

Study of Structural Analysis of Mechanisms - Structural Isomorphism

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ABSTRACT: *In this paper, the adjacency and incidence matrices to determine the topological structure of a mechanism up to structural isomorphism are studied and found that these methods satisfy the uniqueness and decidability conditions but are computationally inefficient. The other methods of identification of structural isomorphism's including the characteristic polynomials, the MAX code, and the degree code, are also presented. The paper is extremely useful for P.G students, research scholar and designer of mechanisms at the conceptual stage of design.*

KEYWORDS: *mechanisms, isomorphism's, matrix, kinematic graphs, degrees of freedom*

I. INTRODUCTION

Structural analysis means the study of the connection among the members of a mechanism kinematic chains and its mobility. Mainly, it is related with the fundamental relationships among the dof, number of links, number and the type of joints used in a mechanism. The structural analysis does not deal with the physical dimensions of the links. The structural analysis only deals with the general functional characteristics of a mechanism. Mostly, graph theory is used as a helping tool in the study of the kinematic structure of mechanisms. The on hand study focused only on mechanisms whose corresponding graphs are planar and contain no articulation points or bridges. A graph having a bridge means that the mechanism is a combination of two mechanisms connected in series with a common link but no common joint, or with a common joint but no common link. Such mechanisms are considered as two separate mechanisms. The topological structure of a mechanism kinematic chain is represented by a graph. An important step in structural synthesis of kinematic chains and mechanisms is the identification of isomorphic structures. Undetected isomorphic structures lead to duplicate solutions, while falsely identified isomorphisms reduce the number of feasible solutions for new designs. Several methods of identification have been proposed. Some are based on visual approaches while others are based on heuristic approaches. Each method has its own advantages and disadvantages. An ideal algorithm, however, should satisfy the following conditions [1-23].

- (a) Uniqueness. There exists a one-to-one correspondence between the kinematic chain and its mathematical representation so that structural isomorphism can be uniquely identified.
- (b) Efficiency. The algorithm for identification of isomorphic mechanisms should be simple and computationally efficient. This is essential for automated identification of structural isomorphisms.
- (c) Decidability. The mathematical representation can be transformed into a unique kinematic chain. This makes it possible for a large number of graphs or mechanisms to be stored in a computer for use by designers.

Both the adjacency and incidence matrices determine the topological structure of a mechanism up to structural isomorphism. They satisfy the uniqueness and decodability conditions. However, they are computationally inefficient. For this reason, in this paper, other methods of identification have been proposed.

II. NOTATIONS USED

The following notations have been used.

c_i : degrees of constraint on relative motion imposed by joint i .

F or dof : degrees of freedom of a mechanism.

f_i : degrees of relative motion permitted by joint i .

j : number of joints in a mechanism, assuming that all joints are binary.

j_i : number of joints with i dof; namely, j_1 denotes the number of 1-dof joints, j_2 denotes the number of 2-dof joints, and so on.

L : number of independent loops in a mechanism.

n : number of links in a mechanism, including the fixed link.

λ : degrees of freedom of the space in which a mechanism is intended to function.

S: spherical kinematic pair (dof =3).

E: plane kinematic pair (dof =3).

G: gear pair (dof =2)

Cp: cam pair (dof =2).

fp: number of passive dof in a mechanism

λ is called motion parameter. $\lambda = 6$ (for spatial mechanisms kinematic chain) and $\lambda = 3$ (for planar and spherical mechanism kinematic chain).

III. DEGREES OF FREEDOM (DOF)

The dof of a mechanism kinematic chain is the first important factor in the study of kinematics and dynamics of mechanisms. The dof of a mechanism speaks about to the number of independent parameters needed to completely describe the configuration of the mechanism in space.

IV. EQUATION FOR DEGREES OF FREEDOM

$$F = \lambda(n - j - 1) + \sum_{i=0}^j fi \text{ -----(1)}$$

Eq.(1) is called *Grübler* or *Kutzbach* criterion [12]. This criterion was also given by Ball [4]. A binary link with two end spherical joints has a redundant dof [see in Figure 1] and this is known as a passive dof as it allows the binary link to rotate freely about a line passing through the centers of the two joints without transferring any torque about that line.

