Crank Rocker Mechanism for Point to Point Path Generation

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Abstract—The Present Work Introduces The Concept Of Orientation Structure Error Of The Fixed Link And Present A New Optimal Synthesis Method Of Crank Rocker Linkages For Path Generation. The Orientation Structure Error Of The Fixed Link Effectively Reflects The Overall Difference Between The Desired And Generated Path. Avoid By Making Point By Point Comparisons Between The Two Paths And Requires No Prescription Of Timing. In The Kinematic Synthesis Of Four Bar Path Generating Linkages, It Is Required To Determine The Linkage Dimensions' So That A Point On The Coupler Link Traces Out The Desired Curve. There Are Two Types Of Task For Path Generation. One Of Them Specified Only Small Number Of Points On The Path, And The Trajectory Between Any Two Specified Point Is Not Prescribed. The Concept Of Orientation Structure Error Of The Fixed Link Is Introduced. A Simple And Effective Optimal Synthesis Method Of Crank –Rocker Path Generating Linkages Is Presented.

Keywords—Four Bar Linkage, Fixed Link, Path Generation, Rocker, Link

I. INTRODUCTION

1.1 Specification of Problem: The Crank Link Length is the most fundamental parameter, controlling all the major designs as well as performance parameters. Crank Rocker Mechanism Must be designed for minimum Crank Link Length, while considering Design Variable R_{max} & R_{min} . Minimization of Crank Link Length is the objective of optimization using Genetic Algorithm.

In order to carry out the optimum design of a Crank-Rocker Mechanism, it is necessary to understand the dynamics of the problem and the interrelation of the parameters involved in Synthesis of Mechanism. These parameters can be grouped as shown in Fig. 1.1 for analysis and optimum design.

1.2 Design Methodology: The most important constraint on the problem is that on the minimization of Input Angle occurring by considering Design Variables. Due to limitations of Design Variable Specification we can calculate the minimum value for crank link Length with optimum value of Input Angle .In this work we can give maximum and minimum value of Variable for Minimization of Objective Function.

1.3 Overview of Non- Conventional Technique : The present work deals with the basic objective to minimize the Crank Link Length .The use of non- traditional algorithm with its efficient search and optimization process results in optimal Crank-Rocker Mechanism Design. In this present work, an attempt is made to use genetic algorithm for optimal design process so that we can arrive for the best possible solution. The present study involves the action of GA technique. The output results in this project work proves that this process of design using non- traditional algorithm is an efficient and effective one.

1.4 Recommended Parameters in GA : Crossover rate: Crossover rate should be high generally, about 80%-95%. (However some results show that for some problems crossover rate about 60% is the best). Mutation rate: On the other side, mutation rate should be very low. Best rates seems to be about 0.5%-1 %.

Population size: Very big population size usually does not improve performance of GA (in the sense of speed of finding solution) Good population size is about 20-30, however sometimes sizes 50-100 are reported as the best. The best population size depends on the size of encoded string (chromosomes). It means that if you have chromosomes with 32 bits, the population should be higher than for chromosomes with 16 bits. Selection: Basic roulette wheel selection can be used, but sometimes rank selection can be better. Encoding: Encoding depends on the problem and also on the size of instance of the problem. Crossover and mutation type: Operators depend on the chosen encoding and on the problem

