*International Journal of Engineering Inventions*
*ISSN: 2278-7461, www.ijeijournal.com*
*Volume 1, Issue 9 (November2012) PP: 13-19*

# Vibration Analysis of a Portal Frame Subjected To a Moving Concentrated Load

Ms. S. J. MODAK[1], H. V. HAZARE[2],
[1]*Assistant Professor in Civil Engg., Ramdeobaba College of Engineering & Management, Nagpur – 440 009*
[2]*Professor of Civil Engg., Priyadarshini College of Engineering, Nagpur – 440 019*

**Abstract:**—The objective of the investigation is to mathematically simulate dynamics and vibrations of a portal frame subjected to a concentrated load moving on it's horizontal member with a certain constant velocity. This portal frame is a basic structure of a low length single span bridge. The emphasis is on an approach to model forced vibrations of the vertical members of the portal frame.

**Keywords:**—Bridges, Columns, Portal Frame, Influence Line, Vibrations.

## I. CONSTRUCTION OF A SHORT LENGTH BRIDGE

Fig. 1 is a schematic presentation of a short length bridge. The length is so short that the basic structure of a bridge is a simple one span portal frame $0_1$ AB $0_2$. The width of the bridge is also fairly small so that it could be considered as a particular case of a girder bridge [1]•. The material of the frame is Mild Steel (M.S.). The philosophy of the analysis is explained through a representative small scale structure with dimensions length of AB = 1005 mm, width = 50 mm and thickness is = 5mm. The vertical members $0_1$A and $0_2$B are geometry wise identical. The material of $0_1$A & that of $0_2$B is also M.S. A vehicle with total weight W is moving on AB with a constant velocity.

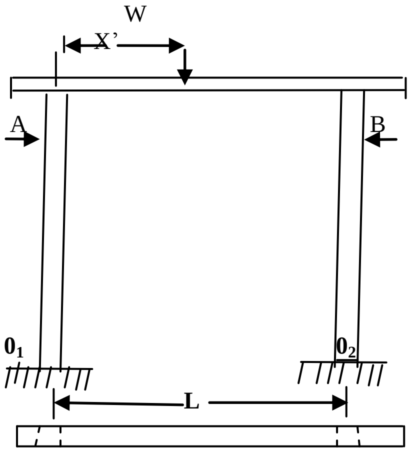


***Fig. 1 :** schematics of a portal frame for a short bridge*

The objective of the investigation is to estimate vibration response of $0_1$A & $0_2$B.

## II. FREE BODY DIAGRAMS OF A PORTAL FRAME

The free body diagrams of the members of the portal frame described in Fig. 1 are detailed in Fig. 2.

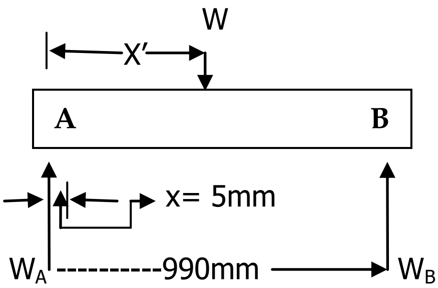


***Figure 2 (a)***

• Numbers in the square brackets denote references listed at the end of the paper.

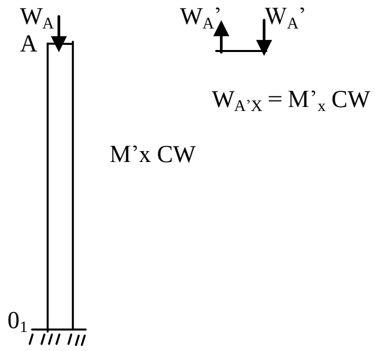


**Fig. 2(b) The net Column action = $W_A + W_A$'**

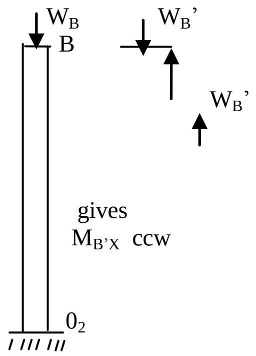


**Fig.2(c) The net column action = $W_B + W_B$'**

**Fig. 2 : free body diagrams of a portal frame**

Fig. 2(a) describes Free Body Diagram of Horizontal beam portion AB of the portal frame. W is a concentrated load (in fact the weight of the vehicle) acting on AB at an arbitrary distance x' from left end A. This x' is all the while changing as the vehicle moves from left to right with a constant velocity V. $W_A$ is a reaction of left support $0_1A$ on AB at A whereas $W_B$ is a reaction of right support $0_2B$ on AB at B.