In principle, a binary link with either *S - S*, *S - E*, or *E - E* pairs as its end joints possesses one passive dof [Table 1].

Table 1 Binary Links with Passive Degrees of Freedom.	
End Joints	Passive Degree of Freedom
S - S	Rotation about an axis passing through the centers of the two ball joints.
S - E	Rotation about an axis passing through the center of the ball and perpendicular to plane of the plane pair.
E - E	Sliding along an axis parallel to the line of intersection of the planes of the two E pairs. If the two planes are parallel, three passive dof exist.

Equation (1) can be represented by eq.(2).

$$F = \lambda(n - j - 1) + \sum_{i=1}^j (fi - fp) \text{ -----(2)}$$

V. LOOP MOBILITY CRITERION

$$\sum_{i=0}^j fi = F + \lambda L \text{ ----- (3)}$$

Equation (3) is known as the loop mobility criterion and is helpful in calculating the number of joint dof required for a mechanism kinematic chain to possess a given number of dof. A multi loop formation chain has been shown in Figure 4.

VI. DESIGNATION OF THE LINKS AND THEIR KINEMATIC GRAPHS

The kinematic graph and kinematic structural representations of the above three links are shown in Figure 5.

VII. CONTRACTED GRAPHS

A binary string of length k as a string of k vertices of degree 2 connected in series by k + 1 edges is represented in Figure 6. The first and last edges of a binary string are necessarily incident to non-binary vertices.

Kinematic graph	Kinematic Structure	Comments
		Binary link
		Ternary link
		Quaternary link

Figure 5: Binary, ternary, and quaternary links and their graph representation.



Figure 6: Binary strings of length one, two and three

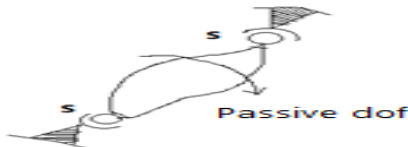


Figure 1: An S-S binary link

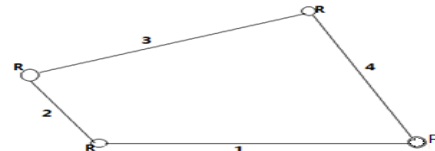


Figure 2: Four bar linkage (3-R,1-P)

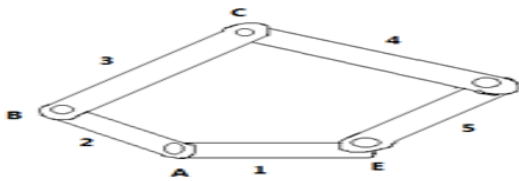
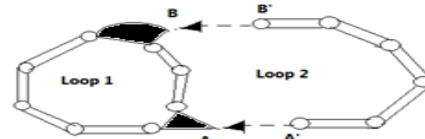


Figure 3: Five-bar linkage



Please note that A and B are two attachment points

Figure 4: Formation of a multiloop chain

A *contracted graph* is achieved by replacing every binary string in a graph with a single edge and therefore a contracted graph has no binary vertices. But, it may contain parallel edges. The process of removing a binary string and replacing it with a single edge is called a *contraction*. Figure 8 represents the contracted graph of Figure 7 in which there are two parallel edges connecting vertices 1 and 2, and two additional parallel edges connecting vertices 3 and 4.

VIII. PARTITION OF BINARY LINK CHAINS

Binary link chains of length 0, 1, 2, and 3 are called the *E*, *Z*, *D*, and *V* chains, respectively, as depicted in Figure 9.

