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Tuning of PID, SVFB and LQ Controllers Using Genetic Algorithms

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Abstract: *The Inverted Pendulum is a very popular plant for testing dynamics and control of highly non-linear plants. In the Inverted Pendulum Control problem, the aim is to move the cart to the desired position and to balance a pendulum at desired location. This paper represents stabilization of pendulum using PID, SVFB and LQR. An advantage of Quadratic Control method over the pole-placement techniques is that the former provides a systematic way of computing the state feedback control gain matrix.LQR In the mathematical model proposed here, a PID, SVFB and LQR controller is designed and all the controllers are tuned using Genetic algorithms. The simulation results of all the controllers are shown. This paper exhibits the capability of genetic algorithm to solve complex and constrained optimization problems and as a general purpose optimization tool to solve control system design problems.*

Keyword: *Inverted pendulum (IP), Mathematical modeling, PID controller, State variable feedback controller (SVFB), LQR, Genetic algorithm (GA).*

I. INTRODUCTION

The inverted pendulum is unstable [8,9,10] in the sense that it may fall any time in any direction unless a suitable control force is applied. If the designer works it right, he can get the advantages of several effects[11]. The control objective of the inverted pendulum is to swing up[4] the pendulum hinged on the moving cart by a linear motor[11] from stable position (vertically down state) to the zero state (vertically upward state) and to keep the pendulum in vertically upward state in spite of the disturbance[6,7]. They make it easy to check whether a particular algorithm [5] yields the requisite results. Several works has been reported on the inverted pendulum for its stabilization. Attempts have been made in the past to control it using classical control [3]. The purpose of the present research is to tune[1,2] the entire controller by soft computing tools. The work was made under MATLAB simulation.

II. MATHEMATICAL MODEL OF THE PLANT

Defining displacement of the cart as x, the angle of the rod from the vertical (reference) line as θ , the force applied to the system as F , centre of gravity of the pendulum rod at its geometric centre and l the half length of the pendulum rod, the physical model of the system is shown in fig (1).

The Lagrangian of the entire system is given as,

L= $\frac{1}{2}(m\dot{x}^2 + 2ml\dot{x}\dot{\theta}\cos\theta + ml^2\dot{\theta}^2 + M\dot{x}^2) + \frac{1}{2}I\dot{\theta}^2$]-$mgl\cos\theta$

The Euler-Lagrange's equation for the system is

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) - \frac{\delta L}{\delta x} + b\dot{x} = F \qquad (1)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}}\right) - \frac{\delta L}{\delta \theta} + d\dot{\theta} = 0 \qquad (2)$$

The dynamics of the entire system using above equation is

$$(I + ml^2)\ddot{\theta} + ml\cos\theta\,\ddot{x} - mgl\sin\theta + d\dot{\theta} = 0 \qquad (3)$$

$$(M + m)\ddot{x} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\,\dot{\theta}^2 + b\dot{x} = F \qquad (4)$$

In order to derive the linear differential equation model, the non linear differential equation obtained need to be linearized. For small angle deviation around the upright equilibrium (fig.2) point, assumption made $\sin\theta = \theta, \cos\theta = 1,\ \dot{\theta}^2 = 0$

Using above relation, equation (5) and (6) are derived.

$$r\ddot{\theta} + q\ddot{x} - k\theta + d\dot{\theta} = 0 \qquad (5)$$

$$p\ddot{x} + q\ddot{\theta} + b\dot{x} = F \qquad (6)$$

Where, $(M + m)$= p, mgl=k, ml=q, $I + ml^2$= r

$x_1 = x + l\sin\theta$

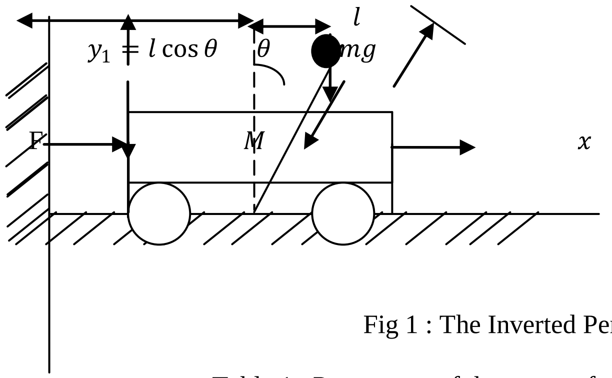

Fig 1 : The Inverted Pendulum System

Table 1. Parameters of the system from feedback instrument .U.K.