At a section of AB at a distance x = 5mm (in this specific case) anticlock wise couple is exerted by support $0_1A$ on AB. x = 5mm because linear dimension of a cross section of $0_1A$ at the top end = 10.0 mm. This meant top section of member $0_1A$ is exerting on section X in the C.C.W. direction a moment because member $0_1A$ is imposing a moment restraint on AB to see that complete portion worth 10mm from end A of the member AB of-course on it's bottom side has to remain straight. Hence, the slope of section x of AB from A at 5mm on account of probable free displacement of neutral axis of AB as a simply supported beam due to concentrated load W is prevented.

In view of above detailed analysis for free body diagram of member AB of the portal frame the free body diagrams of the other members $0_1A$ and $0_2B$ which are vertical supports of AB get deduced logically as described in Figures 2 (b) and 2(c) respectively for $0_1A$ & $0_2B$.

As per the free body diagrams of Figures 2(b) and 2(c) the total axial load on left support $0_1A$ is summation of $W_A$ & $W_A$' whereas the same for the support $0_2B$ is $W_B$ & $W_B$'. The exact quantitative relationships for $W_A$ & $W_A$' and that of $W_B$ & $W_B$' is derived in the following section.

## III. QUANTITATIVE FORCE ANALYSIS OF THE PORTAL FRAME

### 3.1 Analysis of Member AB

Redrawing the Free Body Diagram of Member AB (Fig. 2(a) for the sake of ready reference) as shown.

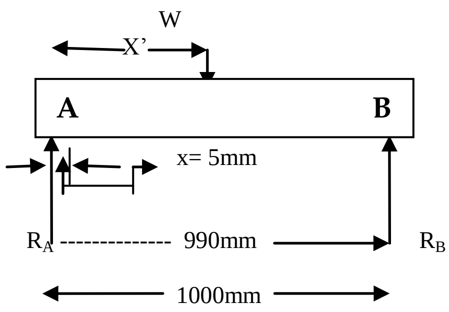


***Fig. 2(a) :** Pepete FDB of AB redrawn for read reference as above*

Refer Fig. 2(a) Pepete. The reactions $R_A$ and $R_B$ would be as under. Taking moments about point A and applying the condition of $\sum$ M=0 of static equilibrium.

$-W_{x'} + R_B \times L = 0$

$$\therefore\ R_B = \frac{W_{x'}}{L} \qquad \text{……………………… (3.1)}$$

Similarly, it can be proved that

$$\therefore\ R_A = W\left(1 - \frac{x'}{L}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.2)$$

The bending moment at x = x' = 5mm in this specific case in which numerically x = x' = 5mm is denoted as $M_{x'}$

$\therefore$ Mx' = $R_A$x

Substituting for $R_A$ from Equation (3.2)

$$\therefore Mx' = -W\left(\frac{L-x'}{L}\right)x \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

Similarly, the bending moment near support B at x = x' = 5mm, in this specific case in which numerically x = x' = 5mm is denoted as Mx' (near support B)

$\therefore$ Mx' (near support B) = $R_B$ x

Substituting the value of $R_B$ from Equation – (3.1) in the above equation

$$Mx'(Near\ support\ B) = \frac{Wx'}{L}x \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

These two bending moments are trying to induce slopes dy/dx at sections x = x' from both the supports A & B respectively in member AB treating AB as a simply supported beam. These moments and associated transverse deflection y at these sections are such that they are trying to turn the cross sections of member AB respectively C.W. and C.C.W. However, these elastic deformations are restrained by members $0_1$A & $0_2$B. Hence, top sides of $0_1$A & $0_2$B are trying to resent these deformations. This is only possible if top side of $0_1$A is exerting C.C. W. moment at x = x' near A and if top side of $0_2$B is exerting C.W. moment at x = x' near B respectively.