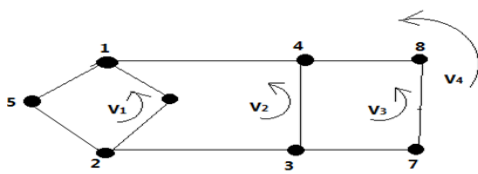


Figure 7: A planar graph [(8,10)graph]

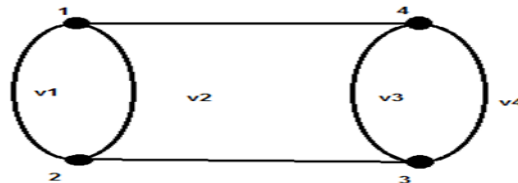


Figure 8: A contracted graph of a graph shown in Figure 7

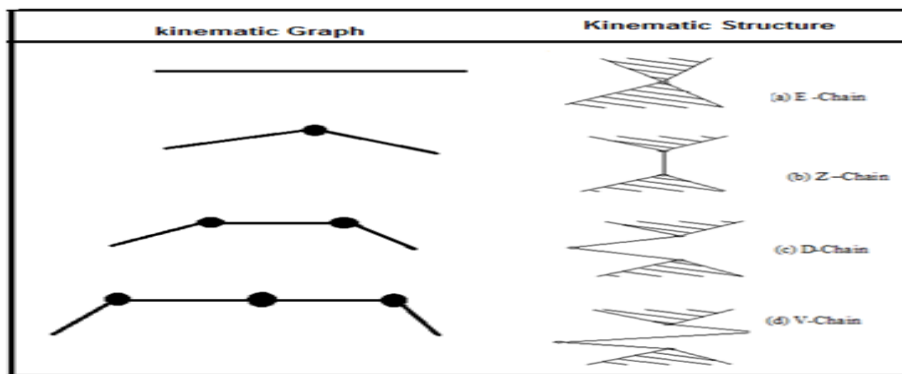


Figure 9 : Various binary link chains

IX. STRUCTURAL ISOMORPHISM

Two kinematic chains or mechanisms are said to be *isomorphic* if they share the same topological structure. In terms of graphs, there exists a one-to-one correspondence between their vertices and edges that preserve the incidence. Mathematically, structural isomorphisms can be identified by their adjacency or

incidence matrices. However, the form of an adjacency matrix is dependent on the labeling of links in a kinematic chain.

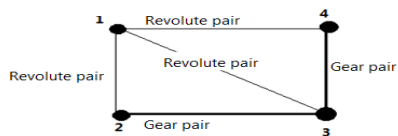


Figure 10. Graph representation of the planetary gear set shown in Figure 2

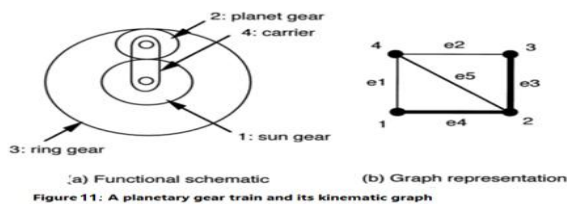


Figure 11: A planetary gear train and its kinematic graph

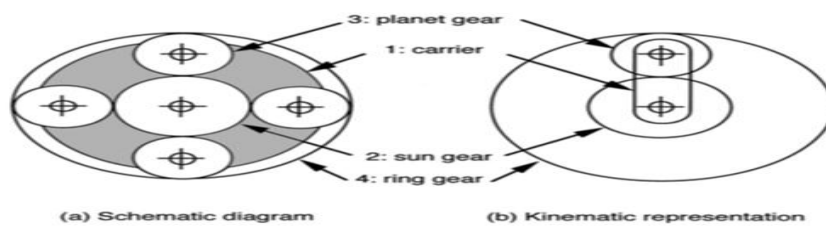


FIGURE 12 :Schematic diagram and kinematic representation.

The graph in Figure 11 is obtained from a relabeling of the vertices of the graph in Figure 10. As a result, the adjacency matrix for figure 10 is given by equation (4) while the adjacency matrix for Figure 11 is represented by eq.(5) .