1.5 Genetic Algorithm Terminology:

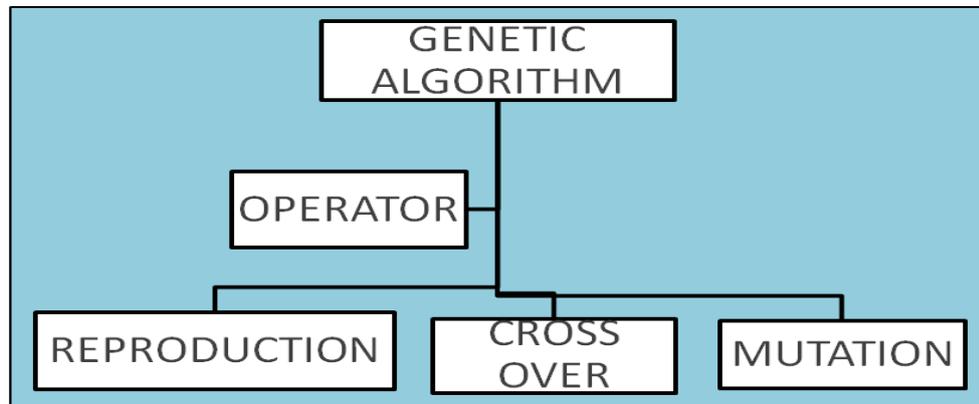


Fig. 1.1 Process of operation of GA

With the help of Fig 1.1, process of operation of GA is explain below-

Encoding: Representation, a linkage between the solutions in the phenotype space and the chromosomes is the genotype space (binary or real). Selection: Darwinian natural evolution, fitter individuals have a greater chance to reproduce offspring “survival of the fittest”.

- Parent selection to create offspring.
- Generational selection from both parents and offspring.

Crossover: Operator to be applied to two chromosomes to generate two offspring. Combines the schemata from two different solutions in various combinations .Mutation: After creation of new individuals via crossover, mutation is applied usually with a low probability to introduce random changes into the population

- replace gene values lost from the population or not initially present.
- evaluate more regions of the search space.
- avoid premature convergence.
- makes the entire search space reachable.

1.6 Plan of action using Genetic Algorithm

- Step 1 Choose a coding to represent problem parameters, a selection operator, a crossover operator, and a mutation operator. Choose population size, n, crossover probability, pc, and mutation probability, pm. Initialize a random population of string of size l. Choose a maximum allowable generation number t max. Set t = 0.
- Step 2 Evaluate each string in the population.
- Step 3 If $t > t_{max}$ or other termination criteria is satisfied. Terminate.
- Step 4 Perform reproduction on the population.
- Step 5 Perform crossover on random pairs of strings.
- Step 6 Perform mutation on every string.
- Step 7 Evaluate strings in the new population. Set $t = t + 1$ and go to Step 3.

1.7 Background of work:

A Crank Link is a mechanism member used to connect the Rocker Link, Follower Link ,Fixed Link and Interrelated Angles of the Linkages , Crank Link Length should be minimum for smooth Rotation of the Crank of the mechanism and should follow the desired curve of the Mechanism. In this work for minimization of structure error , First of all minimize the Crank Link Of the Mechanism by considering some assumption of the parameters.

It is shown that the Crank Link length of the mechanism is the most fundamental parameter, controlling all the major designs as well as performance parameters. The optimum design of Crank-Rocker Mechanism can be obtained, satisfying the constraints on design, by controlling parameter of the Mechanism.Hence it is concluded that the Crank-Rocker Mechanism be designed for minimum Crank Link Length, while satisfying the Design Variables.

1.8 Development of Non- Traditional Search

The development search work describes non-traditional search and optimization methods which are becoming popular in engineering optimization problems in the recent past. We give a brief introduction to the following techniques of optimization:

- Separable programming
- Multi-objective optimization
- Calculus of variations
- Optimal control theory
- Optimality criteria methods

Genetic algorithms
Simulated annealing
Neural - network - based methods
Optimization of fuzzy system

In some practical optimization problem, the objective and constraint functions are separable in the design variables. The separable programming techniques are useful for solving such problems. If an optimization problem involves the minimization of several objective function simultaneously with a specified constraint set, the multi-objective optimization methods can be used for its solution.

If an optimization problem involves the minimization or maximization of a functional subject to the constraints of the same type, the decision variable will not be a number, but it will be a function. The calculus of variations can be used to solve this type of optimization problems. An optimization problem that is closely related to the calculus of variations problem is the optimal control problem. An optimal control problem involves two types of variable: the control and state variables, which are related to each other by a set of differential equations. Optimal control theory can be used for solving such problems. In some optimization problems, especially those related to structural design, the necessary conditions of optimality for specialized design conditions are used to develop efficient iterative techniques to find the optimum solutions. Such techniques are known as optimality criteria methods.

