

Phase Unwrapping Via Graph Cuts

P. Poornachader Rao¹, M. Bhavya²

¹Asst. Professor Marri Laxman Reddy Institute Of Technology, Dundigal, Hyderabad.

²Asst. Professor Christu Jyoti Institute of Technology & Sciences

Abstract: Interferometric synthetic aperture radar (InSAR) is a radar remote sensing technique primarily aimed at measuring terrain altitude. Relevant InSAR features are night-and- day and all-weather operability. In particular, such features spurred InSAR based digital elevation models (DEM) into a wide spread operational use. Phase unwrapping is the inference of absolute phase from modulo- 2π phase. This is a critical step in the InSAR processing chain, yet still one of its most challenging problems; this fact makes of phase unwrapping a crucial problem to InSAR based DEM production. This thesis introduces a new energy minimization framework for phase unwrapping, building on graph cuts based binary optimization techniques. We provide an exact mini- mizer general algorithm, termed PUMF(Phase Unwrapping Max-Flow), considering con- vex pair wise pixel interaction potentials; namely we solve exactly all the phase unwrapping classical minimum L^p norm problems for $p \geq 1$. A set of experimental results illustrates the effectiveness of the proposed algorithm, and its competitiveness with state-of-the-art algorithms.

I. Introduction

This thesis presents a new *energy minimization* framework to phase unwrapping and illustrates its relevance to interferometric synthetic aperture radar (InSAR) applications. InSAR imaging comprises an ensemble of techniques that provide measurements on surface topography and surface deformation. Other characteristics, such as land classifi- cation or motion tracking, can also be obtained using InSAR. Capable of day and night, in all weather, measurements, with an ever improving spatial resolution due to developments in sensors and processing algorithms, InSAR is an increasingly popular remote sensing technique. SAR interferometry¹ utilizes two or more complex-valued images of the same scene to infer the desired information. Those images must differ by a certain feature, like a slight difference in the sensor flight track, difference in the acquisition time, or difference in the used wavelengths. In spite of a possible resemblance with optical stereoscopy, SAR interferometry works with pixel-to-pixel phase differences between the images, instead of intensity (i.e., amplitude) values; this is a crucial distinction that calls for different processing techniques, as well as a complementary set of applications[1].

Like several other imaging technologies, where the information lies in the phase rather than amplitude, in SAR interferometry phase can be observed only in the principal interval $[-\pi, \pi]^2$, i.e., the acquisition system wraps the phase around that interval. A necessary operation is, therefore, the removing of the 2π -multiple ambiguity in order to recover the true (i.e., *absolute*) phase from the *wrapped* phase: the *phase unwrapping* (PU) problem.

Being a critical step in the InSAR processing chain, phase unwrapping is also con- sidered to be one of the most challenging problems for InSAR successful application .With an ongoing wide research and operational set of InSAR applications, e.g., generation of digital elevation models, measurements of glaciers flows, and mapping of earth quakes, volcanoes and subsidence phenomena, phase unwrapping is a worthwhile problem to be addressed in the geographical information science communities

Proposed Approach:

The framework herein presented considers phase unwrapping as an optimization problem. For each pixel a certain 2π multiple is to be found, such that when added to the wrapped phase, it renders, tentatively, the absolute phase. This *estimation* is achieved through the minimization of a so-called *energy* function, using a sequence of binary optimizations, inspired by the $Z\pi M$ algorithm Each of these binary optimizations is solved via a max-flow/min-cut formulation, using recent results on binary energy minimization Accordingly, the algorithm is termed PUMF, for Phase Unwrapping Max-Flow. PUMF competes with state-of-the-art PU algorithms, in a series of shown benchmarking representative problems.

II. SAR Interferometry and Phase Unwrapping

In this chapter, we set the stage by giving some background on SAR interferometry and on phase unwrapping. Without entering into the technicalities of SAR processing, we browse through some of the main

interferometry topics, and we emphasize the critical importance of phase unwrapping for the generation of InSAR products. Next, we give a brief overview of the phase unwrapping problem.

III. Sar Interferometry

Remote sensing systems can be classified as either active or passive, according to whether they provide their own energy source for illumination or not [8]. Most of the active type systems are radar based¹ systems [9], [4]. Radar operates at the microwave range of frequencies (wavelengths between 1cm and 1m), which propagate through clouds, rain, and fog practically without disturbance. Radar systems allow, thus, a 24 hours a day and nearly all-weather operation [4]. Radar imaging spatial resolution is illustrated in Fig. 2.1 (a), which shows a radar platform at velocity V illuminating the ground in a resolution cell having a *ground range* dimension ΔX and an *azimuthal* dimension ΔY . In that sketch the sensor goes on, transmitting radar pulses and retrieving their echoes, using a side looking geometry. Still referring to Fig. 2.1 (a), range r (slant range in SAR jargon) is defined as the distance from the antenna to the target and ground range x as its projection on the ground.