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & g & 0 \\ 1 & g & 0 & g \\ 1 & 0 & g & 0 \end{bmatrix} \text{-----(4)} \quad A^* = \begin{bmatrix} 0 & g & 0 & 1 \\ g & 0 & g & 1 \\ 0 & g & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \text{-----(5)}$$

Although the graphs shown in Figures 10 and Figures 11 represent the same gear train given in Figure 12, their adjacency matrices do not assume the same form. The difference comes from the labeling of the links. In fact, the two adjacency matrices, Equations (4) and (5), are related by a permutation of the rows and the corresponding columns.

If S is a column matrix whose elements represent the labeling of the links of a kinematic chain and S* be another column matrix whose elements correspond to a relabeling of the links of the same kinematic chain. Then there exists a permutation matrix, P, such that

$$S^* = P S \text{-----(6)}$$

Here , A* is related to A by a congruence transformation,

$$A^* = P^T A P \text{-----(7)}$$

Where P^T denotes the transpose of P. Theoretically, the permutation matrix P can be derived by reordering the columns of an identity matrix. It has a positive or negative unit determinant and the transpose is equal to its inverse [17].

For example, if the column matrix for the graphs shown in Figure 10 is given by eq. (8) then the column matrix for the graph shown in Figure 11 will be represented by eq. (9). Therefore, the permutation matrix is given by eq. (10).

$$S = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \text{-----(8)} \quad S^* = \begin{bmatrix} 4 \\ 1 \\ 2 \\ 3 \end{bmatrix} \text{-----(9)}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{-----(10)}$$

clearly, Equations (8), (9), and (10) satisfy Equation (6). Substituting Equations (4) and (10) into Equation (7) yields

$$\begin{aligned}
 A^* = P^T A P &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \dots \dots \dots (11)
 \end{aligned}$$

Equations (6) and (7) constitute the definition of *structural isomorphism*. In other words, two kinematic structures are said to be isomorphic if there exists a one-to one correspondence between the links of the two kinematic chains, Equation (6), and when the links are consistently renumbered, the adjacency matrices of the two kinematic chains become identical, Equation (7).

X. PERMUTATION GROUP AND GROUP OF AUTOMORPHISMS

Here the concept of a permutation group from which a group of automorphisms of a graph is explained. Automorphic graphs are useful for elimination of isomorphic graphs at the outset.

Let there is a set of elements: a, b, c, d, e, and f. These elements may represent the vertices or edges of a graph, or the links or joints of a kinematic chain. Let these elements be arranged in a reference sequence, say (a, b, c, d, e, f). We call an alternate sequence (b, c, a, d, f, e) a permutation of (a, b, c, d, e, f), in which a → b (element a is mapped into b), b → c, c → a, d → d, e → f, and f → e. The reference sequence, (a, b, c, d, e, f), is called the identity permutation.

In a permutation, some elements may map into other elements, whereas others may map into themselves. A mapping of the type a → b → c → a, denoted by (abc), is said to form a cycle. We define the length of a cycle by the number of elements in that cycle. In particular, a cycle of length 1 maps an element into itself; that is, (d) means that d → d.

X.1 Group of Automorphisms

Considering labeled graph when its vertices are labeled by the integers 1, 2, . . . , n. In this regard, a labeled graph is mapped into another labeled graph when the n integers are permuted. For some permutations, a labeled graph may map into itself. The set of those permutations which map the graph into itself form a group called a group of automorphisms. This group of automorphisms is said to be a *vertex-induced* group [9]. Similarly, the edges of a graph may be labeled. We call the group of permutations that maps the graph into itself an *edge-induced* group of automorphisms.