In recent years, some optimization methods that are conceptually different from the traditional mathematical programming techniques have been developed. These methods are based on certain biological, molecular and neurological phenomenon. Methods known as genetic algorithms are based on the principles of natural genetics and natural selection. Simulated annealing is based on the simulation of thermal annealing of critically heated solids. Bother genetic algorithm and simulated annealing are stochastic methods that can find the global minimum with a high probability and are naturally applicable for the solution of discreet optimization problems. In neural- network- based methods, the problem is modeled as a network consisting of several neurons and the network is trained suitably to solve the optimization problem efficiently. In many practical systems, the objective functions, constraints and the design data are known only in vague and linguistic terms. Fuzzy optimization methods can be used for solving such problems.

II. MECHANICAL MODEL OF CRANK-ROCKER MECHANISM

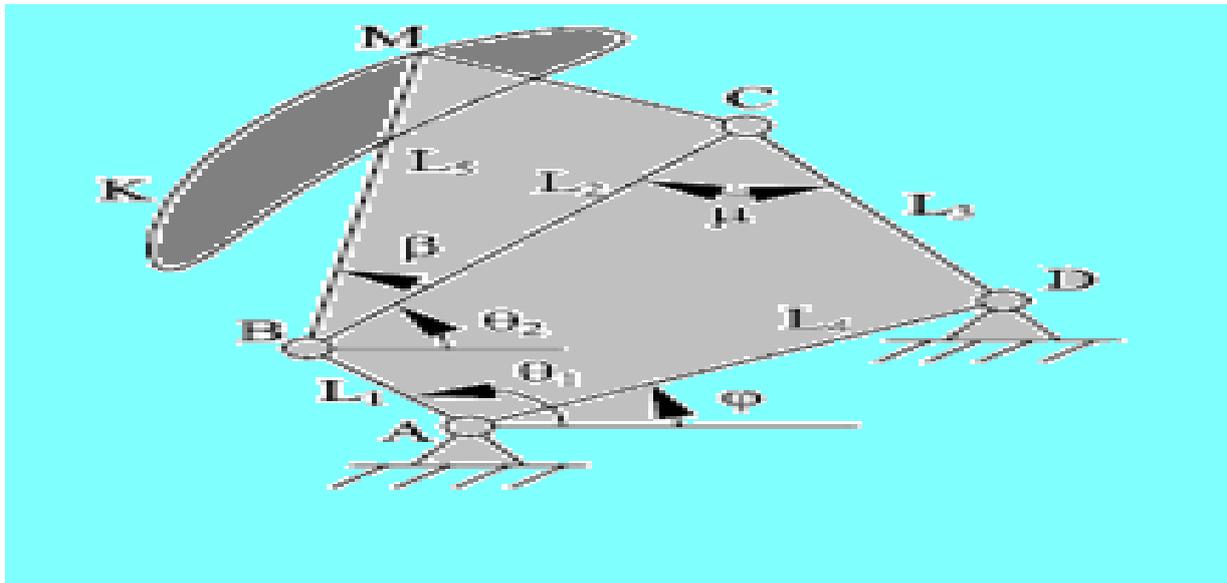


Fig 1.2 Mechanical Model of crank – Rocker Mechanism

As shown in figure 1.2 R_{max} & R_{min} are the longest and shortest distances from point A to the desired curve, respectively. When point A lies outside the desired curve, $L_1 < L_5$; When point A lies inside the desired curve, then $L_1 > L_5$. From Eqs.(1) & (2), we can obtain the values of L_1 and L_5 . The length of coupler link, L_2 , the rocker link L_3 , the Fixed Link L_4 , and the coupler angle β are also taken as Independent Design Variables, which are six in all and do not include the orientation angle Φ of the fixed link. When a set of Design Variables is selected, we make ABCD form a closed loop with point D not Fixed, and Let point M move along the desired curve, then the motion of Dyad ABM is determinant, and so is the other Dyad ADC. When point M completes cycle along the desired curve the variation of orientation angle of link AD is defined as the orientation structure Error of the fixed link. If the candidate linkage can generate the desired curve, the orientation structure error will be zero. The orientation Structural error can effectively Reflect the overall deference between desired and generated paths.