To this action of $0_1$A & $0_2$B on member AB, the member AB will exert equal & opposite reactions on $0_1$A & $0_2$B. With the result, member AB will exert moments Mx' given by Eq. (3.3) on top side of $0_1$A C.W. and member AB will exert moment Mx' given by Eq. (3.4) C.C. W. on top side of member $0_2$B.

These two bending moments are Mx' near A and Mx' (near support B) respectively will have senses C.W. and C.C. W. for supports $0_1$A and $0_2$B as described in Figures 2(b) and 2(c) respectively.

Similarly, the axial forces as exerted by AB on supports $0_1$A and $0_2$B will be both downwards and with magnitudes as given respectively by Equations (3.2) and (3.1) but denoted as $W_A$' and $W_B$' in figures 2(b) and 2(c) respectively.

Thus, total vertically downwards action of AB on $0_1$A is if denoted by $W_{TD01A}$

$$then\ W_{TD01A} = W_A + W_{A'} = W\left(1-\frac{x'}{L}\right) + W\left(1-\frac{x'}{L}\right)\frac{x}{x+1} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.5)$$

Similarly if $W_{TD02B}$ is total downwards action as exerted by AB on $0_2$B then

$$W_{TD02B} = \frac{Wx'}{L} + \frac{Wx'}{L}\left(\frac{x}{x+1}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

In equations (3.5) and (3.6) while deciding force actions of moments exerted by AB on top side of member $0_1$A and $0_2$B these moments are assumed to be equivalent to two forces of equal magnitudes and opposite in senses acting along the axis of members $0_1$A & $0_2$B. These are shown in free body diagrams Fig. 2(b) and Fig.2(c) respectively by forces $W_A$, $W_A$', $W_B$, $W_B$'. The forces $W_A$' and $W_B$ are assumed to have moment arm slightly more than X say more only by 1mm. Hence, they come out be equal to the second terms in both the equations (3.5) and (3.6) given above.

Further Equations (3.5) and (3.6) can be rearranged in the forms given below.

$$W_{TD0_1A} = W\left(1-\frac{x'}{L}\right)\left[1+\frac{x}{x+1}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5.a)$$

and

$$W_{TD0_2B} = \frac{Wx'}{L}\left[1+\frac{x}{x+1}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6.a)$$

In fact if it is assumed that the velocity of the vehicle moving over the bridge is V and it is constant and t is the time elapsed from the instant the vehicle has entered the bridge from left end till the time it has covered the distance x' where of-course 0< x' <L then one can say that

X' = Vt ……………………. (3.7)

Substituting for x' from Equation (3.7) in to equation (3.5.a) and (3.6.a) one gets final action on members $0_1$A & $0_2$B simulated by the below stated equations

$$W_{TD0_1A} = W\left(1-\frac{Vt}{L}\right)\left[1+\frac{x}{x+1}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.8)$$

$$W_{TD0_2B} = W\left(\frac{Vt}{L}\right)\left[1+\frac{x}{x+1}\right] \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.9)$$

## IV. VIBRATION RESPONSE OF $0_1$A AND $0_2$B

The members $0_1$A and $0_2$B the supports of a bridge are subjected to time varying forces as described by Equations (3.8) and (3.9) respectively. Since, the material of supports $0_1$A & $0_2$B is elastic and that it will experience material hysteresis, the supports are subjected to longitudinal vibrations.

The longitudinal vibrations can be ascertained by various approaches as regards various distributed mass, elasticity and damping of the material of column.

(1) Considering entire column represented by single mass, single stiffness, single damper system, popularly abbreviated in the science of vibrations of structures as SDF system

OR

(2) Considering entire column represented by multi mass, multi stiffness, multi damper system say considering 5 Lumped mass system or 5 DOF system.

OR

(3) Considering distributed mass, distributed elasticity, distributed damping system.

In the three cases stated above the column excitation i.e. the external force acting on the two columns is as described in Equations (3.8) and (3.9) respectively.

(4) In the force analysis discussed so far it is considered as if only one vehicle is entering the bridge and it moves over the bridge with constant velocity.