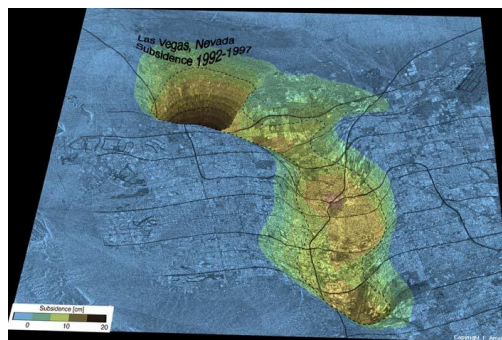
InSAR: Milestones, Concepts and Applications

In this section and in part of the next one, we follow very closely the InSAR review papers by Bamler *et al.* and Rosen *et al.*

Interferometric SAR evolved both from the development of SAR and of interferometric techniques used in radio astronomy. The very first reported application of radar interferometry was made by Rogers and Ingalls [14], in a work on the mapping of the surface reflectivity of Venus, in 1969. In 1972, Zisk published a work on measurements of the moon topography and two years later Graham first reported the first application of InSAR to Earth observation. Therein, he employed an airborne system with an ensemble of two SAR antennas constituting a *cross-track interferometer*, with which he obtained the first InSAR measurements of Earth topography.

DEM Generation sketches a typical InSAR *cross-track interferometry* (XTI) configuration. In this mode, two or more SAR antennas separated by a certain cross-track³ baseline distance acquire images over the same area via slightly different directions. As shown below, in Section 2.1.3, given the two ranges r_1 and r_2 and the two SAR antennas locations, it is possible to recover, by triangulation, the 3-D position for each ground resolution cell, and thus produce a digital elevation model (DEM).

DEM Generation sketches a typical InSAR cross-track interferometry (XTI) configuration. In this mode, two or more SAR antennas separated by a certain cross-track³ baseline distance acquire images over the same area via slightly different directions. As shown below, in Section 2.1.3, given the two ranges r_1 and r_2 and the two SAR antennas locations, it is possible to recover, by triangulation, the 3-D position for each ground resolution cell, and thus produce a digital elevation model (DEM). Unlike classical stereo techniques, which involve the selection of *sparse* homologous points in the image pairs and for which image contrast is needed, InSAR works by measuring phase differences on *every* pixel. This can be so, because as SAR is a coherent system [17], it is possible to retrieve the phase of the electromagnetic radar waves arriving at the sensors and, thus, produce phase images.



The major limitation of PSInSAR is the inhomogeneity of the persistent scatterers spatial distribution, which gives predominance to urban man-made features, or natural rocky areas.

Along-track interferometry:

Some configurations employ two radar antennas displaced in the along-track direction, following each other at a short distance, and usually mounted on the same platform. Time delay between the two acquisitions is very often in the range 10 – 100 ms, and in practice the two antennas acquire images from the same position,

such that phase topographic signature is null. This so-called *along-track interferometry* (ATI) allows the Measurement.

IV. Phase Unwrapping

We have shown above that phase holds the requested In SAR information on ground height. After acquiring two SAR images S_1 and S_2 , and producing the correspondent interferogram $I = S_1 S_2^*$, the interferometric phase can finally be obtained according to expression (2.12). This procedure encloses, however, an ambiguity that is intrinsic to the SAR phase acquisition systems [10]. Instead of the absolute (i.e, the true) phase, only its principal value, which is defined as the remainder of phase after the subtraction of the maximum 2π -multiple that is less or equal than the phase value, plus a further subtraction of a π quantity, can be detected and stored. Correspondingly the principal interval, which is the set of all possible principal values, is $]-\pi, \pi]$ ¹⁴. Accordingly, the ambiguity is also present in the interferometric phase given by (2.12)¹⁵. In mathematical terms, the principal value of the phase is also known as, basically, its *modulo- 2π* value, which can be seen as the outcome of wrapping the phase around a 2π length interval (2π radians corresponds to a full wavelength cycle), resulting in the so-called *wrapped phase*. *Phase unwrapping* (PU) is the inverse process: the recovery of the absolute phase from the wrapped phase. Its goal is, then, to remove the 2π -multiple ambiguity.

Phase unwrapping is an ill-posed problem if no further information is added. In fact, given any wrapped phase image, there is an infinite number of possible corresponding unwrapped phase images¹⁶

Main Phase Unwrapping Approaches & State Of Art Algorithm.