III. OPERATION OF CRANK-ROCKER MECHANISM

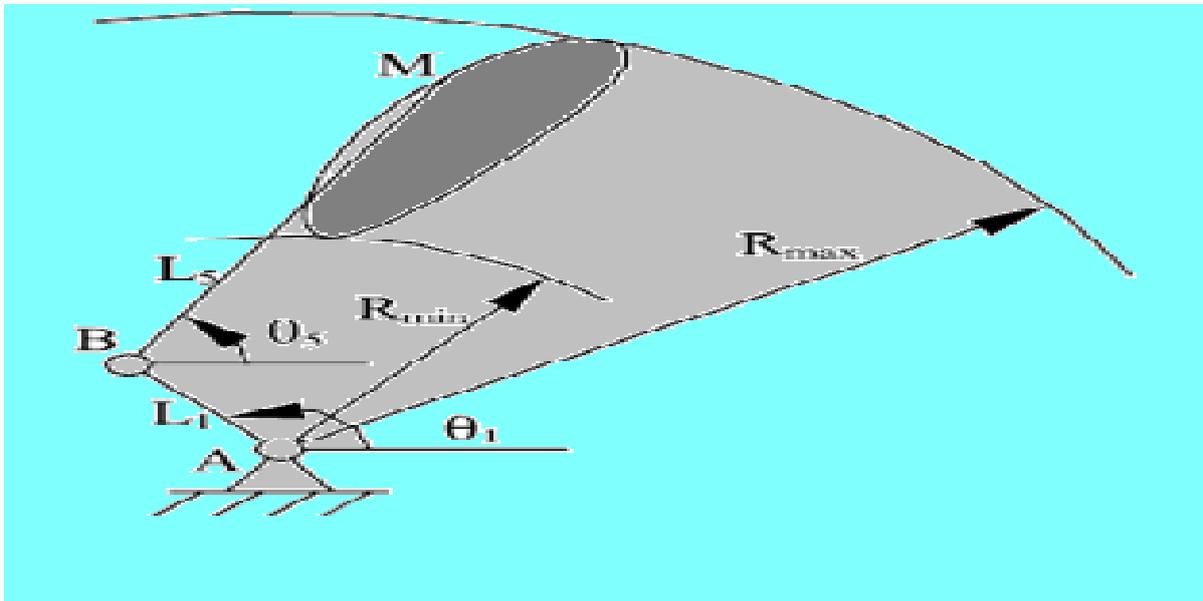


Fig 1.3 Operation Of crank-Rocker mechanism

1. Basically Crank-Rocker Mechanism having four Linkages . It having Crank Link ie. Called as Input Link, Rocker Link, Follower Link and Fixed Link.
2. When the mechanism Performing Dynamic Action then linkages start to move as per by following input Link of the Mechanism at Suitable angles of the Linkages. In this Mechanism Link 4 is fixed with some orientation angles.
3. When the Input Link Start to rotate with input angle, link 2 doing an rocking suitable for Input Link and giving the power to follow up the Link 3 and it is connected to Fixed Link.
4. There should not any clearance in the linkage joints, if there is clearance between the joint then coupler curve can not form desired curve.
5. Now the Crank Link rotate at input angle of the link1 and moving the other linkages.

IV. FORMULATION OF WORK

4.1 Mathematical model of the problem

There are three design variables for the optimal synthesis of the linkages. They are

$$X=[L_2, L_3, L_4]^T$$

The objective function is two minimize the crank length(Input Link). The co-ordinate point of A must insure that point M can generate desired curve when crank AB rotates uniformly

For the synthesis of mechanism, we have to assume the values of co-ordinate of point A(X_a, Y_a) and then we have to decide the Range for R_{max} & R_{min} as per the requirement and then apply the following equations for solving the values of L_1 & L_5 (L_1 =Input Link & L_5 =Died Link)

$$L_1 + L_5 = R_{max} \dots \dots \dots (1)$$

$$L_5 - L_1 = R_{min} \dots \dots \dots (2)$$

On solving the Equations (1) & (2)

We get ,

$$L_5 = (R_{max} + R_{min}) / 2 \dots \dots \dots (3)$$

And

$$L_1 = R_{max} - L_5 \dots \dots \dots (4)$$

Now minimization of Structure Error of the fixed link , means we have to optimize for minimization of Crank Link length.