It may so happen that the entry of vehicles on the bridge may be in a random manner, their weights may be different, their velocities may be different, same may be moving with some acceleration and retardation over the bridge. The accelerations or retardations may be constant or variable. For each one of the above cases the loading pattern on bridge supports $0_1$A and $0_2$B may be different. Further, for all these loading patterns in view of all the three styles of vibrations described earlier, it may become imperative to ascertain vibration response. This may be considered as situation of random excitation & consequently the random vibrations of the columns of a bridge

(5) Harmonic Excitation : One more interesting pattern of vehicle entry could be such that vehicles of the same make are getting admitted from left end with same velocities and with a definite spacing. In this case the loading pattern on $0_1$A and $0_2$B could be as shown below in Fig. 3 and Fig. 4.

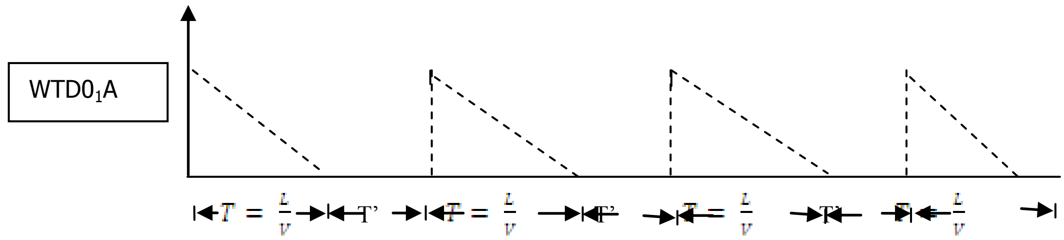


***Fig. 3 : Load pattern on $0_1$A***

T = time to complete travel on bridge
L = Length of the bridge
$V_1$ = Vehicle Velocity
T' = Constant time gap between two consecutive entries.

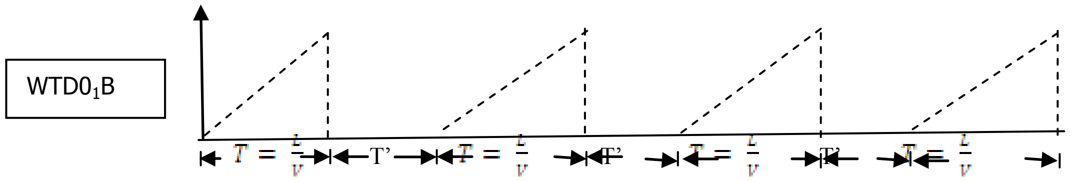


***Fig. 4 : Load pattern on $0_2$B***

T = time to complete travel on bridge
L = Length of the bridge
$V_1$ = Vehicle Velocity
T' = Constant time gap between two consecutive entries

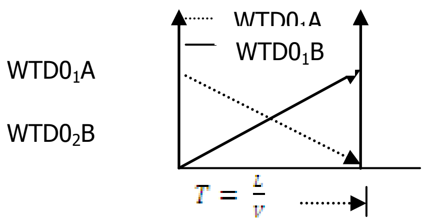


***Figure 5 : Influence lines for the bridge***

As the load patterns described in Fig. 3 and Fig. 4 one can say that the load pattern is a periodically varying function. Such a function can be represented by Harmonic Series [2]. In view of every harmonic component one can decide what could probably be the resonant frequencies. It should then be seen that none of these resonant frequencies are close to the natural frequencies of either $0_1$A or $0_1$B in order to avoid induction of higher value of stress under vibrations [3] due to complete or partial resonance.

This aspect should be checked in view of all the 1 to 3 patterns of assumptions of distribution of mass, elasticity and damping.

The paper essentially addresses the issue of vibrations of bridge column which is much less addressed so far [4 to 69].

## V. CONCLUSION

The first section of the paper details the scope of bridge column vibrations of a fairly small length bridge which may be considered as a portal frame. Sections 2 & 3 respectively detail qualitative and quantitative force analysis of a portal frame with a concentrated load acting only such that the load changes continuously its position from one end to the other. Section 4 details fairly detailed possibility of deciding vibration response of bridge columns. Detailing every possibility of vibration excitation and OR mass-stiffness-damping, distribution of two columns will precipitate individual paper. This is what is planned as a future extension of this work.

In addition the paper includes the influence line [70] of the bridge as depicted in Fig. 5.

