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Wavelength Conversion of a Laser Beam Using a Continuous-Wave Optical Parametric Oscillator

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Abstract: *The nonlinear optical processes are among the most fascinating effects that can be produced in lasers. These processes are almost magical, as they permit light of one colour (wavelength) to be converted into the light of a different colour. Nonlinear optics is a study that deals mainly with various new optical effects and novel phenomena arising from the interactions of intense coherent optical radiation with matter. Second harmonic generation is a nonlinear optical process, which is the most useful and well-developed technique for frequency conversion of laser beam. Optical parametric oscillation is one type of second harmonic process. In the process of optical parametric oscillation the intense input laser beam at frequency ω_p (pump frequency) is passes through a nonlinear material having nonzero value of $\chi^{(2)}$, generates the desired frequencies ω_s (signal frequency) and the frequency ω_i (idler frequency). The amplification is enhanced by placing the optical harmonic (nonlinear) crystal within an optical cavity in which the mirrors are specifically made reflective at either one of these two frequencies, or for both. Here we have converted Nd:YAG laser at wavelength 1064 nm to 1525 nm, using optical parametric oscillation.*

Keywords: *Optical parametric oscillation (OPO), Second harmonic generation, Nonlinear optics, Phase matching.*

I. Introduction

Nonlinear optics is a study that deals mainly with various new optical effects and novel phenomena arising from the interactions of intense coherent optical radiation with matter. There is a historical reason why this new branch of optical physics is termed "*nonlinear optics*". Before 1960's, in the area of conventional optics many basic mathematical equations or formulae manifested a linear feature [1]. In the regime of conventional optics, the electric polarization vector $\boldsymbol{P}$ is simply assumed to be linearly proportional to the electric field strength $\boldsymbol{E}$ of an applied optical wave, i.e. [2][3][4]

$$\boldsymbol{P} = \varepsilon_0 \chi \boldsymbol{E} \tag{1}$$

where ε_0 is the free-space permittivity, χ is the susceptibility of a given medium and a plot of $\boldsymbol{P}$ versus $\boldsymbol{E}$ is a straight line. The relation (1) is valid for the field strengths of conventional sources. The quantity χ is a constant only in the sense of being independent of $\boldsymbol{E}$; its magnitude is a function of the frequency. With sufficiently intense laser radiation this relation does not hold good and has to be generalized to equation (2), which can be written in the vector form, as by a power series

$$\boldsymbol{P} = \varepsilon_0[\chi^{(1)}\boldsymbol{E} + \chi^{(2)}\boldsymbol{EE} + \chi^{(3)}\boldsymbol{EEE} + \text{--------}]$$

or

$$\vec{\boldsymbol{P}}_i(\omega_j) = \varepsilon_0[\sum_j \chi^{(1)}_{ij}(\omega_m)\boldsymbol{E}_j(\omega_m) + \sum_{jk}\sum_{(mn)} \chi^{(2)}_{ijk}(\omega_m,\omega_n)\boldsymbol{E}_j(\omega_m)\boldsymbol{E}_k(\omega_n) + \sum_{jkl}\sum_{(mno)} \chi^{3}_{ijkl}(\omega_m,\omega_n,\omega_o)\boldsymbol{E}_j(\omega_m)\boldsymbol{E}_k(\omega_n)\boldsymbol{E}_l(\omega_0) + ...] \tag{2}$$

where $\chi^{(1)}_{ij}$ is a second – rank (linear) tensor (9 components *xx*, *xy*, *xz*, *yx*,……..), where $\chi^{(2)}_{ijk}$ is a third -rank (nonlinear) tensor (27 components, *xxx*, *xxy*, *xxz*, *xyx*,…….), and $\chi^{(3)}_{ijkl}$ is a forth-rank (nonlinear) tensor (81 components, *xxxx*,*xxxy*, *xxxz*, *xxyx*,……). The values of the tensor coefficients are functions of frequency and temperature. The subscripts *m*, *n*, and *o* etc. denotes different frequency components, and *i*, *j*, *k* and *l* are Cartesian indices that run from 1 to 3 [1]. The polarization is represented by the sum of individual perturbed polarization terms as

$$\boldsymbol{P} = \boldsymbol{P}^{(1)} + \boldsymbol{P}^{(2)} + \boldsymbol{P}^{(3)} + \text{……} + \boldsymbol{P}^{(n)} + \text{….} \tag{3}$$

the general form of the Fourier component of the *n*th-order polarization can be obtained as

$$\boldsymbol{P}^{(n)}(\omega) = \varepsilon_0 \chi^{(n)}(\omega_1,\omega_2,.....,\omega_n)\boldsymbol{E}(\omega_1)\boldsymbol{E}(\omega_2).....\boldsymbol{E}(\omega_n) \tag{4}$$

Here a summation relation is fulfilled:

$$\omega = \omega_1 + \omega_2 + ... + \omega_n = \sum_{m=1}^{n} \omega_m \tag{5}$$

Here $\chi^{(n)}(\omega_1,.....,\omega_n)$ is called the *n*th-order susceptibility of the medium and it is a (*n*+1) th-order tensor. For small field strength the polarization is proportional to the electric field $\boldsymbol{E}$ and is accounted for by the polarizatbility tensor $\chi_{ij}^{(1)}$. All of the optics discussed so far has been linear optics encompassed in the term $\varepsilon_0 \chi_{\boldsymbol{ij}}^{(1)}(\omega_{\boldsymbol{m}})\boldsymbol{E}_{\boldsymbol{j}}(\omega_{\boldsymbol{m}})$. The term $\chi_{\boldsymbol{ijk}}^{(2)}(\omega_{\boldsymbol{m}},\omega_{\boldsymbol{n}})\boldsymbol{E}_{\boldsymbol{j}}(\omega_{\boldsymbol{m}})\boldsymbol{E}_{\boldsymbol{k}}(\omega_{\boldsymbol{n}})$ is responsible for all of the two-wave effects. This includes second harmonic generation (two fields at ω to make one at 2ω) and parametric oscillation (one field at ω_1 and other field at ω_2 to create fields at ω_1 - ω_2 and ω_1 + ω_2). The nonlinear polarization tensor $\chi^{(2)}$ vanishes in the crystals that have a center of symmetry (i.e. crystal symmetry). In these crystals second harmonic generation is not possible. Thus the second-order polarization and the corresponding monochromatic components of the optical field:

$$\boldsymbol{P}^{(2)}(\omega = \omega_1 + \omega_2) = \varepsilon_0 \chi^{(2)}(\omega_1,\omega_2)\boldsymbol{E}(\omega_1)\boldsymbol{E}(\omega_2) \tag{6}$$

where $\chi^{(2)}$ denotes the second-order susceptibility that is a third-order tensor. For instance, in the second-order optical sum-frequency process $\omega_1 + \omega_2 \Rightarrow \omega_3$ created by $\boldsymbol{E}(\omega_1)$ and $\boldsymbol{E}(\omega_2)$, three involved nonlinear polarization component can be written as

$$\left.\begin{aligned} \boldsymbol{P}^{(2)}(\omega_3 = \omega_1 + \omega_2) &= \varepsilon_0 \chi^{(2)}(\omega_1,\omega_2)\boldsymbol{E}(\omega_1)\boldsymbol{E}(\omega_2) \\ \boldsymbol{P}^{(2)}(\omega_1 = \omega_3 - \omega_2) &= \varepsilon_0 \chi^{(2)}(\omega_3,-\omega_2)\boldsymbol{E}(\omega_3)\boldsymbol{E}^{*}(\omega_2) \\ \boldsymbol{P}^{(2)}(\omega_2 = \omega_3 - \omega_1) &= \varepsilon_0 \chi^{(2)}(\omega_3,-\omega_1)\boldsymbol{E}(\omega_3)\boldsymbol{E}^{*}(\omega_1) \end{aligned}\right\} \tag{7}$$

The susceptibility tensors for a given medium must remain unchanged upon the symmetric operation allowed for this medium. According Guang & Song [1], in general the following permutation-symmetry relations of tensor elements for second and third order susceptibilities hold

$$\chi_{ijk}^{(2)}(\omega_1,\omega_2) = \chi_{ikj}^{(2)}(\omega_2,\omega_1) \tag{8}$$

and

$$\chi_{ijkl}^{(2)}(\omega_1,\omega_2,\omega_3) = \chi_{ikjl}^{(2)}(\omega_2,\omega_1,\omega_3) = \chi_{iljk}^{(2)}(\omega_3,\omega_1,\omega_2) = ... \tag{9}$$

Desmond [5] simplified $\chi_{ijk}^{(2)}$, and replaced by a nonlinear optical coefficient d_{il} (Coulomb/Volt^2), according to the following relationship:

$$d_{il} = \varepsilon_0 \chi_{ijk}^{(2)} \qquad \begin{cases} l = 1,2,3,4,5,6 \\ jk = xx, yy, zz, yz, zx, xy \end{cases} \tag{10}$$

And according to Kuhn [2] $d_{ij} = \frac{1}{2}\chi_{ijk}^{(2)}$, here,$\varepsilon_0$, is the permittivity of free space, some authors excludes ε_0 from the *d* coefficient, in this case *d* [As/V^2] = 8.855 × 10^{-12}*d* [m/v]. The conversion from the cgs system to MKS units becomes *d* [As/V^2] = 3.68 × 10^{-15}*d* [esu]. According to Desmond [5], Boyd and Kleinman [6], the tensor d_{ijk} can be defined in terms of the field and polarization amplitudes as

$$P_i^{\omega_3} = d_{ijk}^{\omega_3=\omega_1+\omega_2} E_j^{\omega_1} E_k^{\omega_2} \tag{11}$$

and it follows that $d_{ijk} = d_{ikj}$, and using the matrix notation, for *jk*: *xx* =1; *yy* = 2; *zz* = 3; *yz* = *zy* = 4; *zx* = *xz* = 5; *xy* = *yx* = 6, and therefore:

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{pmatrix} \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_zE_y \\ 2E_zE_x \\ 2E_xE_y \end{pmatrix} \tag{12}$$

The 3× 6 d_{il} matrix operates on a column vector $(EE)_l$ given by

$$\left.\begin{array}{lll}(EE)_1 = E_x^2 & (EE)_2 = E_y^2 & (EE)_3 = E_z^2 \\ (EE)_4 = 2E_yE_z & (EE)_5 = 2E_xE_z & (EE)_6 = 2E_xE_y\end{array}\right\} \quad (13)$$

Under some conditions it turns out that $d_{ijk} = d_{kij} = d_{jik}$ as well; i.e, all subscripts can be permuted. This is known as *Kleinman's symmetry conjecture* [40], and was originally deduced by the inspection of actual tensor element values. It reduces the 18 independent values for the elements in the d_{ijk} matrix (6 × 3) to just 10 (for example, $d_{12} = d_{xyy} = d_{yyx} = d_{26}$, etc).Finally, further reduction in the complexity of the tensor results from the actual symmetry of the crystal to which it refers. For example, for KDP ($\bar{4}2m$),d_{ijk} is given by

$$\begin{pmatrix} 0 & 0 & 0 & d_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{36} \end{pmatrix}$$

Since $d_{14} = d_{xyz}$ and $d_{36} = d_{zxy}$ we would expect them to be equal on the basis of Kleinman's symmetry conjecture, so that in all such $\bar{4}2m$ crystals, there will be only one independent value. Crystals of the KTP family belong to point group 2*mm*, so the form of their nonlinear tensor in a coordinate system where *x, y*, and *z*, refer to principal axes with refractive-index ordering $n_x < n_y < n_z$ is

$$d = \begin{pmatrix} 0 & 0 & 0 & 0 & d_{xxz} & 0 \\ 0 & 0 & 0 & d_{yyz} & 0 & 0 \\ d_{zxx} & d_{zyy} & d_{zzz} & 0 & 0 & 0 \end{pmatrix}$$

In most practical situations the tensor equations containing d_{ijk} can be simplified to non-tensor form in which d_{ijk} is replaced by d_{eff}, is the effective nonlinear coefficient for the interaction dependent on crystal symmetry and propagation direction in the medium.

II. Second Order Nonlinear Processes

The simplest second-order process is that of second-harmonic generation (SHG). In this process, an intense laser beam of angular frequency ω_1 (= ω) is passed through a crystal having nonzero value of $\chi^{(2)}$, such that the beam emerging from the crystal contains the angular frequencies $\omega_1$of the input beam and also $\omega_2 = 2\omega_1$, twice the frequency of the input beam. This can be shown to occur by considering the second nonlinear polarization term $\boldsymbol{P}^{(2)}$. Based on the nonlinear polarization theory, the source of second-harmonic field is the following nonlinear polarization component [1]:

$$\boldsymbol{P}^{(2)}(2\omega) = \varepsilon_0\chi^{(2)}(\omega,\omega)\boldsymbol{E}(\omega)\boldsymbol{E}(\omega) \quad (14)$$

Here, $\boldsymbol{E}(\omega)$ is the incident fundamental optical field, and $\chi^{(2)}(\omega,\omega)$ is the second-order susceptibility tensor of a given nonlinear medium for SHG [7].