We have seen that Itoh condition immediately provides a phase unwrapping method, which, as explained in the previous chapter, employs a path following concept. Nevertheless, it is unrealistic to expect it to be applicable everywhere, as terrain topography can present a very rich geometry, and therefore induce phase *discontinuities*, i.e., neighbour pixels phase differences larger than π radians, which constitute violations to the Itoh condition. Moreover, decorrelation and noise also introduce phase discontinuities.

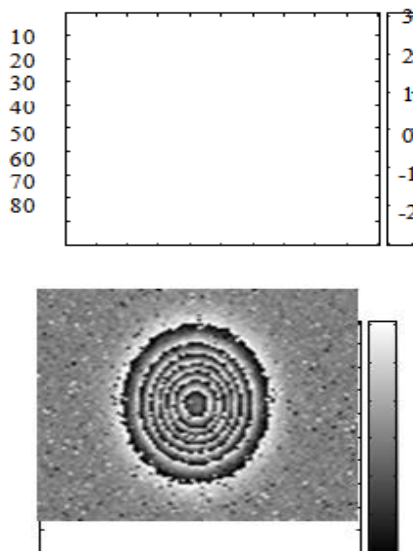
In this scenario, the phase unwrapping problem is rather more difficult and a great number of solving techniques (exact or approximate) have been proposed in the literature. In this chapter, we succinctly overview the main approaches and highlight some of the representative algorithms.

V. Puma Approach

In this chapter we propose a new phase unwrapping method. Our approach is an energy minimization one that generalizes the classical minimum L^p norm formulation introduced in

By changing into a more formal gear, we rigorously formulate the problem to be solved, develop the theoretical backgrounds of the proposed solution and, finally present and discuss our phase unwrapping algorithm.

Solution:



VI. Conclusion

We have presented a new approach to the phase unwrapping problem. Our method employs energy minimization concepts, using a binary-moves optimization scheme adopted from the zrM algorithm [6], jointly with graph cuts techniques for binary optimization.

The proposed algorithm – the PUMF – addresses the phase unwrapping problem entirely in the integers domain, therefore handling a discrete optimization problem for which it is an exact global minimizer. It was shown that, in particular, PUMF exactly solves the minimum L^p norm PU problem, for $p > 1$. Moreover, the PUMF flexibility to admit any $2r$ -periodically convex clique potential, suggests the ability to blindly deal with phase discontinuities. This was confirmed by the experiments presented, which also show the PUMF

Addressing non-convex potentials can be an interesting extension of the current work. Such potentials are well known to possess discontinuity preserving capabilities. In particular, a minimum L^0 norm PU algorithm is commonly regarded as the desirable solution for phase unwrapping, yet an NP -hard problem¹. This leads, then, to research on suitable approximate optimization techniques, which may, possibly, have the additional benefit of speeding up, even more, the PUMF algorithm.

Reference

- [1]. P. Rosen, S. Hensley, I. Joughin, F. LI, S. Madsen, E. Rodriguez, and R. Goldstein. Synthetic aperture radar interferometry. *Proceedings of the IEEE*, 88(3):333–382, March 2000.
- [2]. R. Bamler, M. Eineder, B. Kampes, H. Runge, and N. Adam. SRTM and Beyond: Current Situation and New Developments in Spaceborne InSAR. In *Proceedings of the Joint IS-PRS/EARSeL Workshop: High Resolution Mapping From Space 2003, Hannover, Germany*, 2003.
- [3]. R. Gens. Two-dimensional phase unwrapping for radar interferometry: developments and new challenges. *International Journal of Remote Sensing*, 24(4):703–710, 2003.
- [4]. G. Franceschetti and R. Lanari. *Synthetic Aperture Radar Processing*. CRC Press, 1999.
- [5]. C. Werner, U. Wegmuller, and T. Strozzi. Processing strategies for phase unwrapping for In-SAR applications. In *EUSAR'02 European Conference on Synthetic Aperture Radar*, Cologne, June 2002.
- [6]. J. Dias and J. Leitão. The zrM algorithm for interferometric imager reconstruction in SAR/SAS. *IEEE Transactions on Image Processing*, 11:408–422, April 2002.
- [7]. V. Kolmogorov and R. Zabih. What energy functions can be minimized via graph cuts? *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 26(2):147–159, February 2004.
- [8]. T. Lillesand and R. Kiefer. *Remote Sensing and Image Interpretation*. John Wiley & Sons, 2000.
- [9]. C. Elachi. *Spaceborne Radar Remote Sensing: Applications and Techniques*. IEEE Press, New York, 1988.
- [10]. J. Bioucas-Dias. Radar imaging. Radar Imaging class notes (graduate course given at IST), 2005.
- [11]. M. Soumekh. *Synthetic Aperture Radar Signal Processing with MATLAB algorithms*. Wiley Interscience, 1999.