Parameter	Value	unit
Cart mass(M)	1.206	Kilo gram
Mass of the pendulum(m)	0.2693	Kilo gram
Half Length of pendulum(l)	0.1623	meter
Coefficient of frictional force(b)	0.005	Ns/m
Pendulum damping coefficient(d)	0.005	Mm/rad
Moment of inertia of pendulum(I)	0.099	kg/m^2
Gravitation force(g)	9.8	m/s^2

Eq (5 & 6) is the linear differential equation model of the entire system.
After taking Laplace transform and substituting the parameter value from (table 1), we got

$$\frac{\theta(s)}{F(s)} = \frac{-0.2783\ S^2}{s\ (s+2.026)\ (s-1.978)\ (s+0.03402)} = G_1(S) \quad (7)$$

$$\frac{X(s)}{F(s)} = \frac{0.68843\ (s+2.014)\ (s-1.967)}{s\ (s+2.026)\ (s-1.978)\ (s+0.03402)} = G_2(S) \quad (8)$$

3 STATE SPACE MODELLING

Let, $\boldsymbol{\theta}=\boldsymbol{x_1}$, $\dot{\boldsymbol{\theta}}=\boldsymbol{x_2}=\dot{\boldsymbol{x}}_1$, $\ddot{\boldsymbol{\theta}}=\dot{\boldsymbol{x}}_2=\ddot{\boldsymbol{x}}_1$ **and,** $\boldsymbol{x}=\boldsymbol{x_3}$, $\dot{\boldsymbol{x}}=\boldsymbol{x_4}=\dot{\boldsymbol{x}}_3$, $\ddot{\boldsymbol{x}}=\dot{\boldsymbol{x}}_4=\ddot{\boldsymbol{x}}_3$

From state space modeling, the system matrices are found in matrix form as given below.

$$A=\begin{bmatrix} 0 & 1 & 0 & 0.0000000 \\ 4.0088 & -0.047753952 & 0 & 0.0139155 \\ 0 & 0 & 0 & 1.0000000 \\ 0.116806166 & 0.0939155 & 0 & 0.0344200 \end{bmatrix}$$

and

$$B=\begin{bmatrix} 0 \\ -0.27831 \\ 0 \\ 0.68842 \end{bmatrix} \quad Y=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$, Y be the output equation

III. PID Controller design

PID controllers are a family of controllers. The reason PID controllers are so popular is that using PID gives the designer a large number of options and those options mean that there are more possibilities for changing the dynamics of the system in a way that helps the designer.

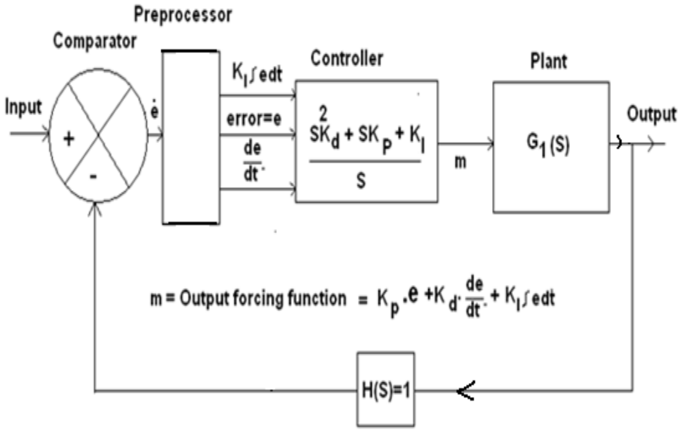

Fig 2: Design of PID controller

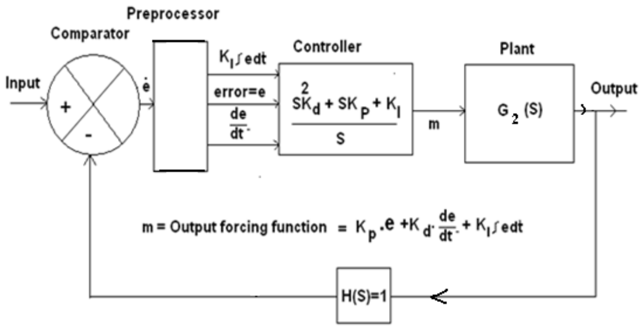

Fig 3:Design of PID controller

IV. LQR DESIGN

A system can be expressed in state variable form as

$\dot{x} = Ax + Bu$

The initial condition is $x(0)$. We assume here that all the states are measurable and seek to find a state-variable feedback (SVFB) control

$u = -Kx + v$

that gives desired closed-loop properties.

The closed-loop system using this control becomes

$\dot{x} = (A- BK)\, x+ Bv= A_c\, x+ Bv$

With A_c the closed-loop plant matrix and v the new command input.