In the second-harmonic generation, considered the combination (addition) of two photons of the same frequency to produce a single photon of twice the frequency. It can now to generalize this process to allow for the case in which the two photons have different frequencies ω_1 and ω_2. In a similar fashion to our treatment of two photons with the same frequency, let us write the expression for the field as

$$\boldsymbol{E} = \boldsymbol{E}_1\boldsymbol{e}^{-i\omega_1 t} + \boldsymbol{E}_1^*\boldsymbol{e}^{i\omega_1 t} + \boldsymbol{E}_2\boldsymbol{e}^{-i\omega_2 t} + \boldsymbol{E}_2^*\boldsymbol{e}^{i\omega_2 t} \quad (15)$$

which includes the complex conjugates of both E_1 and E_2. We again compute the second-order nonlinear polarizability as

$$\begin{aligned}\boldsymbol{P}^{(2)} &= \varepsilon_0\chi^{(2)}\boldsymbol{E}^2 \\ &= \varepsilon_0\chi^{(2)}[\boldsymbol{E}_1^2\boldsymbol{e}^{-i(2\omega)t} + (\boldsymbol{E}_1^*)^2\boldsymbol{e}^{-i(2\omega)t} + \boldsymbol{E}_2^2\boldsymbol{e}^{-i(2\omega)t} + (\boldsymbol{E}_2^*)^2\boldsymbol{e}^{-i(2\omega)t} + 2\boldsymbol{E}_1\boldsymbol{E}_1^* + 2\boldsymbol{E}_2\boldsymbol{E}_2^* \\ &\quad + 2\boldsymbol{E}_1\boldsymbol{E}_2\boldsymbol{e}^{-i(\omega_1+\omega_2)t} + 2\boldsymbol{E}_1^*\boldsymbol{E}_2^*\boldsymbol{e}^{-i(\omega_1+\omega_2)t} + 2\boldsymbol{E}_1\boldsymbol{E}_2^*\boldsymbol{e}^{-i(\omega_1-\omega_2)t} + 2\boldsymbol{E}_1^*\boldsymbol{E}_2\boldsymbol{e}^{-i(\omega_1-\omega_2)t}]\end{aligned} \quad (16)$$

We have conveniently grouped common terms together. These include DC terms, second harmonic terms (involving $2\omega_1$ and $2\omega_2$), and two new terms involving $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$. The new term involving $\omega_1 + \omega_2$ generates a new frequency that is the sum of the two original frequencies and is thus known as *sum frequency generation*. The term involving the difference between the two frequencies, $\omega_1 - \omega_2$, is referred to as *difference*

frequency generation. In the sum frequency generation, when a new photon $\omega_3 = \omega_1 + \omega_2$ is created, the frequencies ω_1 and ω_2 are destroyed. In the difference frequency generation the photon of higher frequency ω_1 is destroyed while both ω_2 and ω_3 are created. Since $\omega_2$is already present as one of the input beams, this suggests that ω_2 is amplified in the process-that is, photons are added to the beam at the frequencyω_2.

Optical parametric oscillation is one type of second harmonic process. In the process of optical parametric oscillation the intense input laser beam at frequency ω_p (pump frequency) is passes through a nonlinear material having nonzero value of $\chi^{(2)}$, generates the desired frequencies ω_s (signal frequency) and the frequency ω_i (idler frequency). The amplification is enhanced by placing the optical harmonic (nonlinear) crystal within an optical cavity in which the mirrors are specifically made reflective at either one of these two frequencies, or for both [8][9]. In the process of difference frequency mixing, the frequency ω_2 is amplified while the frequency ω_3 is being generated. In the process of optical parametric oscillation (OPO) the intense input laser beam at frequency ω_p is known as the *pump* frequency, when passes through a nonlinear material, generates the desired frequencies ω_s (signal frequency) and the frequency ω_i (idler frequency. The amplification can be enhanced by placing the optical harmonic (nonlinear) crystal within an optical cavity in which the mirrors are specifically made reflective at either one of these two frequencies, or for both. Thus the intensity at those frequencies will build up within the cavity, by Fabry-Perot interferometer. Such an amplification process is known as an optical parametric oscillator (OPO). Of course, either ω_s or ω_i can be tunable laser to generate amplified tunable output. This process is used most often in the infrared frequency range, where tunable lasers are not as readily available as in the visible portion of the frequency spectrum. [10].

The output of an optical parametric oscillator (OPO) is similar to that of a laser. It is highly monochromatic and exhibits laser speckle. The spectrum is formed of one or several longitudinal modes. The transverse mode spectrum is often TEM_{00} and propagates with Gaussian-like properties [1]. The energy conservation requires that

$$\omega_p = \omega_s + \omega_i \tag{17}$$

Here ω_p, ω_s, and ω_i are the frequencies of the pump, signal and idler wave. It is clear that the frequencies of the two emitted photons cannot be uniquely determined on the basis of the energy conservation condition,(17) alone. For a given ω_p, there can be a continuous range of choices of ω_s and ω_i. This, in fact, is the origin of the tunability of the optical parametric oscillator. The specific pair of frequencies that will be emitted is dictated by the momentum conservation condition, or phase matching condition: $\boldsymbol{k}_p = \boldsymbol{k}_s + \boldsymbol{k}_i$, that must also be satisfied in order to ensure that the signal waves generated in different parts of the nonlinear crystal are in phase and add coherently [11]. For collinearly propagating waves this may be written [12][13][14]][15] [16] [6]

$$\left.\begin{aligned} \frac{n_p}{\lambda_p} &= \frac{n_s}{\lambda_s} + \frac{n_i}{\lambda_i} \\ \omega_p n_p &= \omega_s n_s + \omega_i n_i \end{aligned}\right\} \tag{18}$$

Here n_p, n_s and n_i are the refractive indices of the pump, signal and idler wave and λ_p, λ_s and λ_i there corresponding wavelengths respectively. The pump signal is usually provided by a laser and, therefore λ_p is fixed. However, if the refractive indices are varied, the signal and idler frequencies will tune. Under an appropriate arrangement for the angle (or temperature) of a given nonlinear crystal, the above two requirements (Eq.(17) &(18)) can be satisfied and oscillations at two different frequencies ω_s, and ω_i can be achieved. Based on this working condition, if we slightly change the angle or temperature of the crystal, the refractive index relation between these three waves will be changed; therefore the oscillating frequencies will be smoothly tuned to different values [4][17]. The requirements of nonlinear crystals for optical parametric oscillation are essentially the same as that for SHG. In other words, the nonlinear materials must be non-centrosymmetrical crystals, highly transparent for pump, signal, and idler beams, able to fulfill the phase matching by using angle-tuning or temperature-tuning.