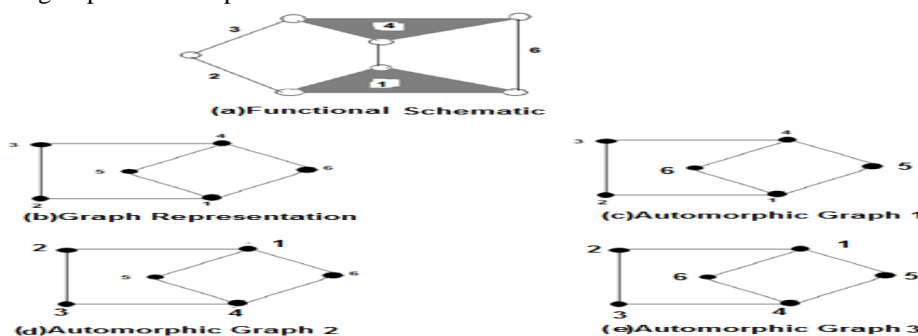


Figure 8:---Stephenson Chain, its Graph Representation, and Automorphic Graph

XI. IDENTIFICATION OF STRUCTURAL ISOMORPHISM

XI.1 Identification of Structural Isomorphism by Classification

Kinematic chains (or graphs) can be classified into families according to the number of links, number of joints, various link assortments, etc. Clearly, kinematic chains of different families cannot be isomorphic with one another. This fact has been used for classification and identification of the topological structures of kinematic chains. For example, Buchsbaum and Freudenstein [6] classified the graphs of epicyclic gear trains according to their (1) number of vertices, (2) number of edges, and (3) *vertex degree listing*. Yan and Hwang [20, 21] expanded the above classification method to include other attributes such as joint assortments, and so on.

Kinematic chains can also be classified by their corresponding contracted graphs. Obviously, two kinematic chains that belong to two different contracted graphs cannot be isomorphic. For example, Figure 13 shows two (11, 14) graphs. Both graphs contain 11 vertices and 14 edges. In addition, they share the same vertex degree listing of 6410. Hence, up to this level of classification, it appears that they might be isomorphic. But, the contracted graph of the graph shown in Figure 13(a) has been shown in Figure 14 (a), whereas the

contracted graph of the graph shown in Figure 13(b) has been shown in Figure 14 (b). Since both the contracted graphs in Figure 14(a) and 14(b) are different, so, they are not isomorphic.

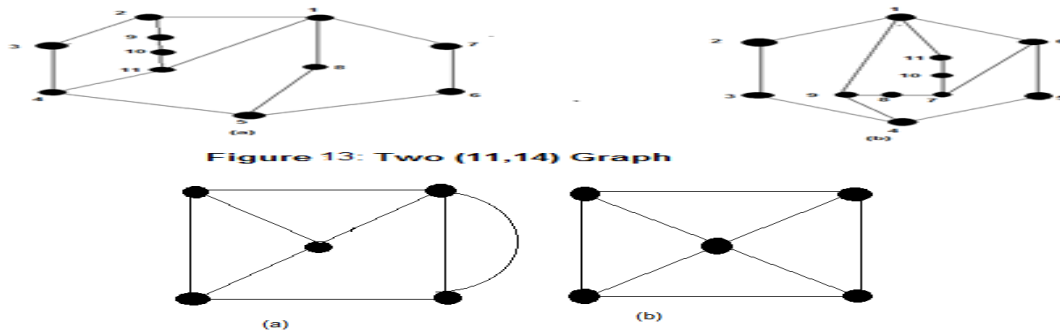


Figure 13: Two (11,14) Graph

Figure 14: Contracted graph of the graph shown in Figure 13 (a) and (b) respectively

XI.2 Identification of Structural Isomorphism by Characteristic Polynomial

We know that the problem of testing structural isomorphism is equivalent to one of determining a permutation matrix P that transforms the A into A^* for the two kinematic chains in question. For an n -link kinematic chain, there are $n!$ possible ways of labeling the links and, therefore, $n!$ possible permutation matrices. Therefore, it is impractical to identify the permutation matrix by trial-and error. Fortunately, there exists a convenient method for the determination of such a permutation matrix.

A well-known theorem of matrix algebra states that the congruence relation given by Equation (7) can exist only if the characteristic polynomials of the two adjacency matrices, A and A^* , are equal to each other; that is,

$$|xI - A| = |xI - A^*| \text{ -----(12)}$$

holds for all x , where x is a dummy variable and I is an identity matrix of the same order as A . We conclude that

Theorem 1

The adjacency matrices of two isomorphic kinematic chains possess the same characteristic polynomial.