## REFERENCES


1. F.T.K.Au, Y. S. Cheng and Y. K. Cheung, Prog. Struct. Engg. Mater; 2001, 3:299-304.
2. P.A. Pipes, “Applied Mathematics for Engineers”, Chapter -…,
3. J.P. Denhertog, “Mechanical Vibrations”
4. Willis R. Appendix to the Report of the Commissioners Appointed to Inquire into the application of Iron to Railway Structures, London, H.M. Stationery Office, 1849.
5. Fryba L., Vibration of Solids and Structures under Moving Loads, London, Thomas Telford, 1999.
6. Oisson M., Finite element, modal co-ordinate analysis of structures subjected to moving loads, Journal of Sound and Vibration 1985:99(I), 1-12.
7. Yang YB & Lin BH, Vehicle-bridge interaction analysis by dynamic condensation method, Journal of Structural Engineering 1995, 121 (II), 1636-163.
8. Yang YB & Yau JD, Vehicle-bridge interaction element of dynamic analysis, Journal of Structural Engineering, 1997, 123 (II), 1512-1518.
9. Yau JD, Yang YB & Kuo SR, Impact response of high speed rail bridges and riding comfort of rail cars, Engineering Structures 1999, 21, 836-844.
10. Cheng YS, Vibration analysis of bridges under moving vehicles and trains, Ph.D. Thesis, The University of Hong Kong, 2000.
11. Henchi K, Fafard M, Dhatt G & Talbot M., Dynamic behavior of multi-span beams under moving loads, Journal of Sound and Vibration 1997, 199(I), 33-50.
12. Wang RT, Vibration of multi-span Timoshenko beams to a moving force, Journal of Sound and Vibration 1997, 207(5), 731-742.
13. Zeng DY, Cheung YK, Au FTK & Cheng YS, Vibration of multi-span non-uniform beams under moving loads by using modified beam vibration functions, Journal of Sound and Vibration 1998, 212(3), 455-467.
14. Wang RT & Lin JS, Vibration of multi-span Timoshenko frames due to moving loads, journal of Sound and Vibration 1998, 212 (3), 417-434.
15. Lee HP, Dynamic response of a beam with a moving mass, Journal of Sound and Vibration 1996, 198(2), 249-256.
16. Esmailzadeh E & Ghorashi M., Vibration analysis of a Timoshenko beam subjected to a travelling mass, Journal of Sound and Vibration, 1997, 199(4), 615-628.
17. Cheung YK, Au FTK, Zheng DY & Cheng YS, Vibration of multi-span non-uniform bridges under moving vehicles and trains by using modified beam vibration functions, Journal of Sound and Vibration, 1999,228(3), 611-628.
18. Coussy O, Said M & Hoove JPV, Influence of random surface irregularities on the dynamic response of bridges under suspended moving loads, Journal of Sound and Vibration, 1989, 130(2), 313-320.
19. Chaterjee PK, Datta TK & Surana CS, Vibration of continuous bridges under moving vehicles, Journal of Sound and Vibration, 1994, 169 (5) 619-632.
20. Huang DZ & Wang TL, Vibration of highway steel bridges with longitudinal grades, Computers and Structures, 1998, 69(2), 235-245.
21. Au FTK, Cheng YS & Cheung YK, Effects of random road surface roughness and long-term deflection of prestressed concrete girder and cable-stayed bridges on impact due to moving vehicles, Computers and Structures, 2001, 79(8), 853-872.
22. Ricciardi G, Random vibration of beam under moving loads, Journal of Engineering Mechanics, 1994, 120(II), 2361-2380.
23. Zibdeh HS, Stochastic vibration f an elastic beam due to random moving loads and deterministic axial forces, Engineering Structures, 1995, 17, 530-535.
24. Muscolino G & Sidoti A, Dynamics of the bridges subjected to moving stochastic mass and velocity, In Fryba L & Naprstek J (eds) Structural Dynamics, Proceedings of the 4th European Conference on Structural dynamics, EURODYN’99, Rotterdam, Balkema, 1999, 711-716.