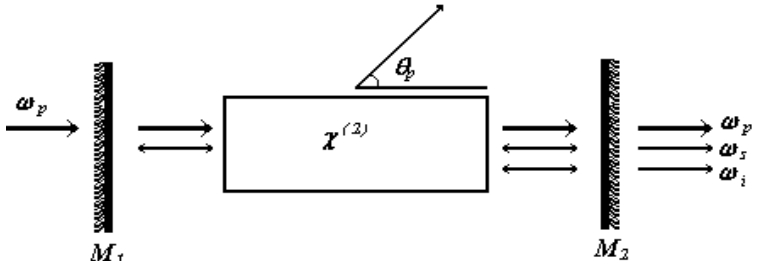

Figure-1 Singly-Resonant Optical Parametric Oscillator

A possible simple implementation of the optical parametric oscillator is shown schematically in Figure-1. It consists of a suitably oriented nonlinear optical crystal in a Fabry-Perot cavity. The cavity mirrors are coated to transmit the pump wave and reflect either the signal wave only or both the signal and idler waves. In the former case, the oscillator is known as the singly resonant oscillator, and, in the latter case, it is known as the doubly resonant oscillator. After passing through the output-coupling mirror the transmitted pump beam is blocked by a filter. The further separation between the signal beam and idler beam can be done by using appropriate spectral filters or optical dispersive elements. Various optical cavity designs, including stable, unstable, or metastable cavity configurations, can be employed for OPO purpose. The criteria of selection of cavity designs are same as that for laser cavity devices. Since we assume for optical parametric oscillation in a second-order nonlinear crystal, the three waves are linearly polarized and propagating along *z* axis:

$$\left.\begin{aligned} E(\omega_p, z) &= a_p A_p(z) e^{jk_p z} \\ E(\omega_s, z) &= a_s A_s(z) e^{jk_s z} \\ E(\omega_i, z) &= a_i A_i(z) e^{jk_i z} \end{aligned}\right\} \tag{19}$$

Accordingly, the nonlinear polarization sources of these waves can be expressed as the following forms:

$$\left.\begin{aligned} P^{(2)}(\omega_p, z) &= \varepsilon_0 \chi^{(2)}(\omega_s, \omega_i) a_s a_i A_s(z) A_i(z) e^{j(k_s + k_i)z} \\ P^{(2)}(\omega_s, z) &= \varepsilon_0 \chi^{(2)}(\omega_p, -\omega_i) a_p a_i A_p(z) A_i^*(z) e^{j(k_p - k_i)z} \\ P^{(2)}(\omega_i, z) &= \varepsilon_0 \chi^{(2)}(\omega_p, -\omega_s) a_p a_s A_p(z) A_s^*(z) e^{j(k_p - k_s)z} \end{aligned}\right\} \tag{20}$$

thus the nonlinear coupled-wave equations leads

$$\left.\begin{aligned} \frac{\partial A_p(z)}{\partial z} &= \frac{ik_p}{2n_p^2}\left[a_p\ \chi^{(2)}(\omega_s, \omega_i) a_s a_i\right] A_s A_i e^{-j\Delta kz} \\ \frac{\partial A_s(z)}{\partial z} &= \frac{ik_s}{2n_s^2}\left[a_s\ \chi^{(2)}(\omega_p, -\omega_i) a_p a_i\right] A_p A_i^* e^{j\Delta kz} \\ \frac{\partial A_i(z)}{\partial z} &= \frac{ik_i}{2n_i^2}\left[a_i\ \chi^{(2)}(\omega_p, -\omega_s) a_p a_s\right] A_p A_s^* e^{j\Delta kz} \end{aligned}\right\} \tag{21}$$

where $\Delta k = k_p - k_s - k_i$ is the phase mismatch factor. The fundamental coupled equations for parametric oscillation are similar to those of second harmonic generations. Following the procedure of Harris [18] and Qiang Liu et. al, [19] these equations can be written as

$$\left.\begin{aligned} \frac{d\boldsymbol{E}_s}{dz} &= -j\eta_s \omega_s d_{eff} \boldsymbol{E}_p \boldsymbol{E}_i^* e^{-j\Delta kz} \\ \frac{d\boldsymbol{E}_i}{dz} &= -j\eta_i \omega_i d_{eff} \boldsymbol{E}_p \boldsymbol{E}_s^* e^{-j\Delta kz} \\ \frac{d\boldsymbol{E}_p}{dz} &= -j\eta_p \omega_p d_{eff} \boldsymbol{E}_s \boldsymbol{E}_i e^{-j\Delta kz} \end{aligned}\right\} \tag{22}$$

where $\boldsymbol{E}_s$ is the electric field of the signal, $\boldsymbol{E}_i$ is the electric field of the idler, $\boldsymbol{E}_p$ is the electric field of the pump, the plane-wave impedance $\eta_x = 377/n_x$ in ohms, d_{eff} is the nonlinear coefficient of the nonlinear tensor, in MKS system unit is As/V^2. The ratio between the signal and idler powers can be obtained by solving these coupled equations (22), as for second harmonic generation. According to Eq.(9) the following relation holds:

$$\chi_e = a_p\ \chi^{(2)}(\omega_s, \omega_i) a_s a_i = a_s\ \chi^{(2)}(\omega_p, -\omega_i) a_p a_i = a_i\ \chi^{(2)}(\omega_p, -\omega_s) a_p a_s \tag{23}$$