For example, the linkage characteristic polynomial for the graph shown in Figure 10 is given by eq.(13) and the linkage characteristic polynomial for the graph shown in Figure 11 is given by eq.(14).

$$p(x) = \begin{vmatrix} x & -1 & -1 & -1 \\ -1 & x & -g & 0 \\ -1 & -g & x & -g \\ -1 & 0 & -g & x \end{vmatrix} = x^4 - (3 + 2g^2)x^2 - 4gx \text{ -----(13)}$$

$$p^*(x) = \begin{vmatrix} x & -g & 0 & -1 \\ -g & x & -g & -1 \\ 0 & -g & x & -1 \\ -1 & -1 & -1 & x \end{vmatrix} = x^4 - (3 + 2g^2)x^2 - 4gx \text{ -----(14)}$$

Since $p(x) = p^*(x)$, the two graphs are most likely isomorphic.

It should be noted that the above theorem is a necessary, but not a sufficient condition for two kinematic chains to be isomorphic. Although this condition is not completely discriminatory, it can successfully distinguish bar linkages with up to eight links. As the number of links increases, however, the probability of failing to detect structural isomorphism increases. Counter examples have been found where two nonisomorphic kinematic chains share the same characteristic polynomial [14].

For example,

Figure 15 shows two (10, 13) nonisomorphic graphs sharing the characteristic polynomial:

$$p(x) = x^{10} - 13x^8 + 53x^6 - 8x^5 - 82x^4 + 26x^3 + 39x^2 - 16x \text{ ----- (15)}$$

Figure 16 shows two (11, 14) nonisomorphic graphs sharing the characteristic polynomial:

$$p(x) = x^{11} - 14x^9 + 65x^7 - 13x^5 + 112x^3 + 32x \text{ -----(16)}$$

This method of identification requires the derivation of characteristic polynomials. Uicker and Raicu [17] presented a computer method for derivation of the coefficients numerically. Yan and Hall [18, 19] developed a set of rules for determination of the polynomials by inspection. Tsai [16] suggested the use of the random number technique to improve the computational efficiency.

Because the characteristic polynomial cannot fully identify structural isomorphisms, the method is often augmented by other techniques such as classification of kinematic chains according to their contracted graphs. For example, the graph shown in Figure 15(a) belongs to the contracted graph shown in Figure 14 (a), whereas the one shown in Figure 15(b) belongs to the Figure 14 (b) contracted graph. Therefore, these two graphs cannot

be isomorphic although they share a common characteristic polynomial. Similarly, the graphs shown in Figure 16(a) and Figure 16(b) cannot be isomorphic because they belong to two different contracted graphs shown in Figure 14(a) and Figure 14 (b) respectively.