Ackermann's formula gives a SVFB K that places the poles of the closed-loop system at desired locations. To design a SVFB that is optimal, we may define the performance index J as

$J = \frac{1}{2}\int_0^\infty X^T\,(Q + K^T RK)X dt$

We assume that input v(t) is equal to zero since our only concern here are the internal stability properties of the closed loop system.The objective in optimal design is to select the SVFB K that minimizes the performance index J.The two matrices Q and R are of appropriate dimension. One should select Q to be *positive semi-definite* and R to be *positive definite*. Since the plant is linear and the performance index is quadratic, the problem of determining the SVFB K to minimize J is called the *Linear Quadratic Regulator (LQR)*.To find the optimal feedback gain matrix K we proceed as follows. Suppose there exists a constant matrix P such that

$$\frac{d}{dt}(x^T Px) = -x^T(Q + K^T RK)X$$

After some mathematical manipulation, the equation becomes,

$J = -\frac{1}{2}\int_0^\infty \frac{d}{dt}(x^T Px)dt = \frac{1}{2}x^T(0)Px(0)$

Where, we assumed that the closed-loop system is stable so that X(t) goes to zero as time t goes to infinity. Substituting the values we get,

$x^T(A_c^T P + PA_c + Q + K^T RKx)x = 0$

It has been assumed that the external control v(t) is equal to zero. Now note that the last equation has to hold for every X^T. Therefore, the term in brackets must be identically equal to zero. Thus, proceeding one sees that

$A^T P + PA + Q + K^T RK - K^T B^T P - PBK = 0$

Exactly as for the scalar case, one may complete the squares. Though this procedure is a bit complicated for matrices, suppose we select

$K = R^{-1} B^T P$

Then, this results with

$A^T P + PA + Q - PBR^{-} B^T P = 0$

This is algebraic matrix Riccati equation

V. GENETIC ALGORITHM

Genetic algorithms (GA) are search procedures inspired by the laws of natural selection and genetics. They can be viewed as a general-purpose optimization method and have been successfully applied to search, optimization and machine learning tasks. GA has the ability to solve difficult, multi dimensional problems with little problem-specific information and hence has been chosen as the optimization technique to solve various problems in control systems.It has been shown that compared with other traditional heuristic optimization method, Genetic Algorithm is likely to be more computationally efficient. The controller parameters are usually determined by trial-and-error through simulation. In such case, the paradigms of GA appear to offer an effective way for automatically and efficiently searching for a set of control performance.

The flow chart for implementing GA for tuning controller parameters is shown below.

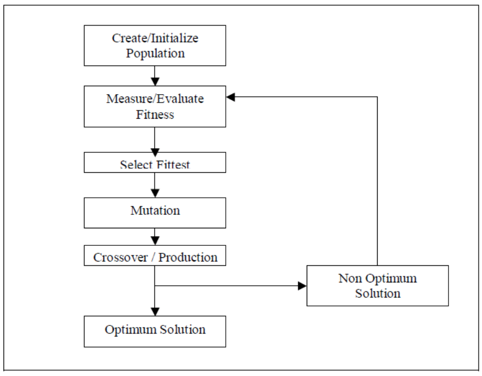

Fig 4:Flow chart of Genetic Algorithm

The steps involved in creating and implementing a genetic algorithm:

(a) Generate an initial, random population of individuals for a fixed size.
(b) Evaluate their fitness.
(c) Select the fittest members of the population.
(d) Reproduce using a probabilistic method (e.g., roulette wheel).
(e) Implement crossover operation on the reproduced chromosomes
(f) Execute mutation operation with low probability.
(g) Repeat step 2 until a predefined convergence criterion is met.

VI. SIMULATION AND RESULTS

6.1 GA based PID controller:

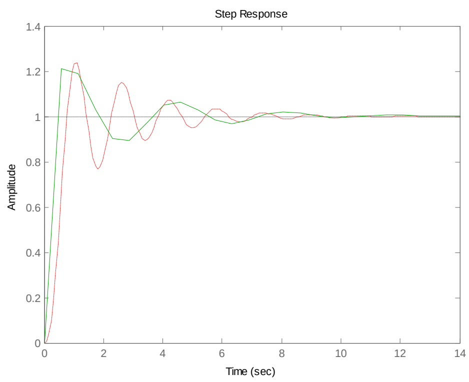

Fig 5 GA based tuning with and without of PID Controller

____ Response of plant with PID Controller
____ Response of plant with GA tuning
Results:- For 100 Population
Kp = 50.8689 Kd = -58.9523 Ki = -149.2234