Here, χ_e is the effective nonlinear susceptibility value for this process. Assuming the phase-matching condition of $\Delta k \Rightarrow 0$ is satisfied, the initial boundary conditions are

$$\left.\begin{aligned} A_p(z=0) &= A_p(0) \\ A_s(z=0) &= A_s(0) = 0 \\ A_i(z=0) &= A_i(0) = 0 \end{aligned}\right\} \tag{24}$$

and the depletion of $A_p(z)$ in a short propagation distance can be nearly neglected in small-signal approximation, then Eq.(21) can be simplified as

$$\left.\begin{aligned}\frac{\partial A_s(z)}{\partial z} &= \frac{ik_s}{2n_s^2}\chi_e A_p(0)A_i^*(z)\\ \frac{\partial A_i(z)}{\partial z} &= \frac{ik_i}{2n_i^2}\chi_e A_p(0)A_s^*(z)\end{aligned}\right\} \tag{25}$$

In order to achieve optical parametric oscillation, the net gain for a round-pass within the cavity is determined by

$$\exp\left[\left(\gamma_0 - 2\alpha_0\right) l\right] \tag{26}$$

where

$$\gamma_0 = \frac{\pi\chi_e}{\sqrt{\lambda_s\lambda_i n_s n_s}} A_p(0) \tag{27}$$

and α_0 is the linear attenuation coefficient, l is the optical-path length of the crystal. One can see that the attenuation length is 2l, while the effective gain length is only l because the backward signal and idler beams reflected from the output coupling mirror is R, the threshold condition for optical parametric oscillation is determined by

$$R\ \exp\left[\left(\gamma_0 - 2\alpha_0\right) l\right] \geq 1 \tag{28}$$

The above expression can be written as

$$\gamma_0 \geq \frac{1}{l}\left(\ln\frac{1}{R} + 2\alpha_0 l\right) \tag{29}$$

Finally, substituting the γ_0 expression of Eq.(27) into Eq.(29) leads to the following threshold condition:

$$A_p(0) \geq \frac{\sqrt{\lambda_s\lambda_i n_s n_i}}{\pi\chi_e}\frac{1}{l}\left(\ln\frac{1}{R} + 2\alpha_0 l\right) \tag{30}$$

Since $A_p(0) \propto \sqrt{I_p(0)}$, the above condition implies that for a given nonlinear medium and interaction length l, only as the input pump beam intensity is higher than a certain threshold the optical parametric oscillation can be achieved. The threshold for oscillation of the singly resonant oscillator is [20][21].

$$I_{th} = \frac{a_s\varepsilon_0 c^3}{\omega_s\omega_i\dfrac{d_{eff}^2 l^2}{n_p n_s n_i}}, \qquad (SRO) \tag{31}$$

and that for the doubly resonant oscillator is

$$I_{th} = \frac{a_s a_i\varepsilon_0 c^3}{2\omega_s\omega_i\dfrac{d_{eff}^2 l^2}{n_p n_s n_i}}, \qquad (DRO) \tag{32}$$

where I_{th} is the threshold intensity; a_s and a_i are the fractional round-trip power losses of the signal and idler waves; d_{eff} is the effective nonlinearity of the OPO crystal; l is the crystal length; n_p, n_s, and n_i are the refractive indices of the pump, signal, and idler waves, respectively; ε_0 is the permittivity of free space, and c is the speed of light. Clearly, the threshold for the singly resonant oscillator will be higher than that for the doubly resonant oscillator. The trade-off is that singly resonant oscillator tends to be more stable than the doubly resonant oscillator, because of the more stringent requirement that two resonant conditions must be satisfied simultaneously in the cavity of the doubly resonant oscillator.

The threshold pump intensity I_{th} for a singly resonant OPO having a reflection of the pump beam from OPO output coupler is given by [22]

$$I_{th} = \frac{1.8}{\kappa g_s l_{eff}(1+\gamma)^2}\left(\frac{25L}{t_p c} + 2\alpha l + \ln\frac{1}{\sqrt{R}} + \ln 2\right)^2 \tag{33}$$

where $$\kappa = \frac{8\pi^2 d_{eff}^2}{\lambda_s \lambda_i n_p n_s n_i \varepsilon_0 c} \tag{34}$$

is the coupling constant, t_p is the pulse width (FHWM) of the pump pulse, and $\gamma = 1$(at threshold) is the ratio of the backward to forward pump amplitude in the OPO cavity, $2\alpha L$ is the round trip loss of the OPO crystal (≈ 0.01), L is the optical length of OPO cavity, l_{eff} is the effective length of the nonlinear crystal, R is the output coupler reflectivity, g_s is the mode coupling coefficient (≈ 0.85), n_p, n_s and n_i are the refractive indices of pump, signal and idler wavelengths (n_p =1.748, n_s = 1.737, n_i = 1.771 [23]), λ_s and λ_i are the wavelengths of signal and idler waves [24] [25] [26] [27].

III. Optics of Uniaxial Crystals

In *uniaxial* crystals a special direction exists called the *optic axis* (*z*-axis). The plane containing the *z*-axis and the wave vector ***k*** of the light wave is termed the *principal plane*. The light beam whose polarization (i.e., the direction of the vector ***E*** oscillations) is normal to the principal plane is called an *ordinary beam* or an *o-beam*. The beam polarized in the principal plane is known as the *extraordinary beam* or *e-beam*. The refractive index of the o-beam does not depend on the propagation direction, whereas for the e-beam it does. Thus, the refractive index in anisotropic crystals generally depends both on light polarization and propagation direction. The difference between the refractive indices of the ordinary and extraordinary beam is known as *birefringence* Δn. The value of Δn is zero along the optic axis z and maximum in the direction normal to this axis. The refractive indices of the ordinary and extraordinary beams in the plane normal to the z axis are termed the *principal values* and are denoted by n_o and n_e, respectively.