Once we get the optimal value of crank link length , then we can calculate following parameters

Optimal Synthesis Of Crank Rocker Mechanism For Point To Point Path Generation

$$\theta_1 \text{ (Input Angle)} = \tan^{-1}[(y_m - y_a)/(x_m - x_a)] + \cos^{-1}[(L_1^2 + (x_m - x_a)^2 + (y_m - y_a)^2 - L_5^2) / (2L_1 \sqrt{(x_m - x_a)^2 + (y_m - y_a)^2})] \dots (5)$$

$$\theta_5 \text{ (Died Link Angle)} = \tan^{-1}[(y_m - y_a - L_1 \sin \theta_1) / (x_m - x_a - L_1 \cos \theta_1)] \dots (6)$$

$$\theta_2 \text{ (Rocker Link)} = \theta_5 - \beta \dots (7)$$

$$L_2 \text{ (Rocker Link)} = L_5 / \cos \beta \dots (8)$$

$$x_c \text{ (Co-ordinate of Point C)} = x_a + L_1 \cos \theta_1 + L_2 \cos \theta_2 \dots (9)$$

$$y_c \text{ (Co-ordinate of Point C)} = y_a + L_1 \sin \theta_1 + L_2 \sin \theta_2 \dots (10)$$

$$L_3 \text{ (Follower Link)} = \sqrt{(y_c - y_d)^2 + (x_c - x_d)^2} \dots (11)$$

$$L_4 \text{ (Fixed Link)} = \sqrt{(y_d - y_a)^2 + (x_d - x_a)^2} \dots (12)$$

$$\Psi \text{ (Orientation Angle)} = \tan^{-1}[(y_{c1} - y_a)/(x_{c1} - x_a)] + \cos^{-1}[(L_4^2 + (x_{c1} - x_a)^2 + (y_{c1} - y_a)^2 - L_3^2) / (2L_4 \sqrt{(x_{c1} - x_a)^2 + (y_{c1} - y_a)^2})] \dots (13)$$

$$\text{Structurer Error of the fixed link, } E_s = \Psi_{\max} - \Psi_{\min} \dots (14)$$

$$\text{Average Orientation Angle, } \Psi_{\text{avg}} = [\Psi_{\max} + \Psi_{\min}] / 2$$

4.2 Optimization of Crank- Rocker Mechanism using Genetic Algorithm

- Objective Function : To minimize Crank Length(Input Link Length)
(Crank Length) $L_1 = R_{\max} - L_5$
Variable: Longest Distance from point A to Desired Curve(R_{\max})
& Shortest Distance From point A to Desired Curve
(i) $5 < R_{\max} < 10$ (ii) $3 < R_{\min} < 7$
As the objective is to minimize the function :-
(Crank Length) $L_1 = R_{\max} - L_5$, in the interval $5 < x_1 < 10$, $3 < x_2 < 7$

STEP 1 Binary coding to represent variable x_1 and x_2
Chose binary coding , 12 bits for variable x_1 , x_2 as shown in Table 4.4

0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1
211	210	29	28	27	26	25	24	23	22	21	20
2048	1024	512	256	128	64	32	16	8	4	2	1

Table 4.4 Binary coding table

- Now the total string length = $12 + 12 = 24$
- For 12 bits, solution accuracy is given below :
- $(\text{Upper limit} - \text{initial interval}) / (2^{\text{no. of bit}} - 1)$
- Minimum count = 0 and Maximum count = $4096 - 1 = 4095$
- The details of coding variable is shown in Table 4.5
Table 4.5 Details of coding variable

Rmax Coding	Rmin coding
Range of Rmax = $R_{\max} - R_{\min}$ $10 - 5 = 5$	Range of Rmin = $R_{\max} - R_{\min}$ $7 - 3 = 4$
$2^n = \text{Range} / \text{Accuracy}$	$2^n = \text{Range} / \text{Accuracy}$
Accuracy for Rmax = 0.0024	Accuracy for Rmin = 0.0017
Number of digits/population size (n) = 12	Number of digits/population size (n) = 12

Possible combination=212=4096	Possible combination=212=4096
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STEP-2 Iteration

Total 10 Iterations for minimization of Crank- link Length.