25. Yoshimura T, Hino J, Kamata T & Ananthanarayan N, Random vibration of a nonlinear beam subjected to a moving load: a finite element method analysis, Journal of Sound and Vibration, 1998, 122(2) 317-329.
26. Lee U, Revisiting the moving mass problem: oneset of separation between the mass and beam, Journal of Vibration and Acoustics, 1996, 118, 517-521.
27. Lee U, Separation between the flexible structure and the moving mass sliding on it, Journal of Sound and Vibration, 1998, 209(5), 867-877.
28. Cheng YS, Au FTK, Cheung YK & Zheng DY, On the separation between moving vehicles and bridge, Journal of Sound and Vibration, 1999, 222 (5), 781-801.
29. Fryba L, Dynamics of Railway Bridges, London, Thomas Telford, 1996.
30. Yang YB, Yau JD, Vehicle-bridge interactions and applications to high speed rail bridges, Proceedings of the National science Council, Republic of China, Part A, Physical Science and Engineering 1998, 22(2), 214-224.
31. Yang YB, Yau JD & Hsu LC, Vibration of simple beams due to the trains moving at high speeds, Engineering Structures, 1997, 19(11), 936-944.
32. Yang YB, Chang CH & Yau JD, An element for analyzing vehicle-bridge systems considering vehicle's pitching effect, International Journal for numerical methods in Engineering, 1999, 46, 1031-1047.
33. Chu KH, Garg VK & Wang TL, Impact in railway prestressed concrete bridges, Journal of Structural Engineering, 1986, 112(5), 1036-1051.
34. Wang TL, Garg VK & Chu KH, Railway bridge/vehicle interaction studies with new vehicle model, Journal of Structural Engineering, 1991, 117(7), 2099-2116.
35. Zhang QL, Vrouwenvelder A & Wardenier J, Numerical simulation of train bridge interactive dynamics, Computers and Structures 2001, 79(10), 1059-1075.
36. Le R, Ripke B & Zecher M, Ballast mats on his speed bridges, In Fryba L & Napstek J (eds)Structural Dynamics, Proceedings of the 4th European Conference on Structural Dynamics, EURODYN'99, Rotterdam, Balkema, 1999, 699-703.
37. Au FTK, Wang JJ & Cheung YK, Impact study of cable-stayed bridge under railway traffic using various models, Journal of Sound and Vibration 2001, 240 (3), 467-481.
38. Gbadeyan JA & Oni ST, Dynamic behaviour of beams and rectangular plates under moving loads, Journal of Sound and Vibration , 1995, 182 (5), 677-695.
39. Takabatake H, Dynamic analysis of rectangular plates with stepped thickness subjected to moving loads including additional mass, Journal of Sound and Vibration, 1998, 213(5), 829-842.
40. Cheung YK & Tham LG, Finite Strip Method, Boca Raton, CRC Press, 1998.
41. Wu JS, Lee ML & Lai TS, The dynamic analysis of a flat plate under a moving load by the finite element method, International Journal for Numerical Methods in Engineering, 1987, 24, 743-762.
42. Taheri MR & Ting EC, Dynamic response of plate to moving loads, finite element method, Computers and Structures, 1990, 34(3), 509-521.
43. Yener M & Chompooming K, Numerical method of lines for analysis of vehicle-bridge dynamic interaction, Computers and Structures, 1994, 53(5), 709-726.
44. Humar JL & Kashif AH, Dynamic response analysis of Slab-type bridges, Journal of Structural Engineering, 1995121(i), 48-62.
45. Henchi K, Fafard M, Talbot M & Dhatti G, An efficient algorithm for dynamic analysis of bridges under moving vehicles using a coupled modal and physical components approach, Journal of Sound and Vibration, 1998, 212(4), 663-683.
46. Cheung YK, Au FTK, Cheng YS & Zheng DY, Vibration analysis of a slab bridge under moving vehicles using the plate-vehicles strip, In:Ong JH & Liew KM (eds), Proceedings of the Asia-Pacific Vibration Conference'99, Singapore, 13-15, December 1999, 460-465.
47. Taheri MR & Ting EC, Dynamics response of plate to moving loads, structural impedance method, Computers and Structures 1989, 33(6), 1379-1393.
48. Cheung YK, Au FTK, Cheng YS & Zheng DY, Vibration of slab bridges under moving vehicles by structural impedance method and finite strip method, In:Jones N & Ghanem R (eds), Proceedings of the 13th ASCE Engineering Mechanics Conference (CDROM), Baltimore, 13-16 June, 1999.
49. Zheng DY, Vibration and stability analysis of plate type structures under moving loads by analytical and numerical methods, Ph. D. Thesis, The university of Hong Kong, 1999.
50. Wang RT & Lin TY, Vibration of multispan Mindlin plates to a moving load, Journal of the Chinese Institute of Engineers, Series A, 1996, 19(4), 467-477.
51. Wang RT & Lin TY, Random vibration of multi-span Mindlin Plate due to moving load, Journal of the Chinese Institute of Engineers, Series A, 1998, 21(3), 347-356.
52. Fleming JF & Egeselis EA, Dynamic Behaviour of a cable-stayed bridge, Earthquake Engineering and Structural Dynamics, 1980, 8(I), 1-16.
53. Zaman M, Taheri MR & Khanna A., Dynamic response of cable-stayed bridges to moving vehicles using the structural impedance method, Applied Mathematical Modelling, 1996, 20(2), 875-889.
54. Wilson JC & Gravelle W., modeling of a cable-stayed bridge for dynamic analysis, Earthquake Engineering and Structural Dynamics, 1991, 20, 707-721.
55. Wang TL & Huang DZ, Cable-stayed bridge vibration due to road surface roughness, Journal of Structural Engineering, 1992, 118(5), 1354-1374.