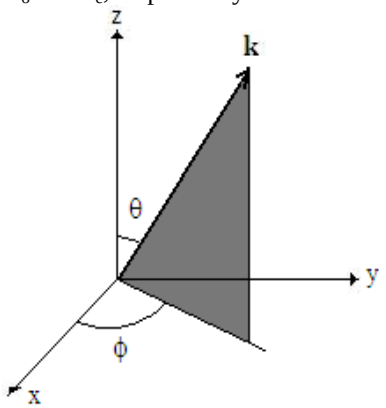

Figure-2. Polar coordinate system for description of refractive properties of uniaxial crystal

The refractive index of the extraordinary wave is, in general, a function of the polar angle θ between the z axis and the vector ***k*** (Figure-2). It is determined by the equation (index e in this case is written as a superscript):[28][29][30][1][3][1]

$$n^e(\theta) = n_o[(1+tan^2\theta)/(1+(n_o/n_e)^2 tan^2\theta)]^{\frac{1}{2}} \tag{35}$$

The following equations are evident:

$$\left.\begin{aligned} n^o(\theta) &\equiv n_o \\ n^e(0^0) &= n_o \\ n^e(90^0) &= n_e \\ \Delta n(0^0) &= 0 \\ \Delta n(90^0) &= n_o - n_e \\ \Delta n(\theta) &= n_o - n^e(\theta) \end{aligned}\right\} \tag{36}$$

If $n_o > n_e$, the crystal is *negative*; if $n_o < n_e$, it is *positive*. The quantity n^e does not depend on the azimuthal angle ϕ (the angle between the projection of ***k*** onto the *xy*-plane perpendicular to the *z*-axis and the *x*-axis (Fig.2). The indicatrix of the refractive indices is a sphere with radius n_o for an ordinary beam and an ellipsoid of rotation with semiaxes n_o and n_e for an extraordinary beam (the axis of the ellipsoid of rotation is the *z*-axis).

In the *z*-axis direction the sphere and ellipsoid are in contact with each other. In a negative crystal the ellipsoid is inscribed in the sphere, whereas in a positive crystal the sphere is inscribed in the ellipsoid. When a plane light wave propagates in a uniaxial crystal, the direction of propagation of the wave phase (vector ***k***) generally does not coincide with that of the wave energy (vector ***s***). The direction of ***s*** can be defined as the normal to the tangent drawn at the point of interaction of vector ***k*** with the $n(\theta)$ curve. For an ordinary wave $n(\theta)$ dependence is a sphere with radius n_o. Therefore, the normal to the tangent coincides with the wave vector ***k***. For an extraordinary wave the normal to the tangent (with the exception of the cases $\theta = 0^0$ and $\theta = 90^0$) does not coincide with the wave vector ***k*** but is rotated from it by the *birefringence angle* or *walk-off-angle:*

$$\rho(\theta) = \pm \ arc \ tan[(n_o/n_e)^2 tan\theta] \mp \theta \tag{37}$$

where the upper signs refer to a negative crystal and lower signs to a positive one. Thus, by rotating the crystal in the plane of the optic axis and the incident beam (the direction of *k* vector) it is possible to select a range of value of n_e from which the condition of phase matching can be satisfied

When the phase matching is achieved by choosing a direction of propagation which is not at 90^0 to the crystal optic axis is called '*critical phase matching*' [31][32][33][34]. In a uniaxial medium, the power flow of the e-wave is not along the direction of *k*. Thus as the fundamental (o-wave) and the second harmonic propagate along the crystal, the power generated in the second harmonic will separate from the fundamental leading to what is referred to as '*walk off*'. For phase matching at $\theta_m = 90^0$, $\varphi = 0$ and there is no walk off. Hence when possible, 90^0 phase matching is desirable. This is referred to as '*noncritical phase matching*' [31][32][33][34]. According to Desmond Smith [5], having chosen a particular phase-matching geometry, which can then determine the value of d_{eff} .

IV. Calculation of Phase-Matching Angle in Biaxial Crystals:

For biaxial crystals the dependence or refractive indices on the light propagation direction and its polarization corresponds to a much more complex surface than for uniaxial crystals. The surface has a bilayer structure with four points of interlayer contact through which two optic axes pass [35]. For simplicity the case of light propagation in the *principle planexz*, *xz*, and *yz*. In these planes the dependences of the refractive index on the propagation direction of two waves with orthogonal polarizations represent a combination or an ellipse and a circle (Figure-3 a, b).

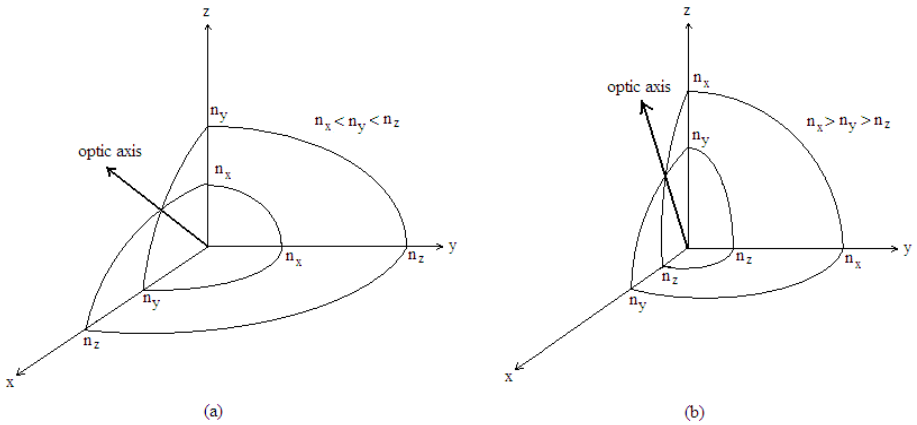

Figure-3 Indicatrices of refractive indices in biaxial crystals

We shall relate *crystallophysical* (X, Y, Z)and *crystallographic* (a, b, c) axes in a biaxial crystal in such a way that the optic axes, whose directions are given by the intersection points of the ellipse and circle, will always be in a plane *xz*. Consider one of two possible cases: $n_x < n_y < n_z$ (Figure-3a), where n_x, n_y, and n_z are the principal values of the refractive indices. The angle θ_z formed by one of the optic axes with the axis z can be found from the expression

$$\sin\theta_z = \frac{n_z}{n_y}\left(\frac{n_y^2 - n_x^2}{n_z^2 - n_x^2}\right)^{1/2} \tag{38}$$

In the plane *xy* the refractive index of the wave polarized normally to this plane is constant and equals n_z, and that of the wave polarized in this plane changes from n_y to n_x with φ varying from 0^0 to 90^0. Hence, a biaxial crystal with $n_x < n_y < n_z$ in the plane *xy* is similar to a negative uniaxial crystal with $n_o = n_z$ and

$$n^e(\varphi) = n_y\left(\frac{1+\tan^2\varphi}{1+\left(n_y/n_x\right)^2\tan^2\varphi}\right)^{1/2} \tag{39}$$