6.2 Genetic Algorithm based State feedback Controller:

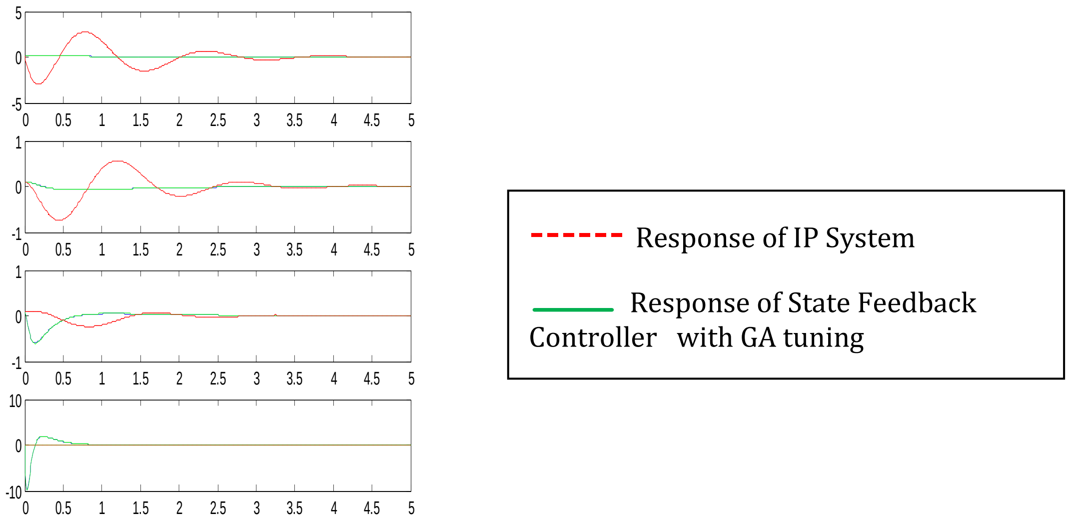

Fig 6 GA based tuning with and without of State Feedback Controller

6.3 Genetic Algorithm based LQR Controller

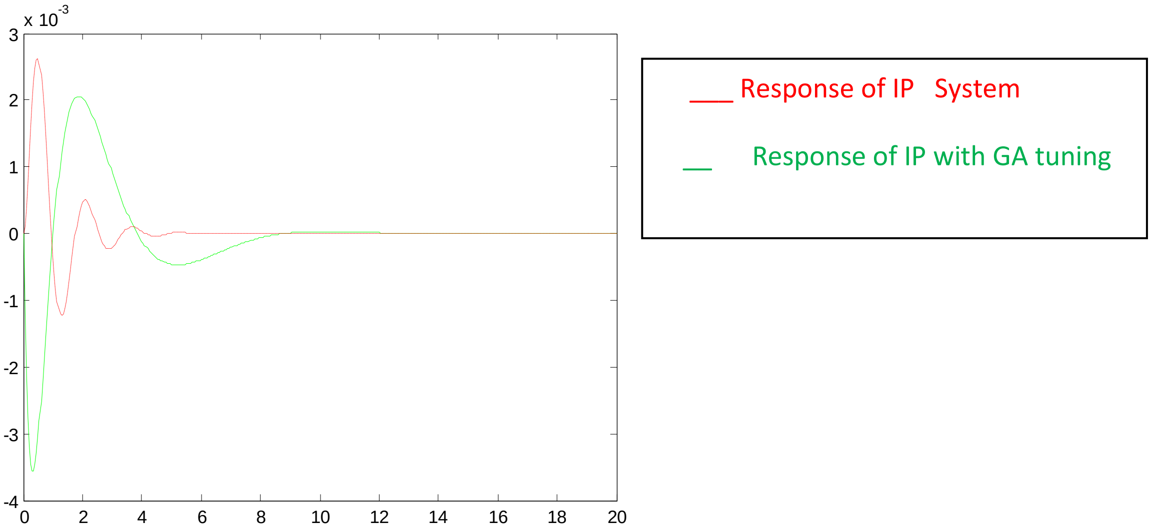

Figure 7 GA based tuning with and without of LQR

VII. CONCLUSION

Modeling of inverted pendulum shows that system is unstable with non-minimum phase zero. Results of applying PID controllers show that the system can be stabilized. While PID controller method is cumbersome because of selection of constants of controller, Constant of the controllers can be tuned by some Genetic Algorithm technique for better result. Results with GA tuned controller are better in respect of rise time and overshoot.Fuzzy Logic Controllre may be used for best result.

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