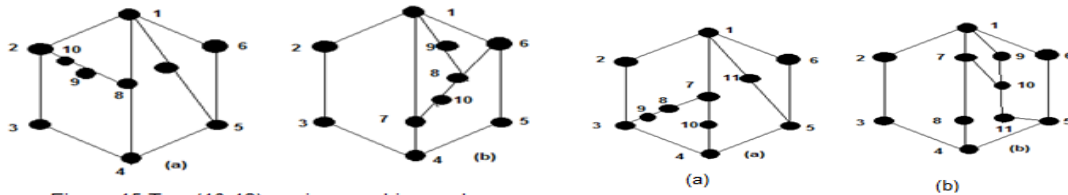


Figure 15: Two (10, 13) nonisomorphic graphs

Figure 16: Two (11, 14) nonisomorphic graphs

XI.3 Identification of Structural Isomorphism by Optimum Code

Despite the extraordinary degree of discrimination, the method of characteristic polynomial encounters difficulties in three respects. (1) The method is not decodable. (2) It is not a sufficient condition for identification of structural isomorphism. (3) The computational efficiency is poor. To overcome these difficulties, Ambekar and Agrawal [1, 2, 3] suggested a method of identification called the *optimum code*. In contrast to the characteristic polynomial method, the optimum code guarantees the decodability and positive identification of structural isomorphism. The method involves a technique for labeling the links of a kinematic chain such that a binary string obtained by concatenating the upper triangular elements of the adjacency matrix row by row, excluding the diagonal elements, is maximized. This is called the *MAX code*. To illustrate the concept, we consider the six-link Stephenson chain shown in Figure 8 (a). The corresponding labeled graph is shown in Figure 8 (b). Let the labeling of the vertices shown in Figure 8 (b) be denoted by an identity permutation $a_1 = (1)(2)(3)(4)(5)(6)$. Then the adjacency matrix is given by eq. (17).

$$A1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \text{-----(17)}$$

Excluding the diagonal elements, there are 15 binary elements in the upper triangular adjacency matrix, namely 10011, 1000, 100, 11, and 0. Writing these elements in sequence we obtain a binary string of 100111000100110, which can be converted into a decimal number as follows.

$$1001110001001102 = 214 + 211 + 210 + 29 + 25 + 22 + 21 = 20006 .$$

Figure 17(a) shows a different labeling of the vertices that corresponds to the permutation $a_2 = (1)(245)(36)$. For this labeling, the adjacency matrix becomes eq. (18).

$$A2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \text{-----(18)}$$

Therefore, the upper triangular elements of A2 form a binary string of 1110000100100 11, which is equal to a decimal number of 28819. It can be shown that, among all possible labelings of the graph, the labeling shown in Figure 17(a) leads to a maximum number. We call the number 28819 the *MAX code* of the Stephenson chain. We note that several labelings of a graph may lead to the same MAX code due to the existence of graph automorphisms. Alternately, we can also search for a labeling of the Stephenson chain that minimizes the binary string of the upper triangular elements. We call the resulting decimal number the *MIN code*. Intuitively, for the permutation $a_3 = (15) (3) (246)$ shown in Figure 17(b), we obtain a minimum binary string of 000110011101100, which gives 3308 as the MIN code.

Using the above method, the problem of testing structural isomorphism is converted into a problem of comparing the optimum codes of two kinematic chains in question.

XI.4 Identification of Structural Isomorphism by Degree Code

There is a heuristic algorithm called the *degree code* [15].

In the degree code, the vertex degrees are used as a constraint for labeling the links of a kinematic chain. Links of the same degrees are grouped together and the various groups of links are arranged in a descending order according to their vertex degree.

The procedure for finding the degree code of a kinematic chain can be summarized as follows:

1. Identify the degree of each vertex in the graph of a kinematic chain and arrange the vertices of the same degree into groups.
2. Renumber the vertices according to the descending order of vertex degrees.
3. Permute the vertices of the same degree to get a new labeling of the graph. Similar vertices, if any, can be arranged in a subgroup to further reduce the number of permutations.
4. For each permutation, calculate the decimal number of the binary string obtained by concatenating, row-by-row, the upper-right triangular elements of the corresponding adjacency matrix.
5. The maximum number obtained from all possible permutations is defined as the degree code.

For example, the degrees of vertices 1 through 6 of the graph shown in Figure 8(b) are 3, 2, 2, 3, 2, and 2, respectively. Since there are two vertices of degree 3 and four vertices of degree 2, the vertices are divided into two groups: (1, 4) and (2, 3, 5, 6). As a first attempt, we relabel the graph as shown in Figure 18(a) where integers 1 and 2 are assigned to the two vertices of degree 3. Under the new labeling, the two groups consist of (1, 2) and (3, 4, 5, 6). We notice that vertices 1 and 2 are similar. Hence, there is no need to permute these two vertices. Similarly, there is no need to permute vertices 5 and 6. Following the above procedure, it can be shown that among all possible permutations, the permutation shown in Figure 18(b) produces a maximum number. The adjacency matrix is