In the plane yz the refractive index of the wave polarized normally to this plane is constant and equals n_x, whereas for the wave polarized in this plane the refractive index changes form n_y to n_z with θ varying from 0^0 to 90^0. Hence, a biaxial crystal with $n_x < n_y < n_z$ in the plane *yz* is similar to a positive uniaxial crystal with $n_o = n_x$ and

$$n^e(\theta) = n_y\left(\frac{1+\tan^2\theta}{1+\left(n_y/n_z\right)^2\tan^2\theta}\right)^{1/2} \tag{40}$$

We can also see that in the plane *yz* at $\theta > \theta_z$ a biaxial crystal with $n_x < n_y < n_z$ is similar to a positive uniaxial crystal and, at $\theta < \theta_z$, to a negative uniaxial crystal. A biaxial crystal with $n_x > n_y > n_z$ is considered in a similar way (Figure-3b). Here the angle θ_z between the optic axis and the axis *z* is expressed as

$$\cos\ \theta_z = \frac{n_x}{n_y}\left(\frac{n_y^2-n_z^2}{n_x^2-n_z^2}\right)^{1/2} \tag{41}$$

The biaxial crystal is said to be optically positive if the bisectrix of the acute angle between optic axes coincides with n_{max} and optically negative if the bisectrix coincides with n_{min}. Hobden [31] give a general approach to calculation of phase-matching angles in biaxial crystals.

V. Nonlinear Optical Materials

For generating new frequencies from existing lasers via harmonic generation and difference generation, there has been an extensive effort in recent years to identify effective materials for such processes. In addition to having a large nonlinearity, these materials must be transparent not only at the laser frequency but also at the newly generated frequency. They must (1) be resistant to optical damage, (2) have high mechanical hardness, (3) exhibit good thermal and chemical stability, (4) be capable of being grown in useful sizes, and (5) have the appropriate phase-matching properties. The second harmonic crystals must have no inversion symmetry (i.e. non-centrosymmetric). Bulk second-order nonlinear materials are generally inorganic crystals. A number of semiconductors are useful for second harmonic generation when used in waveguides. The nonlinear crystals can be classified into two groups according to their physical properties. Crystals grown from water solutions are fragile, hygroscopic, and sensitive to thermal shock. The crystals of this group, to which KDP and its isomorphs belong, are somewhat difficult to handle because the crystals are soft, and the polished faces may be fogged if they are held with bare hands or exposed to humid atmosphere. On the other hand, the crystals are easy to grow, they are available in large sizes, and they are of excellent optical quality. Crystals grown from the melt are relatively hard, nonhygroscopic and less sensitive to thermal shock. Important members of these group crystals are $LiNbO_3$ and $Ba_2NaNb_5O_{15}$. The optical quality is usually inferior to water grown crystals because of refractive index nonuniformities associated with the crystal growth condition [1][3][29].

Potassium titalyl phosphate or KTP ($KTiOPO_4$) [36][37][38][39] is a mm2 biaxial crystal with orthorhombic symmetry. KTP is a difficult crystal to grow, and is currently grown by hydrothermal and flux growth techniques. However, KTP possesses good optical properties, a large acceptance angle, large temperature acceptance, a large nonlinear coefficient, and high optical damage thresholds. It is a mechanically rugged and nonhygroscopic crystal. However KTP suffers from cumulative photochemical degradation phenomena, termed *grey tracking*, caused by long-term exposure to the intense fundamental and second harmonic radiation. Although this photochemical effect can be reversed by operating the crystal at an elevated temperature, absorption in the crystal due to the grey tracking may damage the crystal beyond repair. KTP is a more recently developed crystal than KDP or $LiNbO_3$, but is emerging as one of the most popular frequency-doubling crystals for Nd:YAG and Nd:glass lasers. KTP is also finding application as an OPO material and in difference frequency application.

Table-1 Properties of some important nonlinear crystals[6]

Property	**KTP**	**BBO**	**LBO**	**CLBO**
Nonlinear coefficient (pm/V)	3.1	1.94	1.16	1.11
Transmission range (μm)	0.35 - 5.5	0.19 - 3.5	0.16 – 2.6	0.16 – 2.6
Damage threshold (GW/cm^2)	> 0.5	1.5	2.5	> 2.5

Angular acceptance (mrad-cm)	20	< 1	2	1.4
Spectral acceptance (mm-cm)	0.5	0.5	0.8	1
Walk-off angle (degree)	1.3	5.5	<1	1.8
Damage resistance to moisture	High	Low	Low	Medium

Table-2 Parameters of nonlinear crystals for critically phase matched second harmonic generation[2]

Property	**β-BaB_2O_4**	**$KNbO_3$**	**$KTiOPO_4$**	**LiB_3O_5**	**$LiNbO_3$**	**$LiIO_3$**
Type	I	I	II	II	I	I
d_{eff} (pm / V)	2.1	11.2	3.3	0.7	5.0	1.9
Angular bandwidth (mrad-cm)*	0.7	0.5	1.7	25.0	0.9	0.5
Spectral bandwidth ($cm^{-1}cm$)*	35	3.9	9.6	200	4.8	9.5
Damage threshold (J cm^{-2}) at 1064 nm	13	2	15	25	10	1

*Angular and spectral bandwidths correspond to the ranges over which phase mismatch varies from $-\pi$ to $+\pi$

Table-3 Nonlinear optical coefficients for primary materials[5]

Material	Formula	Relative nonlinear coefficient d/d_{36} (KDP)	Nonlinear coefficient[a] [10^{-24} As/V^2]	Index of refraction n^o (1.06μm)
Ammonium dihydrogen phosphate (ADP)	$NH_4H_2PO_4$	1.2	d_{36} =5.96	1.50
Potassium dihydrogen phosphate (KDP)	KH_2PO_4	1.0	d_{36} =5.16	1.49
Potassium dideuterium phosphate	KD_2PO_4	1.06	d_{36} =5.43	1.47
Rubidium dihydrogen phosphate (KD*P)	RbH_2PO_4	0.92	d_{36} =3.81	1.49
Rubidium dihydrogen arsenate (RDA)	RbH_2AsO_4	0.64	d_{36} =2.66	1.55
Cesium dihydrogen arsenate (CDA)	CsH_2AsO_4	0.92	d_{36} =3.81	1.55
Cesium dideuterium arsenate (CD*A)	CsD_2AsO_4	0.92	d_{36} =3.81	1.55
Lithium iodate	$LilO_3$	15.0	d_{31} =58.4	1.86
Lithium niobate	$LiNbO_3$	13.4	d_{31} =55.8	2.23
Barium sodium niobate	$Ba_2NaNb_5O_{15}$	38.0	d_{31} =159.4	2.26