STEP-3 Evaluation of each string in population

STEP-4 Reproduction Of Each String in population

STEP-5 Crossover of string in Population

STEP-6 Mutation of string in population

STEP-7 Iteration

Total 10 Iterations for minimization of Crank- link Length after mutation.

V. DESIGN EXAMPLE

5.1 Oval Shape Curve generation A total of 11 points on the desired curve are specified, which are given in table 1.

A cubic parametric spline is used to interpolate the desired path , which passes through the 11 specified points . The cumulative sum of chords length between two neighboring specified points is taken as parameter of the spline. Each chords is divided in to n equal parts .

Table-5.1 Coordinates of specified points of given curve

Order of points	1	2	3	4	5	6	7	8	9	10	11
x-coordinates	4.38	3.84	3.39	3.05	2.84	3.00	3.53	3.97	4.31	4.67	4.89
y-coordinates	3.96	3.82	3.60	3.33	3.02	2.67	2.59	2.84	3.16	3.52	3.94

Table-5.2 for Result of synthesis Example

L1	L2	L3	L4	L5	¥ avg	Es
0.14	1.142	7.66	8.95	1.1125	30.16	1.48

Result of synthesis Example by using Genetic Algorithm in C++Language Programme

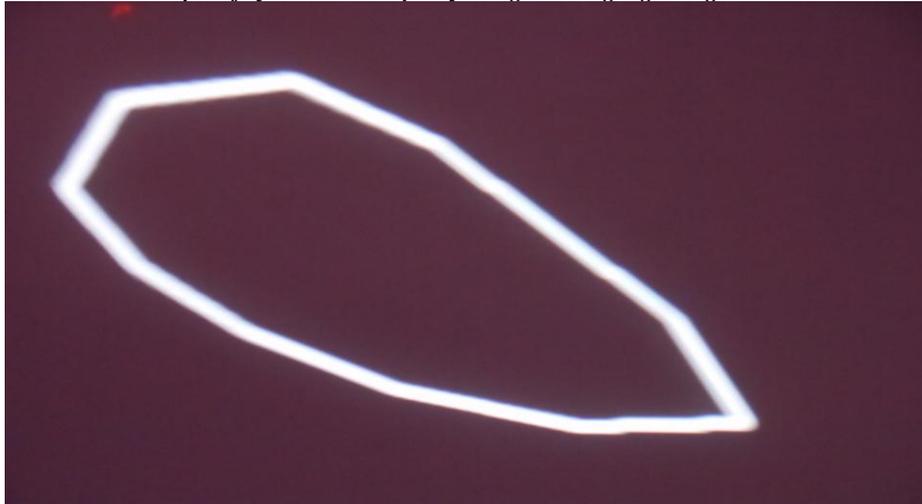
```

C:\ATC3\BIN\TC.EXE
0.161571 0.120385 0.120385 0.2 0
0.384084 0.158793 0.158793 0.5 1
0.179617 0.176755 0.176755 0.1 8
0.21702 0.198457 0.198457 0.7 6
0.300451 0.228502 0.228502 0.9 7

create NEW GENERATION ??
1-YES 0-NO0

PROGRAM TERMINATING -----
press any key
Link 1 -> 9.47033
Link 2 -> 1.292011
Link 3 -> 7.43893
Link 4 -> 8.9453
Link 5 -> 1.143834
¥ min -> 32.595706
¥ max -> 33.284004
ES -> 0.688297
¥ -> 32.932855
    
```

Output of synthesis Example by using C++Language Programme



5.2 Eight Curve generation

A total of 11 points on the desired curve are specified, which are given in table 5.4. A cubic parametric spline is used to interpolate the desired path, which passes through the 11 specified points. R_{max} & R_{min} range is $6 < R_{max} < 15$ and $4 < R_{min} < 10$. The cumulative sum of chords length between two neighbouring specified points is taken as parameter of the spline. Each chord is divided into n equal parts.