$$A_4 = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (19)$$

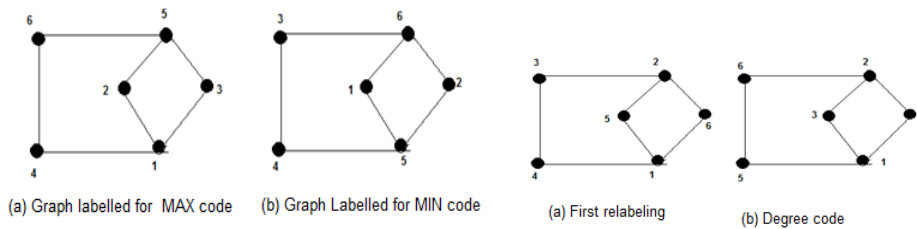


Figure 17: Stephenson chain labeled for the MAX and MIN codes

Figure 18: Stephenson chain labeled for the degree code

Therefore, the binary string is 011101101000001, which is equal to a degree code of 15169. We note that the degree code is smaller than the MAX code, because permutations of the links for the degree code are confined within each group of vertices of the same degree.

Finally, we note that the degree codes for the graphs shown in Figures 15(a) and Figures 15(b) are 28055807549442 and 28055872017416, respectively. Although the two graphs share a common characteristic polynomial, their degree codes are unequivocally different.

XII. ILLUSTRATIVE EXAMPLES

XII.2 Example 1: Group of Automorphisms of Stephenson Chain Consider the Stephenson chain shown in Figure 8(a), where the six links are labeled from 1 to 6. The corresponding graph is shown in Figure 8(b). Let this labeling of the graph be the identity permutation, $a_1 = (1)(2)(3)(4)(5)(6)$. Figures 8(c) through e show three permutations of the labeling that can be denoted as $a_2 = (1)(2)(3)(4)(56)$, $a_3 = (14)(23)(5)(6)$, and $a_4 = (14)(23)(56)$, respectively. In the following we show that the above four labeled graphs form a group of automorphisms.

Following the definition of group operation, we can construct a multiplication table:

	a_1	a_2	a_3	a_4
a_1	a_1	a_2	a_3	a_4
a_2	a_2	a_1	a_4	a_3
a_3	a_3	a_4	a_1	a_2
a_4	a_4	a_3	a_2	a_1

We conclude that every product is an element of the group; the associative law holds; a_1 is the identity element; and every element is its own inverse. Therefore, $a_1, a_2, a_3,$ and a_4 form a group of automorphisms.

Two vertices of a graph are said to be similar if they are contained in the same cycle of a permutation of a vertex-induced group of automorphisms. In the above example, vertices 1 and 4 are similar. Vertices 5 and 6, and 2 and 3 are also similar. Similar vertices have the same vertex degrees and their adjacent vertices also have the same vertex degrees. In other words, similar vertices possess the same attributes. Automorphic graphs are by

definition isomorphic. Analogously, two edges of a graph are said to be *similar* if they are contained in the same cycle of a permutation of an edge-induced group of automorphisms.

XIII. CONCLUSIONS

The basic concept of graph theory is essential for structural analysis and synthesis of mechanisms kinematic chains. The topological structure of a kinematic chain can be represented by kinematic graph easily with the help of graph theory. The Grubler equation for determination of dof does not consider the concept of redundant dof. Although, the redundant dof has no role in torque transfer from input to output link but the precision and accuracy of design depends upon it (see Table 1). There are many mechanisms (over-constrained mechanisms) those do not obey Grubler criterion. We can determine the dof of a mechanism kinematic chain using loop mobility criterion also. The light also has been thrown on the partition of binary links. Several methods of identification of structural isomorphism among mechanisms kinematic chains like identification by-classification, characteristic polynomials, optimum code and degree code has been studied. These structural characteristics are extremely useful for the development of computer algorithms for systematic enumeration of mechanisms. It is suggested that a designer should use the concept of contracted graphs to determine the structural isomorphism as it is a reliable, easy to compute method. The study is extremely helpful for the U.G./P.G. students, research scholars and designers in their early age of learning at the conceptual stage of design.

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