56. Au FTK, Wang JJ & Cheung YK, Dynamic response of cable-stayed bridges under railway traffic, In:Wang CM, Lee KH & Ang KK (eds), Computational Mechanics for the Next Millennium, Proceedings of the 4th Asia-Pacific Conference on Computational Mechanics, APCOM'99 Elsevier Science, Oxford, UK, 1999, 499-504.
57. Abdel-Ghaffar AM & Khalifa MA, Importance of cable vibration in dynamics of cable-stayed bridged, Journal of Engineering Mechanics 1991, 117(II), 2571-2589.
58. Khalifa MA, Parametric study of cable-stayed bridge responses due to traffic-induced vibration, Computers and Structures, 1993, 47(2), 321-339.
59. Yang FH & Fonder GA, Dynamic response of cable-stayed bridges under moving loads, Journal of Engineering Mechanics, 1998, 124(7), 741-747.
60. Hayashikawa T & Watanabe N, Suspension bridge response to moving loads, Journal of Engineering Mechanics Division (ASCE),1982, 108(EM6), 1051-1066.
61. Hayashikawa T., Effects of shear deformation and rotary inertia on suspension bridge response under moving loads, Transactions of the Japan Society of Civil Engineering, 1985, 15, 126-129.
62. Chaterjee PK, Datta TK & Surana CS, Vibration of suspension bridges under vehicular movement, Journal of Structural Engineering 1994, 120(3), 681-703.
63. Arzoumanidis SG & Bieniek MP, Finite element analysis of suspension bridges, Computers and Structures, 1985, 21(6), 1237-1253.
64. Brownjohn JMW, Observations on non-linear dynamic characteristics of suspension bridges, Earthquake Engineering and Structural Dynamics, 1994, 23 (12), 1351-1367.
65. Bryja D & Sniady P, Random vibration f a suspension bridge due to highway traffic, Journal of Sound and Vibration, 1988, 125(2), 379-387.
66. Bryja D & Sniady P, Specially coupled vibrations of a suspension bridge under random highway traffic, Earthquake Engineering and structural Dynamics, 1991, 20, 999-1010.
67. Yang YB, Yau J.D. & Wu. Y.S., ***"Vehicle –bridge interaction dynamics***", World Scientific Publishing Company Pvt. Ltd., Singapore, 2004.
68. FTK Au, Y.S.Cheng, & Y.K.Cheung, Vibration analysis of bridges under moving vehicles and trains: An overview, Prog. Struct. Engg. Mater : 2001, 3, pp 299-304.
69. Mario Par, "Structural Dynamics, Theory and Computation", 2nd Edition, C-135, Publishers & Distributors Pvt.Ltd., New Delhi, 2005
70. Keval Pujara, Vibrations for Engineers, Dhanpat Rai & Sons, Nai Sarak, Delhi
71. A reference on influence line