Table-5.3 Coordinates of specified points of given curve

Order of points	1	2	3	4	5	6	7	8	9	10	11
x-coordinates	4.15	4.50	4.53	4.13	3.67	2.96	2.67	2.63	2.92	3.23	3.49
y-coordinates	2.21	2.18	1.83	1.68	1.58	1.33	1.06	0.82	0.81	1.07	1.45

Table 5.4 for Result of synthesis Example

L1	L2	L3	L4	L5	¥ avg	Es
0.7295	1.9108	6.9020	8.9453	1.6917	32.69	0.4631

Result of synthesis Example by using Genetic Algorithm in C++Language Programme

```
C:\ATC3\BIN\ATC.EXE
0.077289  0.059376  0.059376  0.2  0
0.212714  0.080647  0.080647  0.5  1
0.08577  0.089224  0.089224  0.1  8
0.103312  0.099556  0.099556  0.7  6
0.14998  0.114554  0.114554  0.9  7

      create NEW GENERATION ??
      1-YES  0- NO0

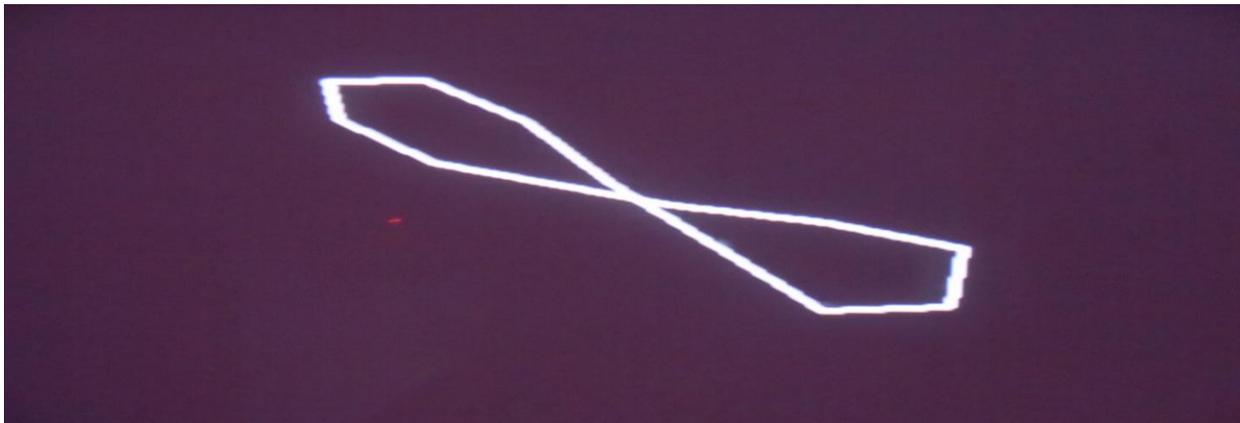
      PROGRAM TERMINATING .....
press any key
Link 1 -> 0.729548
Link 2 -> 1.910846
Link 3 -> 6.902008
Link 4 -> 8.9453
Link 5 -> 1.691697

¥ min -> 32.458957
¥ max -> 32.922035
ES -> 0.463077
¥ -> 32.690496
```

Output of synthesis Example by using C++Language Program

VI. CONCLUSIONS

The salient conclusions drawn from this analysis are :-



- Optimal design of mechanism is closely related to the Input Link Of the Mechanism .
- The Rmax & Rmin are more important for synthesis of Mechanism.
- In this study, the Crank Rocker mechanism optimization problem is solved efficiently using the non-conventional method which is genetic algorithm.
- The results obtained by GA outperformed the conventional method. The efficiency of non-conventional techniques demonstrated in this study suggests its immediate application to other design optimization problems.
- Hence we can conclude that minimization of objective function can be achieved by increasing number of iterations
- Also, GA approach handles the constraints more easily and accurately.
- Genetic algorithms can be applied to a wide range of problems Their flexibility comes at the cost of more function evaluations. There are many variants of genetic algorithms available, each with its own strengths and weaknesses.
- It is observed that optimized value obtained by genetic Algorithm are closer and they are much better than the values obtained by the conventional method.

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