[a]Absolute value of nonlinear coefficients is based on d_{36} (KDP) = 1.15×10^{-9} [esu].
Conversion: d [As/V^2] = 8.855 15 $\times 10^{-12} d$ [m/V] = $3.68 \times 10^{-15} d$ [esu]

VI. Conclusion

The OPO used here employed a KTP crystal as the non-linear element. The KTP crystal is antireflection coated at both the pump and OPO waves. KTP exhibits effective non-linear coefficient and high damage threshold (around 11 MW /cm^2), and has a relatively wide acceptance angle. The OPO consists of a 15 × 10 × 10 mm KTP crystal was placed within the 4.5 cm OPO resonator consisting plane mirrors M_3 and M_4. Mirror M_3 is highly reflective (100 %) for the signal wavelength, while mirror M_4 is 85 % reflective for the signal wavelength and highly reflective for pump and idler wavelengths. The KTP OPO was pumped by this 1.064 μm, diode-pumped Nd:YAG laser with a TEM_{00} transverse mode. The laser beam is set normal to the z-axis (*c*-axis), which indicates $\theta = 90^0$. The laser output was focused by a 100 cm focal lens. The KTP crystal was cut so as to achieve type-II non-critical phase matching *x*-cut ($\theta = 90^0$ and $\phi = 0^0$) as it maximizes effective non-linear coefficient and has large angular/thermal acceptance angles, for a pump wavelength of 1.064 μm (Nd:YAG). Type-II phase matching in KTP is chosen because of a combination of relatively high d_{eff} and high damage threshold. The effective nonlinear coefficient for the Type-II KTP crystal non-critical-phase-matching OPO is given by [22]

$$d_{eff} = (d_{24} - d_{15}) \sin 2\phi \sin 2\theta - (d_{15}\sin^2\phi + d_{24}\cos^2\phi)\sin\theta$$

For $\theta = 90^0$ and $\phi = 0^0$, the value of $d_{eff} = d_{24}$. Here θ is the angle of propagation with respect to the z-axis and φ is the angle to the *x*-axis in the x-y plane. An output coupler with a reflectivity of R = 85 % at the signal wavelength was used for this measurements. The input pump energy was fixed at 40 ± 0.6 mJ and the pump beam had a diameter of 2 mm ($1/e^2$ intensity point), a divergence of 0.8 mrad and pulse duration of 15.6 ns. The largest signal energy is obtained at the point of normal incidence, at phase matching angle of $\theta = 90^0$ and $\phi = 21^0$.For this configuration, the direction of propagation is along *x*-axis. The polarization of the pump wave

and signal wave is along y-axis (o-wave) whereas idler wave is polarized in the x-z plane (e-wave). This propagation angle is very close to the value calculated using the Sellmeier coefficient for parametric generation. While the beam polarized along the z-axis experience no walk-off, the calculated walk-off angle associated with the beams polarized along the fast axis are relatively small. It is difficult to find a signal set of Sellmeier coefficients that is accurate throughout the transparency range of a nonlinear crystal and valid for different $\chi^{(2)}$ processes. The output energy at 1525 nm is 8 mJ, corresponding to an energy conversion efficiency of 21 %. Figure-.4 shows the calculated signal wavelength as a function of phase-matching angle θ for $\phi = 0$ in type-II KTP OPO pumped at 1.064 μm. The polarization in the x-z plane (e-wave).The OPO tuning curve, shown in Figure-5.4, making the output wavelengths quite insensitive to rotation of the crystal. In practice the crystal could be rotated by 3 degrees with no measurable change in the output wavelength (angstrom resolution). In the non-critically phase-matched configuration the refractive indices are $n_s = 1.73$, $n_p = 1.73$, and $n_i = 1.82$. The round trip loss of the OPO crystal is $2\alpha l = 0.01$, included losses due to the optical coatings on the crystal faces. The mode-coupling coefficient g_s is approximately unity at threshold. In the present case of non-critical phase matching, the walk-off is negligible.

For pump wavelength of 1064 nm, it generates a signal wave at **1525** nm and idler beam at **2130** nm. The input and output faces of the KTP crystal were antireflection coated at 1064 nm and 1525 nm. For low divergence output beam, a plane parallel resonator configuration was employed for OPO cavity. The OPO conversion efficiency depends on the intensity of the pump beam inside the crystal. For focusing the pump laser beam (1064 nm) inside the crystal a convex lens of focal length 100 cm was employed. The focused pump beam spot size within the crystal was measured to be 2 ± 0.1 mm. The focusing lenses reduce the diameter of the pump beam that has a Gaussian-like spatial profile almost in a 1.6-mm-diameter beam ($1/e^2$ intensity point) with a divergence of 0.8 mrad. The OPO output principally is a signal beam at **1525** nm, it is also detected other harmonics for different orientations. This wavelength is beyond the ocular region, thus the output laser is safe for eye. The OPO output with centre wavelength of signal at **1525** nm was measured employing a sensor based energy meter. For measuring the pulse width of the pump source, a photodetector was placed behind the rear mirrors M_2 and M_4 to record the temporal profile of pump pulse. The signal and idler wavelengths can also be measured by using the grating monochromator set for approximately 0.2 nm resolution.

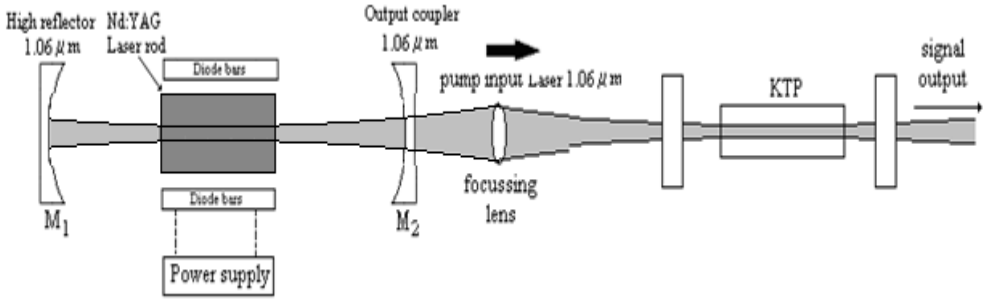

Figure-4 Schematic diagram for the Nd:YAG eye safe laser system

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